CHAPTER 10
THE MORTALITY TABLE

Example 1: *What is the probability that a person aged 30 will attain age 50?*

Solution: *Of the* l_{30} *persons alive at age 30,* l_{50} *will still be alive at age 50:*

$$\frac{l_{50}}{l_{30}} = \frac{166{,}682}{178084} = \mathbf{0.93597}$$

Example 2: *What is the probability that a person aged 20 will die within 1 year?*

Solution: *Of the* l_{20} *persons alive at age 20,* l_{21} *will still be alive at age 21. Therefore* l_{20} *-* l_{21} *persons will die within 1 year, so that the required probability is:*

$$\frac{l_{20} - l_{21}}{l_{20}} = \frac{181{,}322 - 180{,}977}{181{,}322} = 0.0019$$

Example 3: *What is the probability that a person aged 25 will die between ages 60 and 70?*

Solution: *Of the* l_{25} *persons alive at age 25,* l_{60} *will be alive at age 60 and* l_{70} *at age 70. Then, of the* l_{25} *persons,* l_{60} *-* l_{70} *will die between ages 60 and 70 so that the required probability is:*

$$\frac{l_{60} - l_{70}}{l_{25}} = \frac{150{,}281 - 116{,}630}{179{,}627} = 0.18734$$

Example 4*: what is the probability that a 20-year-old man will live to age 50?*

Of a group of 1000- 20-year-old men, what is the predicted number that will live to age 50?

Solution: *The probability that a 20-year-old man will live to age 50 is simply the number still living at age 50 divided by the number living at age 20.*

$$\frac{l_{50}}{l_{20}} = \frac{166{,}682}{181{,}322} = 0.9193$$

Out of 1000 20-year- old men, the predicted number that will live to age 50 is 1000 **×** *0.9193 = 919.*

Example 5: *What is the probability that a 20-year-old man will die before reaching age 50? Of a group of 1000 20-year-old men, what is the predicted number that will die before reaching age 50?*

Solution: *the probability of a man dying between age 20 and age 50 is the number dying during this period divided by the number living at age 20.*

$$\frac{l_{20}-l_{50}}{l_{20}}=\frac{181{,}322-166{,}682}{181{,}322}=0.0807$$

The probability of dying by age 50:

= 1 – 0.9193 = 0.0807.

Out of 1000- 20-year-old men, the predicted number that will die before reaching age 50 is 1000 x 0.0807 = 81.

Although the l_x column is all we need to compute the probabilities used in this book, it is convenient to have another column called the $\mathbf{d_x}$ column. The term d_x represents the number, of the l_x persons attaining precise age x, who die before reaching age $x + 1$. Thus, $\mathbf{d_x = l_x - l_{x+1}}$.

We could have used the $\mathbf{d_x}$ column in solving example 2 above. Thus, of the l_{20} persons alive at age 20, $\mathbf{d_{20}}$ will die within 1 year so that the required probability is:

$$\frac{d_{20}}{l_{20}}=\frac{345}{181322}=\mathbf{0.0019}$$

3-Notation: *The symbol (x) is used to represent a person aged x, or a life aged x.*

${}^{n}\mathbf{p}_x$ denotes the probability that (x) will live n years.

${}^{n}\mathbf{p}_x = \dfrac{l_{x+n}}{l_x}$. If $n = 1$, the left-hand subscript is omitted.

Thus, $\mathbf{p}_x$ denotes the probability that (x) will live 1 year.

$$\mathbf{p}_x = \frac{l_{x+1}}{l_x}.$$

${}^{n}\mathbf{p}_x$ denotes the probability that (x) will die within n years.

$$_{n}q_x=\frac{l_x-l_{x+n}}{l_x}$$

If n = 1, the left-hand subscript is omitted.

