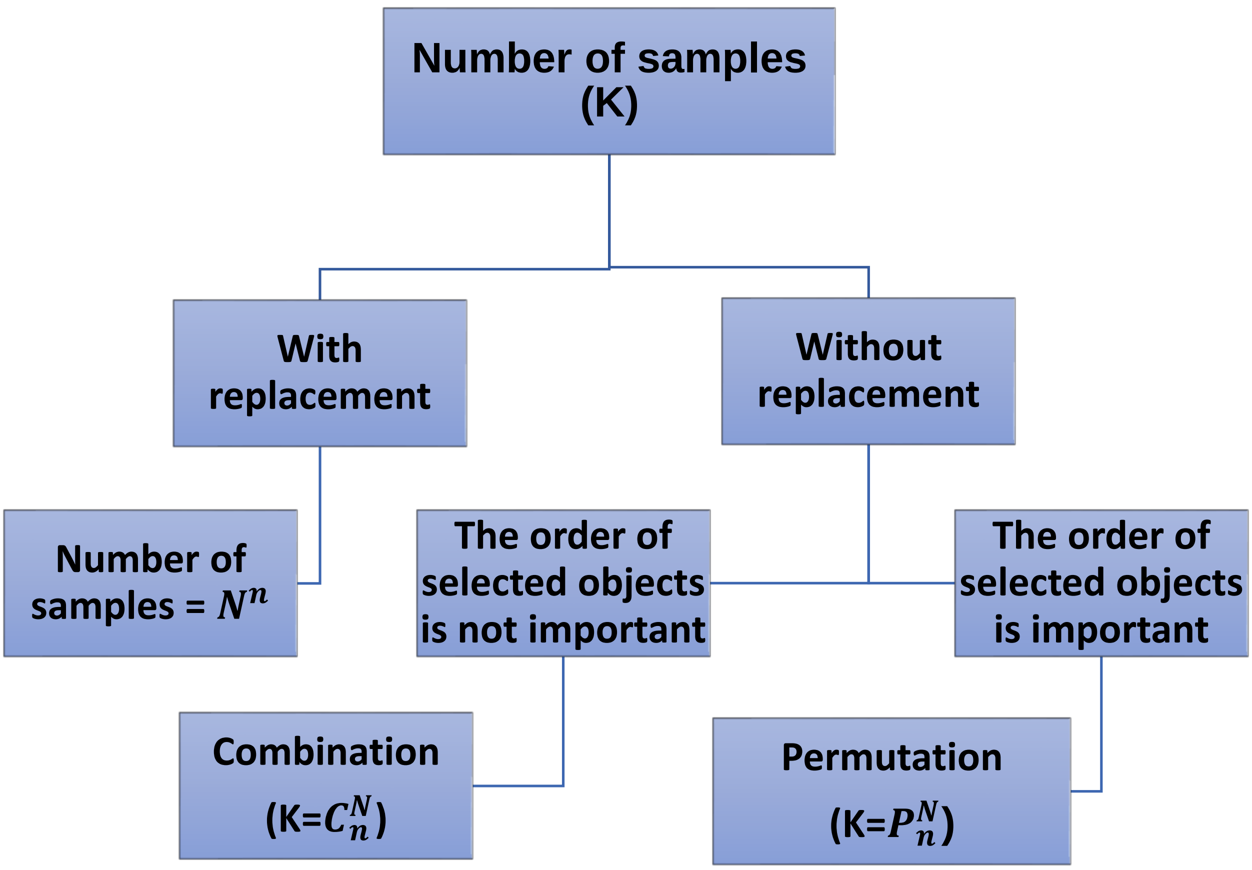
# Chapter (7)
# Sampling Distributions
# Examples

**How to draw sample from population**



**Example (1): textbook (slide 4-9)**

The following data represent age of individuals in a population;

**N=4 .** The ages are **18, 20, 22, 24 years.**

1) Find the population mean.
2) If the order is important, how many different samples of 2 are possible without replacement? Find the mean of all the sample means.
3) How many different samples of 2 are possible without replacement? Find the mean of all the sample means.
4) Are the means equal?
5) What the property called?

