Chapter (5)
A Survey of Probability Concepts
Examples

Example (1):

A box contain five red balls, a ball is drawn at random, what is the possibility that the ball will be red?

$n(R) = 5 \qquad n(\Omega) = 5$

$P(R) = \frac{n(R)}{n(\Omega)} = \frac{5}{5} = 1$ (Certain event)

Example (2):

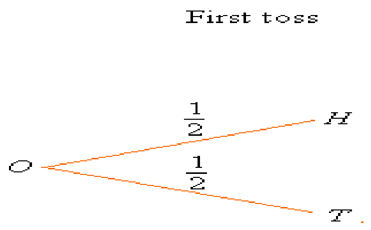
A box contain five red balls, a ball is drawn at random, what is the possibility that the ball will be blue?

$n(B) = 0 \qquad n(\Omega) = 5$

$P(B) = \frac{n(B)}{n(\Omega)} = \frac{0}{5} = 0$ (Impossible event)

Example (3):

An experiment is consisting of tossing (flip) a fair coin once, what is the probability of getting a head?



$\Omega = \{H, T\}$

$n(\Omega) = 2 \qquad n(H) = 1$

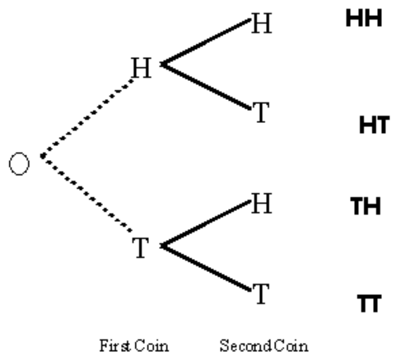
$P(H) = \frac{n(H)}{n(\Omega)} = \frac{1}{2} = 0.50$

Example (4):

If an experiment is consisting of tossing a fair coin twice, find:

1. The Set of all possible outcomes of the experiment.
2. The probability of the event of getting at least one head.
3. The probability of the event of getting exactly one head in the two tosses.
4. The probability of the event of getting two heads.

Solution:



1.

Ω= {HH, HT, TH, TT}

Where,

n(Ω) =2^2 =4

And since the coin is fair, then all of the elementary events are equally likely, i.e.

P(HH) = P(HT) = P(TH) = P(TT) = 0.25

2.

Let

E_1 = {HH, HT, TH} be the event of getting at least one head, then

n(E_1) =3

And hence $P(E_1) = \frac{n(E_1)}{n(\Omega)} = \frac{3}{4} = 0.75$

3.

E_2 = {HT, TH} be the event of getting exactly one head, then

n (E_2) =2 And hence

$$P(E_2) = \frac{n(E_2)}{n(\Omega)} = \frac{2}{4} = 0.5$$

4. Let

E_3 = {HH} be the event of getting two heads, then n (E_3) =1

$$P(E_3) = \frac{n(E_3)}{n(\Omega)} = \frac{1}{4} = 0.25$$

Example (5):

If the experiment is consisting of rolling a fair die once, find:

1. Set of all possible outcomes of the experiment.
2. The probability of the event of getting an even number.
3. The probability of the event of getting an odd number.
4. The probability of the event of getting a four or five.
5. The probability of the event of getting a number less than 5.

Solution:

1.

Ω={1,2,3,4,5,6 }n(Ω)=6

Since the coin is fair, then all events are equally likely, i.e.

$$P(1) = P(2) = \cdots = P(6) = \frac{1}{6}$$

2.Let,

E_1= {2, 4, 6} be the event of getting an even number, then

n (E_1) = 3

$$P(E_1) = \frac{n(E_1)}{n(\Omega)} = \frac{3}{6} = 0.50$$

3.

