Chapter (7)
Continuous Probability Distributions
Examples

The uniform distribution

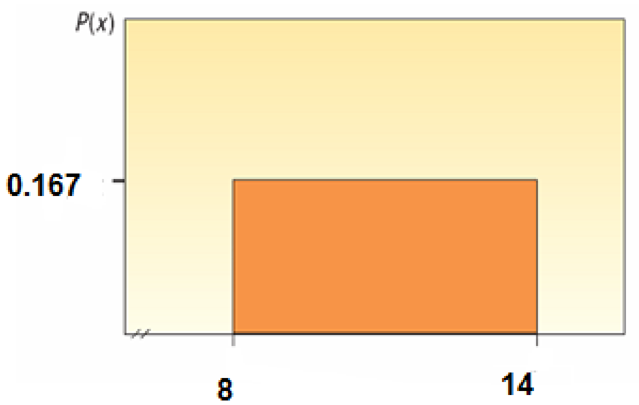
Example (1)

Australian sheepdogs have a relatively short life .The length of their life follows a uniform distribution between 8 and 14 years.

1. Draw this uniform distribution. What are the height and base values?
2. Show the total area under the curve is 1.
3. Calculate the mean and the standard deviation of this distribution.
4. What is the probability a particular dog lives between 10 and 12 years?
5. What is the probability a particular dog lives greater than 10?
6. What is the probability a dog will live less than 9 years?

Solution:

1.



$$The\ height = \frac{1}{b-a} = \frac{1}{14-8} = \frac{1}{6} = 0.167$$

Maximum=14

Minimum=8

2.

$$The\ total\ area = \frac{1}{14-8}(14-8) = \frac{1}{6}(6) = 1$$

3.

$$\mu = \frac{a+b}{2} = \frac{8+14}{2} = \frac{22}{2} = 11$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(14-8)^2}{12}} = \sqrt{\frac{6^2}{12}} = \sqrt{\frac{36}{12}} = \sqrt{3} = 1.73$$

4.

$$p(10 < X < 12) = \frac{1}{14-8}(12-10) = \frac{1}{6}(2) = \frac{1}{3} = 0.3333$$

5.

$$p(X > 10) = p(10 < X < 14) = \frac{1}{14-8}(14-10) = \frac{1}{6}(4) = \frac{4}{6} = 0.667$$

6.

$$p(X < 9) = p(8 < X < 9) = \frac{1}{14-8}(9-8) = \frac{1}{6}(1) = 0.167$$

Normal probability distribution

How to find the area under the normal curve?