**Solution:**

1) The population mean= $\mu = \frac{\sum X}{N} = \frac{18+20+22+24}{4} = \frac{84}{4} = 21$

2) The number of samples selected without replacement when the order is important is:

$$K = P_n^N = \frac{N!}{(N-n)!} = \frac{4!}{(4-2)!} = 12$$

| $1^{st}$ obs | $2^{nd}$ observation | | | |
|---|---|---|---|---|
| | 18 | 20 | 22 | 24 |
| 18 | - | **(18, 20)** | **(18, 22)** | **(18, 24)** |
| 20 | **(20, 18)** | - | **(20, 22)** | **(20, 24)** |
| 22 | **(22, 18)** | **(22, 20)** | - | **(22, 24)** |
| 24 | **(24, 18)** | **(24, 20)** | **(24, 22)** | - |

| **Sample number(i)** | **Samples** | $\overline{X}$ | $\overline{X}_i - \mu$ | $(\overline{X}_i - \mu)^2$ |
|---|---|---|---|---|
| 1 | 18,20 | 19 | -2 | 4 |
| 2 | 18,22 | 20 | -1 | 1 |
| 3 | 18,24 | 21 | 0 | 0 |
| 4 | 20,18 | 19 | -2 | 4 |
| 5 | 20,22 | 21 | 0 | 0 |
| 6 | 20,24 | 22 | 1 | 1 |
| 7 | 22,18 | 20 | -1 | 1 |
| 8 | 22,20 | 21 | 0 | 0 |
| 9 | 22 ,24 | 23 | 2 | 4 |
| 10 | 24,18 | 21 | 0 | 0 |
| 11 | 24, 20 | 22 | 1 | 1 |
| 12 | 24,22 | 23 | 2 | 4 |
| | | $\sum \overline{X} = 252$ | 0 | 20 |

The samples mean is: $\mu_{\bar{x}} = \frac{\sum_{i=1}^{N} X_i}{k} = \frac{252}{12} = 21$

3) The number of samples selected without replacement when the order is not important is:

$$K = C_n^N = C_2^4 = \frac{4*3}{2} = 6$$

| 1st obs | 2nd observation | | | |
|---|---|---|---|---|
| | 18 | 20 | 22 | 24 |
| 18 | **-** | **(18, 20)** | **(18, 22)** | **(18, 24)** |
| 20 | **-** | **-** | **(20, 22)** | **(20, 24)** |
| 22 | **-** | **-** | **-** | **(22, 24)** |
| 24 | **-** | **-** | **-** | **-** |

| Number of sample | samples | $\bar{X}$ |
|---|---|---|
| 1 | (18,20) | 19 |
| 2 | (18,22) | 20 |
| 3 | (18,240 | 21 |
| 4 | (20,22) | 21 |
| 5 | (20,24) | 22 |
| 6 | (22,24) | 23 |
| | | $\sum \bar{X} = 126$ |

The samples mean is: $\mu_{\bar{x}} = \frac{\sum \bar{X}}{K} = \frac{126}{6} = 21$

4) Yes, the means are equal. $\mu = \mu_{\bar{x}} = 21$

5) This property is called the unbiased property of the sample mean.

## Example (2):

Random samples of size 3 were selected (with replacement) from populations' size 6 with the mean 10 and variance 9. Find the number of all possible samples, the mean and standard deviation of the sampling distribution of the sample mean.

**Solution:**

**a.** $N = 6 \quad n = 3 \quad \mu = 10 \quad \sigma^2 = 9 \ , \sigma = 3 \ (with\ replacement)$

The number of samples $= k = N^n = 6^3 = 216$

$\mu_{\bar{x}} = \mu = 10$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{3}} = 1.73$

**Example (3)**

Given a normal distributed with μ = 30, σ = 12, and n = 25. What is the probability that $\overline{X}$ is:

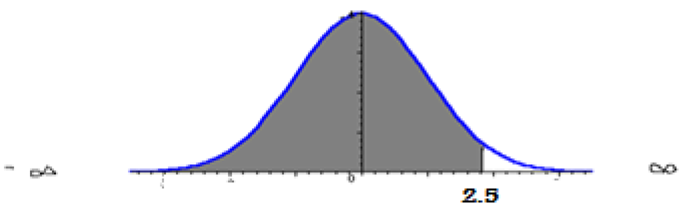
1. Less than 36?
2. Between 27 and 34?
3. Less than 27?
4. The probability is 95% that the sample mean will be between what two values symmetrically distributed around the population means?
5. Probability that $\overline{X}$ is?
6. Greater than 27?
7. Between 23 and 27?
8. There is a 72% chance that $\overline{X}$ is above what value?
9. There is a 72% chance that $\overline{X}$ is below what value?
10. Greater than 34?
11. Between 33 and 34?
12. There is a 43% chance that $\overline{X}$ is above what value?
13. There is a 43% chance that $\overline{X}$ is below what value?
14. Between 30 and 34?
15. Between 27 and 30?