E_2= {1, 3, 5} be the event of getting an odd number, then

n(E_2) =3

$$P(E_2) = \frac{n(E_2)}{n(\Omega)} = \frac{3}{6} = 0.50$$

4.Let,

E_3= {4, 5} be the event of getting a four or five, then

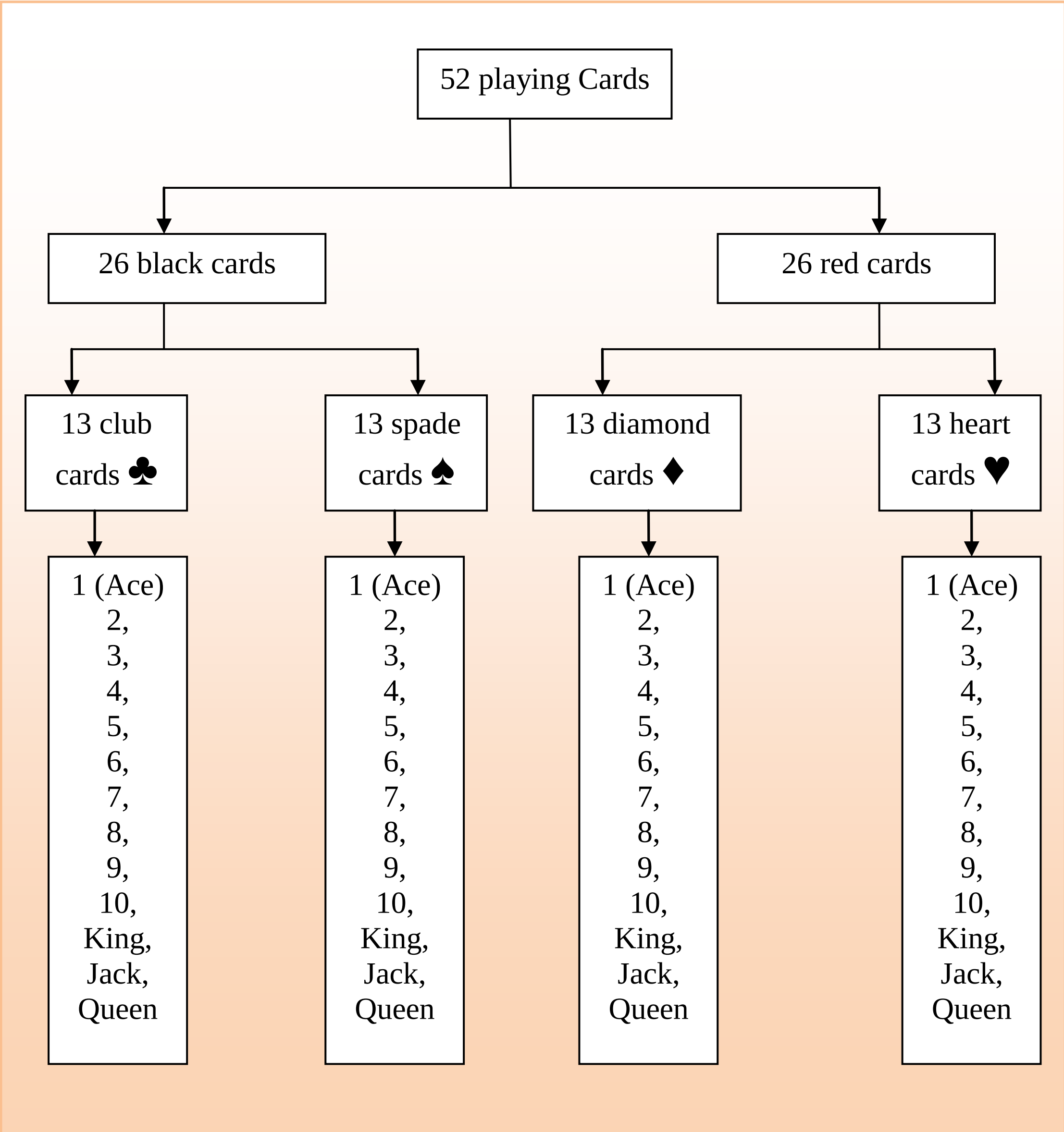
n (E_3) =2

$$P(E_3) = \frac{n(E_3)}{n(\Omega)} = \frac{2}{6} = 0.33$$

5.Let,

E_4= {1, 2, 3, 4} be the event of getting a number less than 5, then

$$P(E_4)=\frac{n(E_4)}{n(\Omega)}=\frac{4}{6}=0.67$$



Example (6):

When one card is drawn from a well-shuffled deck of 52 playing cards, what are the probabilities of getting?