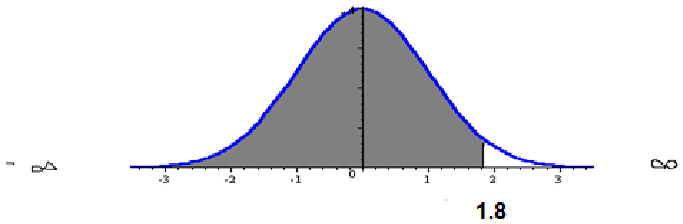
If $\mu = 50 \quad \& \quad \sigma = 6$

Find

$$(1)P(X < 60.8) = P\left(Z < \frac{60.8 - 50}{6}\right)$$

$$= P\left(Z < \frac{10.8}{6}\right) = p(Z < 1.8) = 0.5 + P(0 < Z < 1.8)$$

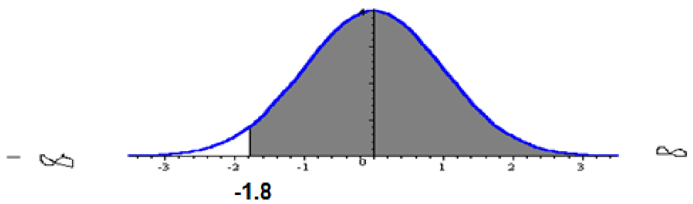
$$= 0.5 + 0.4641 = 0.9641$$



$$(2)P(X > 39.2) = P\left(Z > \frac{39.2 - 50}{6}\right)$$

$$= P\left(Z > \frac{-10.8}{6}\right) = p(Z > -1.8) = 0.5 + P(-1.8 < Z < 0)$$

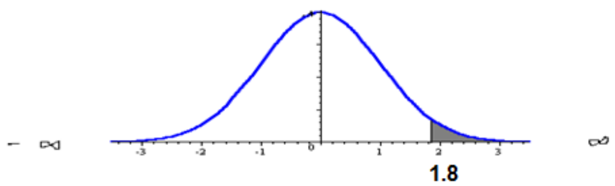
$$= 0.5 + 0.4641 = 0.9641$$



$$(3)P(X > 60.8) = P\left(Z > \frac{60.8 - 50}{6}\right)$$

$$= P\left(Z > \frac{10.8}{6}\right) = p(Z > 1.8) = 0.5 + P(0 < Z < 1.8)$$

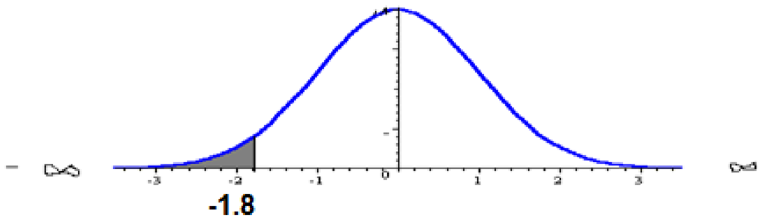
$$= 0.5 - 0.4641 = 0.0359$$



$$(4)\, P(X < 39.2) = P\left(Z < \frac{39.2 - 50}{6} \right)$$

$$= P\left(Z < \frac{-10.8}{6} \right) = p(Z < -1.8) = 0.5 + P(-1.8 < Z < 0)$$

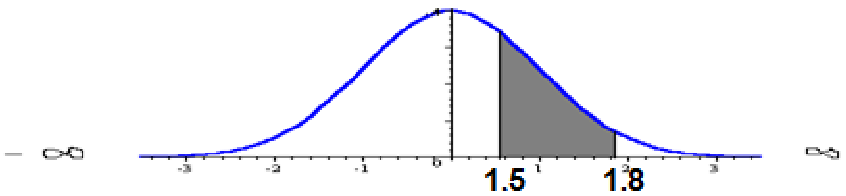
$$= 0.5 - 0.4641 = 0.0359$$



$$(5)\, P(59 < X < 60.8) = P\left(\frac{59 - 50}{6} < Z < \frac{60.8 - 50}{6} \right)$$

$$= P\left(\frac{9}{6} < Z < \frac{10.8}{6} \right) = p(1.5 < Z < 1.8) = P(0 < Z < 1.8) - P(0 < Z < 1.5)$$

$$= 0.4641 - 0.4332 = 0.0309$$

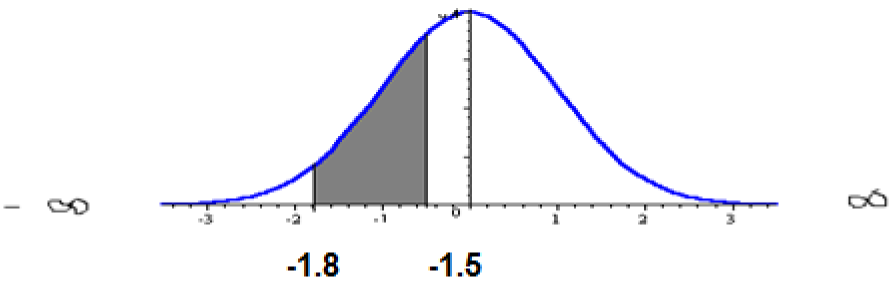


$$(6)\, P(39.2 < X < 41) = P\left(\frac{39.2 - 50}{6} < Z < \frac{41 - 50}{6} \right)$$