**Solution:**

1) Less than 36?

$$P(\overline{X} < 36) = P\left(Z < \frac{36-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(Z < \frac{6}{2.4}\right) = p(Z < 2.5) = P(2.5) = 0.9938$$

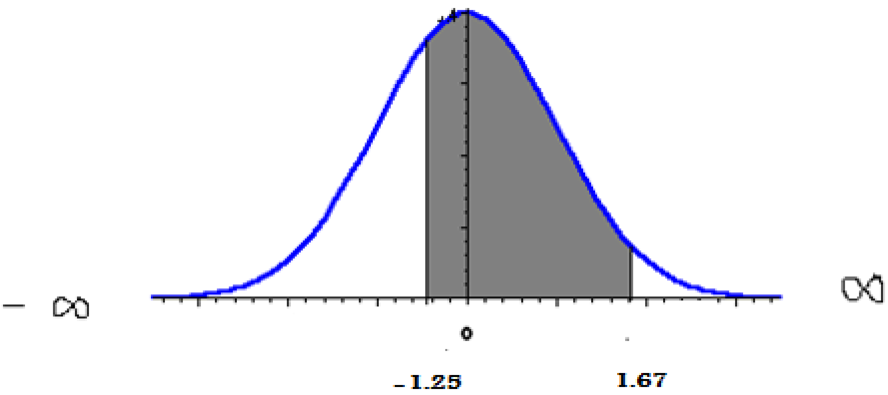


2) Between 27 and 34?

$$P(27 < \bar{X} < 34) = P\left(\frac{27-30}{\frac{12}{\sqrt{25}}} < Z < \frac{34-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(\frac{-3}{2.4} < Z < \frac{4}{2.4}\right) = p(-1.25 < Z < 1.67) = P(1.67) - P(-1.25)$$

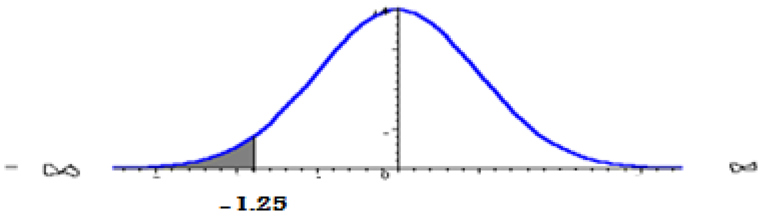
$$= 0.9525 - 0.1056 = 0.8469$$




3) Less than 27?

$$P(\bar{X} < 27) = P\left(Z < \frac{27-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(Z < \frac{-3}{2.4}\right) = p(Z < -1.25) = P(-1.25) = 0.1056$$




4) The probability is 95% that the sample mean will be between what two values symmetrically distributed around the population means.

Since the interval contains 95% of the sample means (1-0.95 = 0.05)
0.05 of the sample means will be outside the interval
Since the interval is symmetric, 0.05 will be above the upper limit and
0.025 will be below the lower limit.

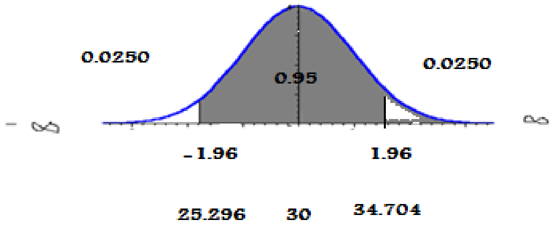
From the standardized normal table, the Z score with 2.5% (0.0250) below it is -1.96 and the Z score with 2.5% (0.95+0.0250=0.9750) above it is 1.96.