1. A black card**.**

$$P(Black\ card)=\frac{26}{52}=0.5$$

2. Number 2.

$$P(Number\ 2)=\frac{4}{52}=0.08$$

3. A black king $=\frac{2}{52}=0.038$.

4. Number 3, 4, 5.

$$P(Number\,3,4,5)=\frac{12}{52}=0.23$$

5. A heart card.

$$P(A\,heard\;card)=\frac{13}{52}=0.25$$

6. A jack or queen or king or ace.

$$P(A\cup B\cup C\cup D)=P(A)+P(B)+P(C)+P(D)$$

$$=\frac{4}{52}+\frac{4}{52}+\frac{4}{52}+\frac{4}{52}=\frac{16}{52}$$

7. A card number 7 spade.

$$P(A)=\frac{1}{52}$$

Example (7):

Suppose that P (A) = 0.4 and P (B) = 0.2 . If events A and B are mutually exclusive:

- What is the probability of either A or B occurring.
- What is the probability of neither A nor B will happen.

Solution:

$$P(A\cup B)=P(A)+P(B)=0.4+0.2=0.6$$

$$P(\sim(A\cup B))=1-P(A\cup B)=1-.6=0.4$$

Example (8):

The probabilities of the events A and B are 0.20 and 0.25, respectively. The probability that both A and B occur is 0.10.What is the probability of either A or B occurring.

Solution:

$$P(A\,or\,B)=P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=0.20+0.25-0.10=0.45-0.10=0.35$$

Example (9):

Suppose P (A) =0.3 and P (B) =0.15 .What is the probability of A and B**?**

Solution:

$$P(A\,and\,B)=P(A\cap B)=P(A)\times P(B)=0.30\times 0.15=0.045$$

Example (10): Suppose P (A) =0.45 &P (B\A) =0.12.What is the probability of A and B**?**

Solution:

$$P(A\, and\, B) = P(A \cap B) = P(A)P(B \setminus A) = 0.45 \times 0.12 = 0.054$$

Example (11):

Suppose that P (A) =0.7and P (A ∩ B) =0.21, find:

1. The value of P (B\A)
2. If P (B) = 0.3 are events A and B independent?

Solution:

1. $P(B \setminus A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.21}{0.7} = 0.30$

2. $\because P(B|A) = 0.30 \;\; and P(B) = 0.3$

$\therefore A \text{ and } B \text{ independent}$

Example (12):

The events A and B are mutually exclusive. If P (A) =0.2 P (B) = 0.5 Find the probability of:

1. Either A or B2. Neither A nor B

Solution:

1. $\because P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B) = 0.2 + 0.5 = 0.7$

2. $P(\sim (A \cup B)) = 1 - P(A \cup B) = 1 - 0.70 = 0.30$

Example (13):

If A and B two events .Let $P(A) = 0.2$, $P(B) = 0.5$, $P(A \cap B) = 0.1$. Find:

1. Either A or B2.Neither A nor B

Solution:

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$

2. $P(\sim(A\cup B)) = 1 - P(A\cup B) = 1 - 0.6 = 0.5$

Example (14)

The events A and B are independent .Let **P (A) =0.2   P (B) = 0.5**

Find:

1. **P (A and B)**
2. **P (~ (** $A\cap B)$**)**
3. **Either A or B**
4. **Neither A nor B**

Solution:

1. $P(A\cap B) = P(A)P(B) = (0.2)(0.5) = 0.1$

2. $P(\sim(A\cap B)) = 1 - P(A\cap B) = 1 - 0.1 = 0.9$

3. $P(A\cup B) = P(A) + P(B) - P(A\cap B) = (0.2) + (0.5) - 0.1$
$= 0.7 - 0.1 = 0.6$

4. $P(\sim(A\cup B)) = 1 - P(A\cup B) = 1 - 0.6 = 0.4$

Example (15)

The events A and B are dependent. Let **P (A) =0.2 P (B\A) =0.4. Find:**

1. P (A and B)   2. P (~ ($A\cap B)$**)**

1. $P(A\cap B) = P(A)P(B\setminus A) = (0.2)(0.4) = 0.0.8$

2. $P(\sim(A\cap B)) = 1 - P(A\cap B) = 1 - 0.08 = 0.92$

(Special rule of multiplication)

Example (16)

A box contains eight red balls and five white balls, two balls are drawn at random, find:

1. The probability of getting both the balls white, when the first ball drawn is replace.
2. The probability of getting both the balls red, when the first ball drawn is replace
3. The probability of getting one of the balls red, when the first drawn ball is replaced back.