$$= P\left(\frac{-10.8}{6} < Z < \frac{-9}{6} \right) = p(-1.8 < Z < -1.5)$$

$$= P(0 < Z < -1.8) - P(0 < Z < -1.5)$$

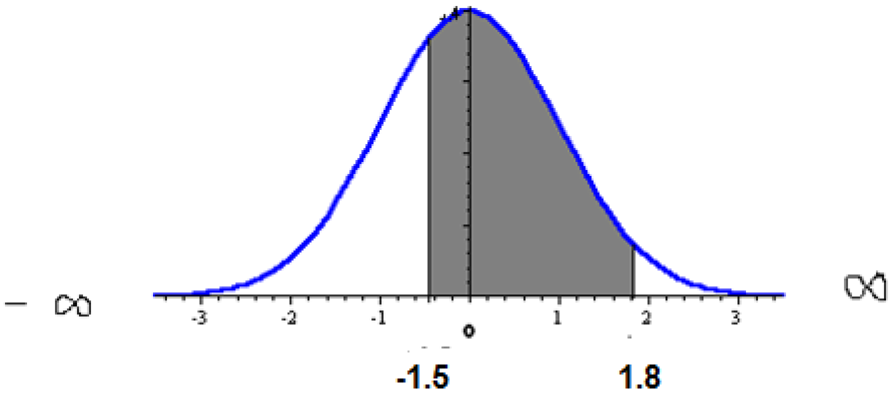
$$= 0.4641 - 0.4332 = 0.0309$$



$$(7)\, P(41 < X < 60.8) = P\left(\frac{41-50}{6} < Z < \frac{60.8-50}{6}\right)$$

$$= P\left(\frac{-9}{6} < Z < \frac{10.8}{6}\right) = p(-1.5 < Z < 1.8) = P(0 < Z < 1.8) - P(-1.5 < Z < 0)$$

$$= 0.4641 + 0.4332 = 0.8973$$



Example (2)

Let X is a normally distributed random variable with mean 65 and standard deviation 13. Find the standard normal random variable (z) for P(X >80)

Solution:

$$P(X > 80) = P(Z > \frac{80-65}{13}) = P(Z > \frac{15}{13}) = P(Z > 1.15)$$

$$0.5 - 0.3749 = 0.1251$$

Example (3)

If the mean = 65 and standard deviation =13. Find x from the following:

1. z = 0.6
2. z = -1.93

Solution:

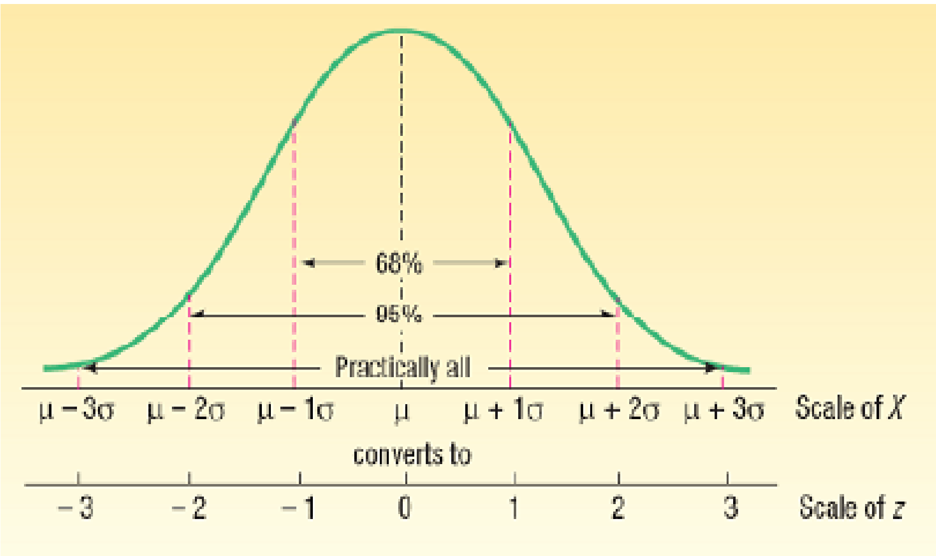
1. $z = 0.6$

$x = 65 + (0.6)13 = 65 + 7.8 = 72.8$

2. $z = -1.93$

$x = 65 + (-1.93)13 = 65 - 25.09 = 39.91$

The Empirical Rule



Example (4)

A sample of the rental rates at University Park Apartments approximates a systematical, bell- shaped distribution. The sample mean is $500; the standard deviation is $20.Using the Empirical Rule, answer these questions: About 68 percent of the monthly food expenditures are between what two amounts?

1. About 95 percent of the monthly food expenditures are between what two amounts?

1. About all of the monthly (99.7%) food expenditures are between what two amounts?

Solution

1- $\bar{X} \pm 1\sigma = 500 \pm 1(20) = 500 \pm 20$

$\$480, \$520$

About 68 percent are between \$480 and \$520.

2- $\bar{X} \pm 2\sigma = 500 \pm 2(20) = 500 \pm 40$

$\$460, \$540$

About 95 percent are between \$460 and \$540.

3- $\bar{X} \pm 2\sigma = 500 \pm 3(20) = 500 \pm 60$

$\$440, \$560$

About 99.7 percent are between \$440 and \$560.