Thus q_x denoted the probability that (x) will die within 1 year.

$$q_x = \frac{l_x - l_{x+1}}{l_x} = \frac{d_x}{l_x}$$

${}_{m|n}q_x$ denotes the probability that (x) will live m years but die in the next n years; that is, that (x) will die between ages: $x + m$ and $x + m + n$.

$${}_{m|n}q_x = \frac{l_{x+m} - l_{x+m+n}}{l_x}$$

If n = 1, it is omitted form the notation so that ${}_{m|n}q_x$ denotes the probability that (x) dies between ages $x + m$ and $x + m + 1$.

$${}_{m|n}q_x = \frac{l_{x+m} - l_{x+m+n}}{l_x} = \frac{d_{x+m}}{l_x}$$

***The following general observations should be made**:*

1- The right-hand subscript represents the present age of the person in question.

2- The left-hand subscript represents the period of years during which the event (living or dying) is to take place.

3- The letter or number before the bar represents the period of deferment.

*4- The letter **p** is used to denote the probability of an individual's living a given period.*

*5- The letter **q** is used to denote the probability of an individual's dying during given period. The probability of dying within a year for a person age x is indicated by the symbol **q_x**. This probability is obtained by dividing the number dying at age **x** by the number living at age **x**. In symbols, **$q_x = \frac{d_x}{l_x}$**.*

<u>Numerical Illustrations</u>:

${}_{10}p_{20}$ represents the probability that a life aged 20 will live 10 years, i.e., will live to be age 30.

p_{20} represents the probability that (20) will live to be 21.

${}_{5}q_{30}$ represents the probability that (30) will die before reaching age 35.

q_{30} represents the probability that (30) will die before reaching age 31.

$_{10|5}q_{20}$ *represents the probability that (20) will die between ages 30 and 35; that is, that (20) will live 10 years but die during the next 5.*

$_{10|}q_{20}$ *represents the probability that (20) will die between ages 30 and 31.*

4-Expectation of Life: *The average number of complete years to be lived in the future by persons now aged* x *is called the* ***curate expectation of life*** *of (*x*). The words "complete years" imply that, in computing the average, fractions of a year lived will be disregarded. This is the same as assuming that all deaths occur in the instant after a birthday.*

Consider l_x *persons who are alive at age* x*.* l_{x+1} *of these will be alive at age* x*+1, so that a total of* l_{x+1} *years will be lived during the first year following the reference age,* x*. similarly, a total of* l_{x+2} *years will be lived during the second year,* l_{x+3} *during the third year, ad so forth. The total number of years lived by these* l_x *persons will be* $l_{x+1} + l_{x+2} + l_{x+3} + \ldots + l_{\omega-1}$*, so that the average number of full years, or the curate expectation of life, is:*

$$e_x = \frac{l_{x+1} + l_{x+2} + l_{x+3} \ldots\ldots\ldots\ldots + l_{\omega-1}}{l_x}$$

Where: e_x *= expectation of life of a person aged* x*.*

l_x *= number living at age* x*.*

ω *= terminal age of the mortality table.*

The symbol Σ *is the capital Greek letter sigma.*

This standard mathematical symbol is sued to indicate summation of similar terms.

This question can be looked at in another way, by using the concept of mathematical expectation. If we think of (x*) as receiving a reward of 1 for every complete year that he lives, his mathematical expectation for the whole of life would be the probability that he lives 1 year times his reward of 1 plus the probability that he lives 2 years times his reward of 1, and so forth. In symbols,*

$$e_x = p_x + {}_2p_x + {}_3p_x . + \ldots\ldots\ldots\ldots + {}_{\omega-x-1}p_x$$

The two expressions for e_x *are exactly equal.*

The average number of years, including fractions, to be lived in the future by persons now aged x *is called the* ***complete expectation of life*** *of (*x*) and is denoted by* $\mathring{e}_x$*. If we assume that deaths are evenly distributed throughout the year, we can say that on the average a person will be halfway form one birthday to the next at death. Therefore:*

$$\mathring{e}_x = e_x + \frac{1}{2}$$

The concept of expectation of life is useful in two major respects. First, it is useful in comparing in very general terms the level of mortality under various mortality tables. Second, it can be used by experienced actuaries in making a rough analysis of certain types of problems. Contrary to rather popular opinion, this concept is not used in precise actuarial calculations relating to premiums.