Solution:

Let W_1 be the event that the in the first draw is white and W_2. In a similar way define R_1 and R_2. Since the result of the first draw has no effects on the result of the second draw, it follows that W_1 and W_2 are independent and similarly R_1 and R_2 are independent.

1.

$$P(W_1 \cap W_2) = P(W_1)\,P(W_2) = \left(\frac{5}{13}\right)\left(\frac{5}{13}\right) = \frac{25}{169}$$

2.

$$P(R_1 \cap R_2) = P(R_1)\,P(R_2) = \left(\frac{8}{13}\right)\left(\frac{8}{13}\right) = \frac{64}{169}$$

3. Since the first drawn ball is replaced back, then the result of the first draw has no effect on the result of the second draw. Let E be the event that one of the ball is red, then:

$$P(E) = P(R_1)\,P(W_2) + P(W_1)\,P(R_2) = \left(\frac{8}{13}\right)\left(\frac{5}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{8}{13}\right) = \frac{80}{169}$$

Example (17):

Two cards are drawn with replacement from a well-shuffled deck of 52 playing cards. What is the probability of getting king in the first card and ace in the second?

Solution:

Since the first drawn card is replaced back, then the result of the first draw has no effect on the result of the second draw.

$$P(E_1 and\ E_2) = P(E_1 \cap E_2) = P(E_1)\,P(E_2) = \left(\tfrac{4}{52}\right)\left(\tfrac{4}{52}\right) = \tfrac{16}{2704}$$

General rule of multiplication

Example (18)

A box contains seven blue balls and five red balls, two balls are drawn at random without replacement, find:

1. The probability that both balls are blue.
2. The probability that both balls are red.
3. The probability that one of the balls is blue.
4. The probability that at least one of the balls is blue.
5. The probability that at most one of the balls is blue.

Solution:

Let B_1 denote the event that the ball in the first draw is blue and B_2 denote the event that the ball in the second draw is blue. In a similar way define R_1 and R_2.

1.

$$P(B_1 \ and \ B_2) = P(B_1 \cap B_2) = P(B_1)\,P(B_2|B_1)$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{6}{11}\right) = \tfrac{42}{132} = 0.32$$

2.

$$P(R_1 \ and \ R_2) = P(R_1 \cap R_2) = P(R_1)\,P(R_2|R_1)$$

$$= \left(\tfrac{5}{12}\right)\left(\tfrac{4}{11}\right) = \tfrac{20}{132} = 0.15$$

3.

$$P(one\,ball\,is\,blue) =$$

$$P((B_1 and\ R_2)\ or\ (R_1\ and\ B_2)) = P(B_1)\,P(R_2|B_1) + P(R_1)P(B_2|R_1)$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{5}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{7}{11}\right)$$

$$= \tfrac{35}{132} + \tfrac{35}{132} = \tfrac{70}{132} = 0.53$$

4. That at least one of the balls is blue

$$P(At\,least\,one\,ball\,is\,blue) = P((B_1\ and\ B_2)\ or\ (B_1 and\ R_2)\ or\ (R_1\ and\ B_2))$$

$$= P(B_1)\,P(B_2|B_1) + P(B_1)\,P(R_2|B_1) + P(R_1)\,P(B_2|R_1)$$

$$= \left(\tfrac{7}{12}\right)\left(\tfrac{6}{11}\right) + \left(\tfrac{7}{12}\right)\left(\tfrac{5}{11}\right) + \left(\tfrac{5}{12}\right)\left(\tfrac{7}{11}\right)$$

$$= \tfrac{42}{132} + \tfrac{35}{132} + \tfrac{35}{132} = \tfrac{112}{132} = 0.85$$