Example (5)

The mean of a normal probability distribution is 120; the standard deviation is 10.

a. About 68 percent of the observations lie between what two values?

b. About 95 percent of the observations lie between what two values?

c. About 99 percent of the observations lie between what two values?

Solution:

a. $\mu \pm 1(\sigma) = 120 \pm 1(10) = 120 \pm 10$

$130 \;\; and \;\; 110$

b. $\mu \pm 2(\sigma) = 120 \pm 2(10) = 120 \pm 20$

$140 \;\; and \;\; 100$

c. $\mu \pm 3(\sigma) = 120 \pm 3(10) = 120 \pm 30$

$150 \;\; and \;\; 90$

Example (6)

Studies show that gasoline use for compact cars sold in the United States is normally distributed, with a mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. Find the probability of compact cars that get:

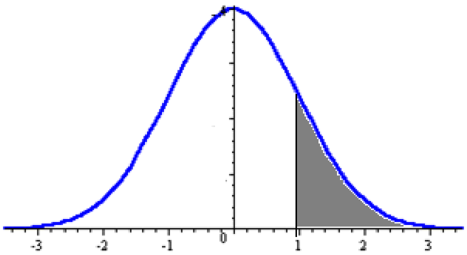
1. 30 mpg or more.
2. 30 mpg or less.
3. Between 30 and 35.
4. Between 30 and 21.

Solution:

$\mu = 25.5 \qquad \sigma = 4.5$

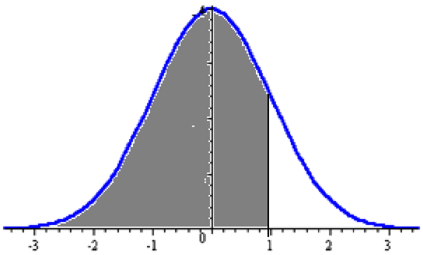
1. $P(x \geq 30) = P(Z \geq \frac{30-25.5}{4.5}) = P(Z \geq \frac{4.5}{4.5}) = P(z \geq 1) = 0.5 - \Phi(1)$

$= 0.5 - 0.3413 = 0.1587$



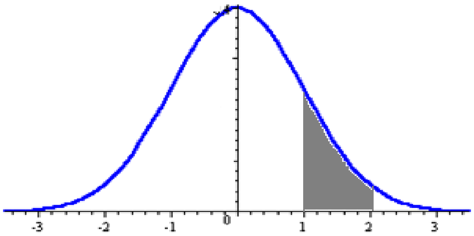
2. $P(x \leq 30) = P\left(Z \leq \frac{30-25.5}{4.5}\right) = P\left(Z \leq \frac{4.5}{4.5}\right) = P(z \leq 1) = 0.5 + \Phi(1)$

$= 0.5 + 0.3413 = 0.8413$



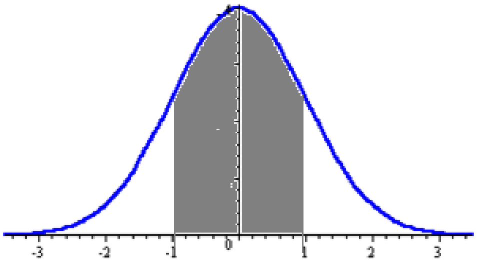
3. $P(30 \leq x \leq 35) = P\left(\frac{30-25.5}{4.5} \leq Z \leq \frac{35-25.5}{4.5}\right) = P\left(\frac{4.5}{4.5} \leq Z \leq \frac{9.5}{4.5}\right)$

$= P(1 \leq z \leq 2.11) = \Phi(2.11) - \Phi(1) = 0.4826 - 0.3413 = 0.1413$



4. $P(21 \leq x \leq 30) = P\left(\frac{21-25.5}{4.5} \leq Z \leq \frac{30-25.5}{4.5}\right) = P\left(\frac{-4.5}{4.5} \leq Z \leq \frac{4.5}{4.5}\right)$

$= P(-1 \leq z \leq 1) = \Phi(1) + \Phi(1) = 0.3413 + 0.3413 = 0.6826$



Example (7)

Suppose that X ~N (3, o.16). Find the following probability:

1. $P(x \geq 3)$.
2. $P(2.8 < x < 3.1)$.