The index of summation t ranges over the integers 1, 2, 3… ω **– χ –** *1. Thus if we want the expectation of life of a person aged χ, we start with the number living 1 year later, add the number living 2 years later, and continue in this way to the end of the mortality table. This sum gives the full years of future lifetime for the group of persons age* ***χ****.*

When this sum is divided by l_χ, the number living at age χ, we get the full years of future life for each person on the average. This summation makes no allowance for fractional years of life. We assume that complete future lifetimes will exceed by half a year the number of complete years, so we add 0.5 year to get the complete future lifetime.

To show how this formula is used, we compute the expectation of life for men aged 90 using the 1980 CSO Table. Calculations for younger people are done in exactly the same way but are more tedious because of the larger number of terms in the summation. We substitute 90 for χ in the formula and find from the mortality table that l_{90} = 468,174. This is the number of men over whom we will average the full years of future lifetime. For t = 1, l_{91} = 361,365; for t = 2, l_{92} = 272,552. We continue in this way to the last term in the summation, which is for t = (100 – 90 – 1) = 9. For t = 9, l_{99} = 6415. Arranging the complete problem in tabular form, we have:

Example 5: *Show that:*

$$e_x = p_x + {}_2p_x + {}_3p_x + \cdots + {}_{\omega - x}p_x$$

Solution: $p_x + {}_2p_x + {}_3p_x + \cdots + {}_{\omega\ x}p_x$

$$= \frac{l_{x+1}}{l_x} + \frac{l_{x+2}}{l_x} + \frac{l_{x+3}}{l_x} + \cdots + \frac{l_\omega}{l_x}$$

$$= \frac{l_{x+1} + l_{x+2} + l_{x+3} + \cdots + l_\omega}{l_x} = e_x$$

Example 6: *Show that:*

$$l_x - l_{x+2} = d_x + d_{x+1} + d_{x+2} + \cdots + d_{x+n-1}$$

Solution:

$$d_x + d_{x+1} + d_{x+2} + \cdots + d_{x+n-1}$$
$$= (l_x - l_{x+1}) + (l_{x+1} - l_{x+2}) + (l_{x+2} - l_{x+3}) + \cdots$$
$$+ (l_{x+n-2} - l_{x+n-1}) + (l_{x+n-1} - l_{x+n})$$
$$= l_x - l_{x+1} + l_{x+1} - l_{x+2} + l_{x+2} - l_{x+3} + \cdots$$
$$+ l_{x+n-2} - l_{x+n-1} + l_{x+n-1} - l_{x+n}$$
$$= l_x - l_{x+n}$$

Example 6: *Show that:* $_{m+n}p_x = {}_mp_x - {}_{m|n}q_x$

Solution:

$${}_mp_x \quad {}_{m|n}q_x = \frac{l_{x+m}}{l_x} \quad \frac{l_{x+m} \quad l_{x+m+n}}{l_x}$$
$$= \frac{l_{x+m+n}}{l_x} = {}_{m+n}p_x$$

22-Complete the following table:

x	l_x	d_x	q_x	p_x	e_x	$\mathring{e}_x$
95	1000					
96	700					
97	400					
98	100					
99	10					
100	0					

23-Complete the following table:

x	l_x	d_x	q_x	p_x
21	1600			
22	1200		0.3	
23	400			0.65
24				
25	344		0.375	

23-If: $l_{36} = 100{,}000$ $q_{36} = 0.02$ $p_{37} = 0.95$

$l_{39} = 83{,}790$ $d_{39} = 16{,}758$ $p_{40} = 0.75$

Construct the mortality table.