Another solution:

$$P(at\ least\ one\ blue\ is\ ball) = 1 - P(zero\ blue\ ball)$$
$$= 1 - P(R_1 and\ R_2)$$
$$= 1 - [(\tfrac{5}{12})(\tfrac{4}{11})]$$
$$= 1 - \tfrac{20}{132} = 1 - 0.15 = 0.85$$

5.

$$P(at\ most\ one\ ball\ is\ blue)$$
$$= ((B_1 and\ R_2)\ or\ (R_1 and\ B_2)\ or\ (R_1 and\ R_2))$$
$$= P(B_1)\ P(R_2|B_1) + P(R_1)P(B_2|R_1) + P(R_1)P(R_2|R_1)$$
$$= (\tfrac{7}{12})(\tfrac{5}{11}) + (\tfrac{5}{12})(\tfrac{7}{11}) + (\tfrac{5}{12})(\tfrac{4}{11})$$
$$= \tfrac{35}{132} + \tfrac{35}{132} + \tfrac{20}{132} = \tfrac{90}{132} = 0.68$$

Another solution:

$$P(at\ most\ one\ blue\ ball) = 1 - P(two\ \ blue\ balls)$$
$$= 1 - P(B_1 and\ B_2)$$
$$= 1 - [(\tfrac{7}{12})(\tfrac{6}{11})]$$
$$= 1 - \tfrac{42}{132} = 1 - 0.32 = 0.68$$

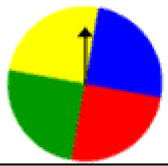
Example (19):

A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

P (A or B) = P (A) + P (B)

P (2) +P (5) = (1/6) + (1/6) = 2/6 =1/3

Example (20) A spinner has 4 equal sectors colored yellow, blue, green, and red. What is the probability of landing on red or blue after spinning this spinner?



$$P(red) = \frac{1}{4}$$

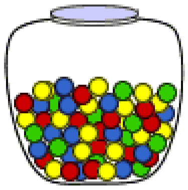
$$P(blue) = \frac{1}{4}$$

$$P(red\ or\ blue) = P(red) + P(blue)$$
$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Example(21): A glass jar contains 1 red, 3 green, 2 blue, and 4 yellow marbles. If a single marble is chosen at random from the jar, what is the probability that it is yellow or green?



$$P(\text{yellow}) = \frac{4}{10}$$

$$P(\text{green}) = \frac{3}{10}$$

P(yellow or green)=P(yellow)+P(green)

$$= \frac{4}{10} + \frac{3}{10}$$

$$= \frac{7}{10}$$

Example (22): In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

P(girl or A)=P(girl)+P(A)-P(girl and A)

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30}$$

$$= \frac{17}{30}$$

Example (23):

On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while talking mobile is 0.32 and probability of a person having a car accident while driving while talking is 0.15. What is the probability of a person driving while talking mobile or having a car accident?

P (talking or accident) =P (talking mobile) + P (accident) - P (talking mobile and accident)
0.32 + 0.09 – 0.15 =0.26

Example (24)

Suppose we roll one die followed by another and want to find the probability of rolling a 4 on the first die and rolling an even number on the second die.

Solution:

Notice in this problem we are not dealing with the sum of both dice. We are only dealing with the probability of 4 on one die only and then, as a separate event, the probability of an even number on one dies.

P (4) =6/36= 1/6
P (even) = 18/36=3/6
So
P ($4 \cap$ even) = (1/6) (3/6) = 3/36 = 1/12

Example (25)

Suppose you have a box with 3 blue marbles, 2 red marbles, and 4 yellow marbles. You are going to pull out one marble, record its color, **put it back** in the box and draw another marble. What is the probability of pulling out a red marble followed by a blue marble?

Solution:

The multiplication rule says we need to find P (red) * P (blue).
P (red) = 2/9
P (blue) = 3/9

P (red $\cap$ blue) = (2/9) (3/9) = 6/81 = 2/27

Example (26)

Consider the same box of marbles as in the previous example. However in this case, we are going to pull out the first marble**, leave it out**, and then pull out another marble. What is the probability of pulling out a red marble followed by a blue marble?