Solution:

$u = 3 \qquad \sigma = 0.4$

1. $P(x \geq 3) = P\left(Z \geq \frac{3-3}{0.4}\right) = P(z \geq 0) = 0.5000$

2. $P(2.8 < x < 3.1) = P\left(\frac{2.8-3}{0.4} < Z < \frac{3.1-3}{0.4}\right) = P(\frac{-0.2}{0.4} < Z < \frac{0.1}{0.4}) =$

$P(-0.50 < z < 0.25) = \Phi(0.5) + \Phi(0.25) = 0.1915 + 0.0987 = 0.2902$

Example (8)

The grades on a short quiz in math were 0, 1, 2…, 10 point, depending on the number answered correctly out of 10 questions. The mean grade was 6.7 and the standard deviation was 1.2. Assuming the grades to be normally distributed, determine:

1. The percentage of students scoring more than 6 points.
2. The percentage of students scoring less than 8 points.
3. The percentage of students scoring between 5.5and 6 points.
4. The percentage of students scoring between 5.5and 8 points.
5. The percentage of students scoring less than 5.5 points.
6. The percentage of students scoring more than 8 points.
7. The percentage of students scoring equal to 8 points.
8. The maximum grade of the lowest 5 % of the class.
9. The minimum grade of the highest 15 % of the class.

Solution:

$\mu = 6.7 \qquad \sigma = 1.2$

1. $P(x > 6) = P\left(Z > \frac{6-6.7}{1.2}\right) = P\left(Z > \frac{-0.7}{1.2}\right) = P(z > -0.58)$

$= 0.5 + \Phi(0.58) = 0.5 + 0.2190 = 0.7190$

2. $P(x < 8) = P\left(Z < \frac{8-6.7}{1.2}\right) = P\left(Z < \frac{1.3}{1.2}\right) = P(z < 1.08)$

$= 0.5 + \Phi(1.08) = 0.5 + 0.3599 = 0.8599$

3. $P(5.5 < x < 6) = P\left(\frac{5.5-6.7}{1.2} < Z < \frac{6-6.7}{1.2}\right) = P\left(\frac{-1.2}{1.2} < Z < \frac{-0.7}{1.2}\right)$

$= P(-1 < z < -.58) = \Phi(1) - \Phi(0.58) = 0.3413 - 0.2190 = 0.1223$

4. $P(5.5 < x < 8) = P\left(\frac{5.5-6.7}{1.2} < Z < \frac{8-6.7}{1.2}\right) = P\left(\frac{-1.2}{1.2} < Z < \frac{1.3}{1.2}\right)$

$= P(-1 < Z < 1.08) = \Phi(1.08) + \Phi(1) = 0.3599 + 0.3413 = 0.7012$

5. $$P(X<5.5)=P\left(Z<\frac{5.5-6.7}{1.2}\right)=P\left(Z<\frac{-1.2}{1.2}\right)$$
$$=P(z<-1)=0.5-\Phi(1)=0.5-0.3413=0.1587$$

6. $$P(x>8)=P\left(Z>\frac{8-6.7}{1.7}\right)=P\left(Z>\frac{1.3}{1.2}\right)=P(z>1.08)$$
$$=0.5-\Phi(1.08)=0.5-0.3599=0.1401$$

7. $P(x=8)=0$

8.
$$0.5-0.05=0.4500$$
$$z=-1.645$$
$$x=6.7-1.645(1.2)=6.7-1.974=4.726\sim 4.7$$

9.
$$0.5-0.15=0.3500$$
$$z=1.04$$
$$x=6.7+1.04(1.2)=6.7+1.248=7.948\sim 7.9$$

Example (9)

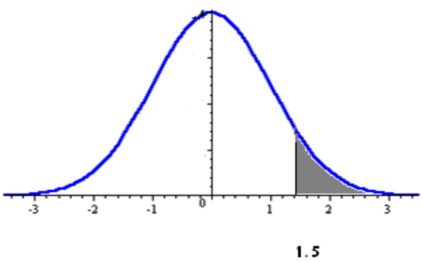
If the heights of 300 students are normally distributed, with a mean 172 centimeters and a standard deviation 8 centimeters, how many students have heights?

1. Greater than 184 centimeters.
2. Less than or equal to 160 centimeters.
3. Between 164 and 180 centimeters inclusive.
4. Equal to 172 centimeters.