Calculating the upper limit of the interval

$$\bar{X}_U = \mu + Z\frac{\sigma}{\sqrt{n}} = 30 + (1.96)\frac{12}{\sqrt{25}} = 30 + 1.96(2.4) = 30 + 4.704 = 34.704$$

Calculating the lower limit of the interval

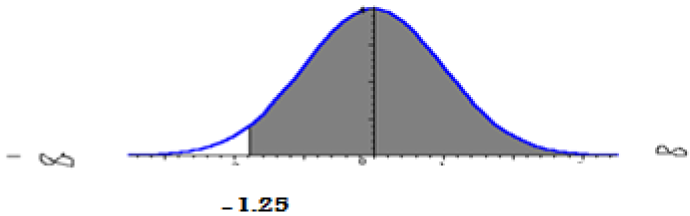
$$\bar{X}_L = \mu + Z\frac{\sigma}{\sqrt{n}} = 30 + (-1.96)\frac{12}{\sqrt{25}} = 30 - 1.96(2.4) = 30 - 4.704 = 25.296$$




5) Greater than 27?

$$P(\bar{X} > 27) = P\left(Z > \frac{27-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(Z > \frac{-3}{2.4}\right) = p(Z > -1.25) = 1 - P(-1.25) = 1 - 0.1056 = 0.8944$$

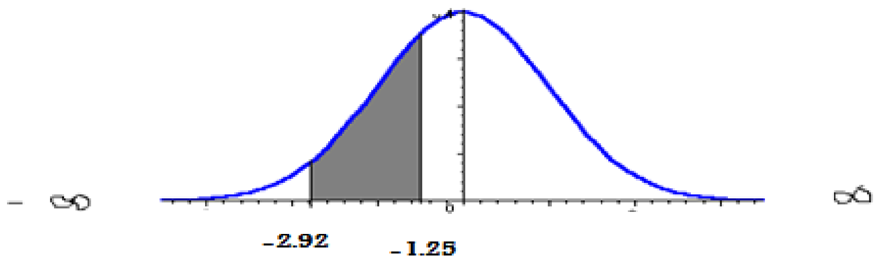


6) Between 23 and 27?

$$P(23 < \overline{X} < 27) = P\left(\frac{23-30}{\frac{12}{\sqrt{25}}} < Z < \frac{27-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(\frac{-7}{2.4} < Z < \frac{-3}{2.4}\right) = p(-2.92 < Z < -1.25) = P(-1.25) - P(-2.92)$$

$$= 0.1056 - 0.0018 = 0.1038$$



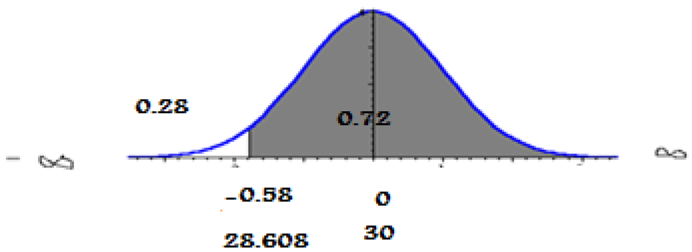

7) There is a 72% chance that $\overline{X}$ is above what value?

1- 0.7200 = 0.2800

Z = -0.58

$$\overline{X} = \mu + Z\sigma_{\overline{X}} = \mu + Z\frac{\sigma}{\sqrt{n}}$$

$$= 30 + (-0.58)\left(\frac{12}{\sqrt{25}}\right) = 30 - 0.58(2.4) = 30 - 1.392 = 28.608$$



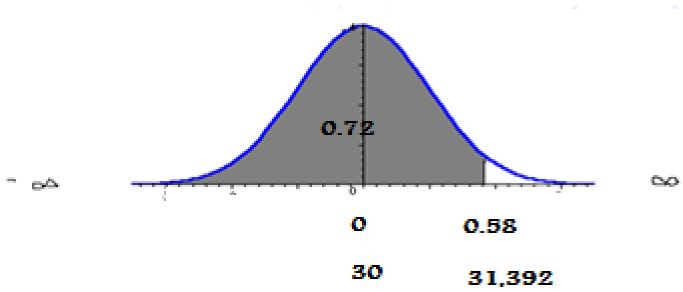

8) There is a 72% chance that $\overline{X}$ is below what value?