Solution:

We can still use the multiplication rule which says we need to find P (red) *P (blue). But be aware that in this case when we go to pull out the second marble, there will only be **8 marbles left in the bag**.
P (red) = 2/9
P (blue) = 3/8

P (red $\cap$ blue) = (2/9) (3/8) = 6/72 = 1/12

Example (27): A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

Solution:

$$P(\text{head}) = \frac{1}{2}$$

$$P(3) = \frac{1}{6}$$

$$P(\text{head and } 3) = P(\text{head}) \cdot P(3)$$

$$= \frac{1}{2} \cdot \frac{1}{6}$$

$$= \frac{1}{12}$$

Example (28): A school survey found that 9 out of 10 students like pizza. If three students are chosen at random **with replacement,** what is the probability that all three students like pizza?

Solution::

$$P(\text{student 1 likes pizza}) = \frac{9}{10}$$

$$P(\text{student 2 likes pizza}) = \frac{9}{10}$$

$$P(\text{student 3 likes pizza}) = \frac{9}{10}$$

$$P(\text{student 1 and student 2 and student 3 like pizza}) = \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = \frac{729}{1000}$$

Example (29):

A committee consists of four women and three men. The committee will randomly select two people to attend a conference in Hawaii. Find the probability that both are women.

Solution:

Let A be the event that first person selected is woman and B be the event that second person selected is woman.

Then P (A) = P (B) = 4/7 as there are 4 women in the committee of 7 people.

Now we selected a woman as the first person to attend the conference, we cannot select her as a second person to attend the conference.
So now there are 6 people left to select from and only 3 of them are women. So to find the probability of selecting both women is

P (*A* and *B*) = P (*A*) * P (*B* | *A*) = (4/7) * (3/6) = 12/42 = 0.2857

Example (30):
The following table classifies 80 individuals according to whether they are employed (E) or unemployed (U) and according to their smoking habits; Smoker (S) and Nonsmoker (N):

	E	**U**	**Total**
S	**20**	**15**	**35**
N	**10**	**35**	**45**
Total	**30**	**50**	**80**

Find: 1. $P(S|E)$ **2.** $P(N|U)$

Solution:

1. $P(S|E) = \frac{P(S \cap E)}{P(E)} = \frac{20}{30} = 0.67$

2. $P(N/U) = \frac{P(N \cap U)}{P(U)} = \frac{35}{50} = 0.70$

Example (31):

The following table shows the classification of 80 employees from Company (A) according to nationality and age.

Nationality **Age**	**Saudi (S)**	**Tunisian (T)**	**Egyptian (E)**	Total
(20–30) (*A*)	**5**	**3**	**4**	12
(30–40) *(B)*	**15**	**5**	**6**	26
(40–50) (C)	**12**	**4**	**9**	25
(50–60) (*D*)	**8**	**2**	**7**	17
Total	40	14	26	80

If an employee selected at random, find:

a. $P(A) = \frac{12}{80} = 0.15$

b. $P(\sim E) = 1 - P(E) = 1 - \frac{26}{80} = 1 - 0.33 = 0.67$

c. $P(A \cap E) = \frac{4}{80} = 0.05$

d. $P(\sim (A \cap E)) = 1 - 0.05 = 0.95$

e. $P(D \cup S) = P(D) + P(S) - P(D \cap S) = \frac{17}{80} + \frac{40}{80} - \frac{8}{80} = \frac{49}{80} = 0.61$

f. $P(D \cup A) = P(D) + P(A) = \frac{17}{80} + \frac{12}{80} = \frac{29}{80} = 0.36$

g. $P(S \cap T) = P(\phi) = 0$

h. $P(B / S) = \frac{P(S \cap B)}{P(S)} = (15/80)/(40/80) = \left(\frac{15}{80}\right)\left(\frac{80}{40}\right) = \frac{15}{40} = 0.38$

i. $P(C / T) = \frac{P(T \cap C)}{P(T)} = \frac{4}{14} = 0.29$