$n=300 \qquad \mu=172 \qquad \sigma=8$

$$P(x>184)=P\left(Z>\frac{184-172}{8}\right)=P\left(Z>\frac{12}{8}\right)$$

1. $$=P(z>1.5)=1-\Phi(1.5)=0.5-0.4332=0.0668$$
$$number\ of\ students\ have\ heights\ greater\ than\ 184=300(0.0668)=20$$

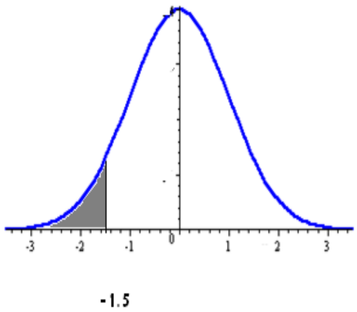


2.

$$P(x<160)=P\left(Z<\frac{160-172}{8}\right)=P\left(Z<\frac{-12}{8}\right)$$

$$=P(z<-1.5)=0.5-\Phi(1.5)=0.5-0.4332=0.0668$$

$$number\ of\ students\ have\ heights\ less\ than\ 160=300(0.0668)=20$$

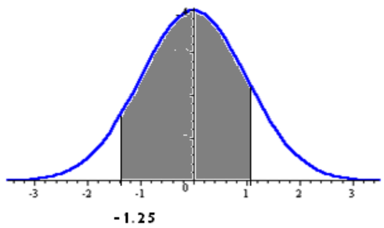


3.

$$P(164<x<180)=P\left(\frac{164-172}{8}<Z<\frac{180-172}{8}\right)=P\left(\frac{-8}{8}<Z<\frac{8}{8}\right)$$

$$=P(-1<z<1)=\Phi(1)+\Phi(1)=0.3413+0.3413=0.6826$$

$$number\ of\ students\ have\ heights\ between\ 164\ and\ 180=300(0.6826)=204.78\sim 205$$



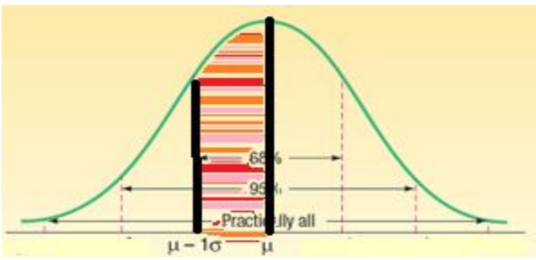
4.

$$P(x=172)=0$$

Example (10)

What is the area under a normal curve that falls between the mean and one standard deviation below the mean?

Solution:



$$p(0\le Z\le 1)=0.3413$$

Example (11)

- If P (Z ≤z1) = 0.9099, what does the (z1) equal?

Solution:

$p(Z \le z_1) = 0.9099$

$0.9099 - .5 = 0.4099$

$Z_{0.4099} = 1.34$

- If P (Z ≥ z1) = 0.9099, what does the (z1) equal?

Solution:

$p(Z \ge z_1) = 0.9099$

$0.9099 - .5 = 0.4099$

$Z_{0.4099} = -1.34$

Example (12)
The mean of a normal probability distribution is 130; the standard deviation is 10.

- **What observation which more than or equal 84.13% percent of the observations?**

Solution:

$p(Z \ge z_1) = 0.8413$

$0.8413 - .5 = 0.3413$

$Z_{0.3413} = -1$

$X = u + Z\sigma = 130 + 10(-1) = 130 - 10 = 120$

- **What observation which less than or equal 84.13% percent of the observations?**

Solution:

$P(Z \le z_1) = 0.8413$

$0.8413 - .5 = 0.3413$

$Z_{0.3413} = 1$

$X = u + Z\sigma = 130 + 10(1) = 130 + 10 = 140$

Example (13)
The mean of a normal probability distribution is 130; the standard deviation is 10.

- **What observation which more than or equal 8.85 % percent of the observations?**

Solution:

$p(Z \geq z_1) = 0.0885$

$0.5 - 0.0885 = 0.4115$

$Z_{0.4115} = 1.35$

$X = u + Z\sigma = 130 + 10(1.35) = 130 + 13.5 = 143.5$

- **What observation which less than or equal 8.85 % % percent of the observations?**

Solution:

$p(Z \leq z_1) = 0.0885$

$0.5 - 0.0885 = 0.4115$

$Z_{0.4115} = -1.35$

$X = u + Z\sigma = 130 + 10(-1.35) = 130 - 13.5 = 116.5$

Z-Table

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990