0.7200

Z = 0.58

$$\overline{X} = \mu + Z\sigma_{\overline{X}} = \mu + Z\frac{\sigma}{\sqrt{n}}$$

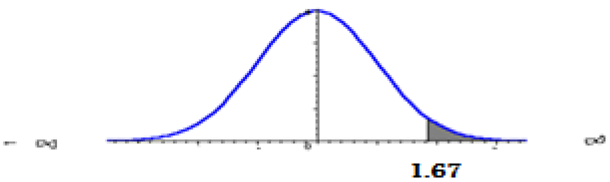
$$= 30 + (0.58)\left(\frac{12}{\sqrt{25}}\right) = 30 + 0.58(2.4) = 30 + 1.392 = 31.392$$




9) Greater than 34?

$$P(\overline{X} > 34) = P\left(Z > \frac{34-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(Z > \frac{4}{2.4}\right) = p(Z > 1.67) = 1 - P(1.67) = 1 - 0.9525 = 0.0475$$

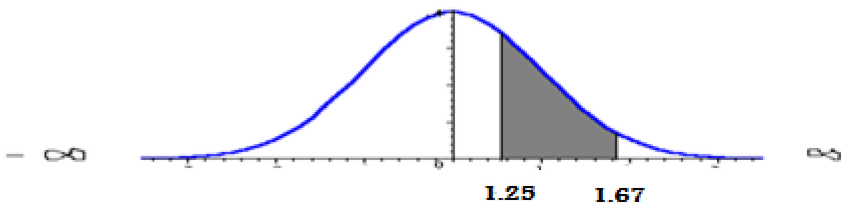



10) Between 33 and 34?

$$P(33 < \overline{X} < 34) = P\left(\frac{33-30}{\frac{12}{\sqrt{25}}} < Z < \frac{34-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(\frac{3}{2.4} < Z < \frac{4}{2.4}\right) = P(1.25 < Z < 1.67) = P(1.67) - P(1.25)$$

$$= 0.9525 - 0.8944 = 0.0581$$

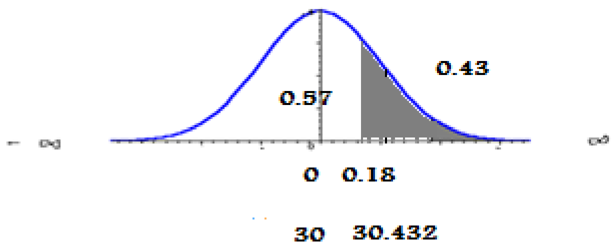



11) There is a 43% chance that $\overline{X}$ is above what value?

1-0.4300=0.5700
Z = 0.18

$$\overline{X} = \mu + Z\sigma_{\overline{X}} = \mu + Z\frac{\sigma}{\sqrt{n}}$$

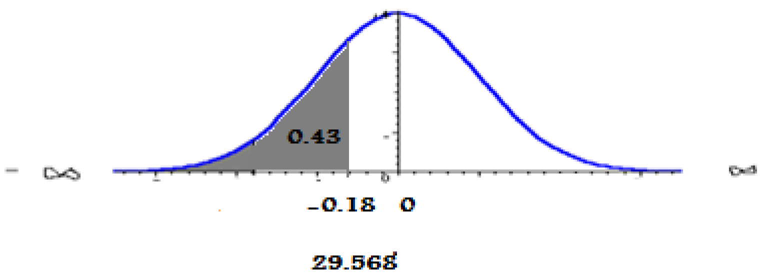
$$= 30 + (0.18)\left(\frac{12}{\sqrt{25}}\right) = 30 + 0.18(2.4) = 30 + 0.432 = 30.432$$




12) There is a 43% chance that $\overline{X}$ is below what value?
0.43
Z =- 0.18

$$\overline{X} = \mu + Z\sigma_{\overline{X}} = \mu + Z\frac{\sigma}{\sqrt{n}}$$

$$= 30 + (-0.18)\left(\frac{12}{\sqrt{25}}\right) = 30 - 0.18(2.4) = 30 - 0.432 = 29.568$$

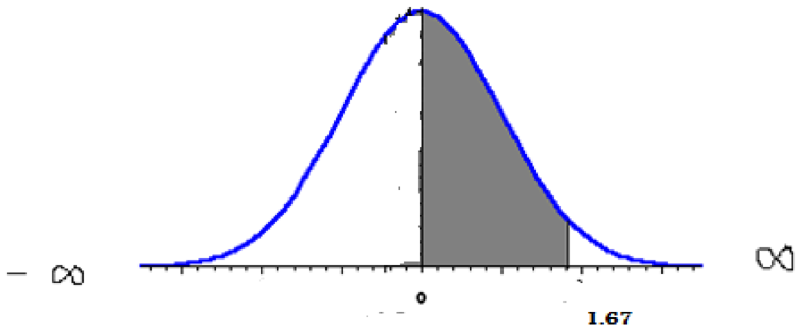


13) Between 30 and 34?

$$P(30 < \overline{X} < 34) = P\left(\frac{30-30}{\frac{12}{\sqrt{25}}} < Z < \frac{34-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(\frac{0}{2.4} < Z < \frac{4}{2.4}\right) = p(0 < Z < 1.67) = P(1.67) - 0.5000$$

$$= 0.9525 - 0.5000 = 0.4525$$

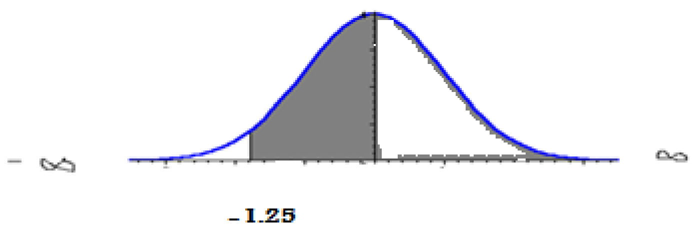



14) Between 27 and 30?

$$P(27 < \overline{X} < 30) = P\left(\frac{27-30}{\frac{12}{\sqrt{25}}} < Z < \frac{30-30}{\frac{12}{\sqrt{25}}}\right)$$

$$= P\left(\frac{-3}{2.4} < Z < \frac{0}{2.4}\right) = P(-1.25 < Z < 0) = 0.5000 - P(-1.25)$$

$$= 0.5000 - 0.1056 = 0.3944$$



## Example (4)

**Suppose that** $n = 100,\ \pi = 0.3$**. Using the normal approximation for the binomial probabilities find:**

1. The sample distribution of the proportion
2. The standard error of the proportion
3. $P(P \geq 0.25)$
4. *P(0.25 ≤ p ≤ 0.30)*
5. $P(P \leq 0.25)$

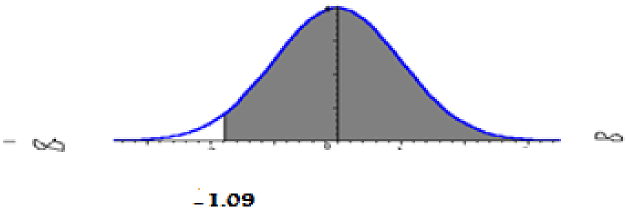
**Solution:**

1) The mean sample distribution of the proportion = $\mu_p = p = \frac{x}{n} = \pi = 0.3$

2) The standard error of the proportion = $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.3(1-0.3)}{100}} = 0.0458$

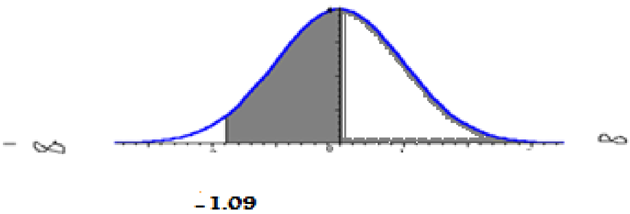
3) $P(P \geq 0.25) = P\left(Z \geq \frac{0.25-0.3}{0.0458}\right) = P(Z \geq -1.09)$

$= 1 - P(-1.09) = 1 - 0.1379 = 0.8621$




4) $P(0.25 \leq P \leq 0.30) = P\left(\frac{0.25-0.30}{0.0458} \leq Z \leq \frac{0.30-0.3}{0.0458}\right) = P(-1.09 \leq Z \leq 0)$

$= 0.5 - p(-1.09) = 0.5 - 0.1379 = 0.3621$




5) $P(P \leq 0.25) = P\left(Z \leq \frac{0.25-0.3}{0.0458}\right) = P(Z \leq -1.09)$

$= 0.1379$