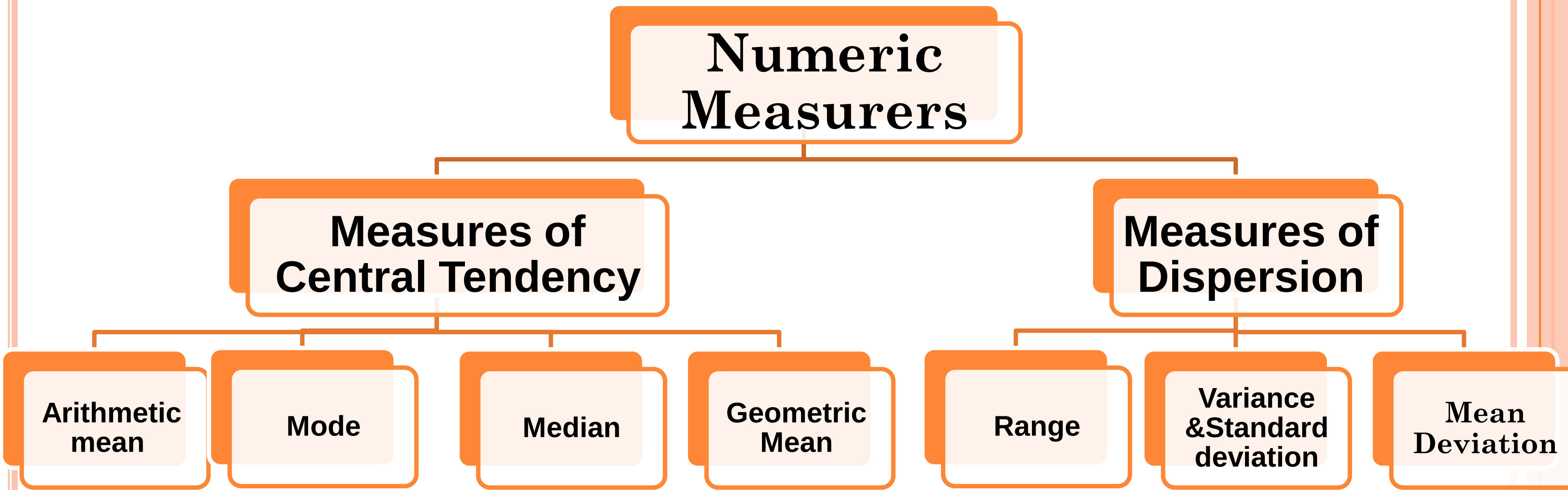
Chapter (3)

Describing Data Numerical Measures Examples


Numeric Measurers
Measures of Central Tendency
Measures of Dispersion
Arithmetic mean
Mode
Median
Geometric Mean
Range
Variance &Standard deviation
Mean Deviation

Example (1)

If King Saud University (girls section) has 10 coffee shops on the campus and if their purchases (in thousands) per month are as follows:

10, 20, 30, 40, 40, 41, 45, 50, 52, 60

Find the Population mean.

Solution:

$$\mu = \frac{\sum_{i=1}^{10} X_i}{10} = \frac{10 + 20 + \cdots + 60}{10} = \frac{388}{10} = 38.8$$

Example (2)

The following are ages (in years) for a sample of students from school A:

15, 10, 18, 17, 18, 11, 11, 15, 14,13,11,10

Find the sample mean.

Solution:

$$\bar{X} = \frac{\sum_{i=1}^{12} X_i}{12} = \frac{15+10+\cdots.+10}{12} = \frac{136}{12} = \mathbf{13.58}$$

Example (3)

The average age of 5 women in a group is 27 years. If two other women aged 40 and 28 years join the group, find, in years, the new average age of the group of women.

Solution:

The average age of 5 women = $\bar{X}$ = **27**

$$\sum X = 135$$

The average age of 7 women = $\frac{135+40+28}{7}$=29

Example (4)

Suppose that you have three samples and their means and sizes are

Sample	Mean	Size
1	20	10
2	22	12
3	24	11

Then weighted the mean of all samples together would be given by

$$\bar{X}_W = \frac{\sum_{i=1}^{n} X_i W_i}{\sum_{i=1}^{n} W_i} = \frac{20(10) + 22(12) + 24(11)}{10 + 12 + 11} = 22.06$$

Example (5) :

A student received an A in English Composition I (3 credits), a C in Introduction to Psychology (3 credits), a B in Biology I (4 credits), and a D in Physical Education(2 credits). Assuming A=4 grade points, B=3 grade points, C=2 grade points, D=1 grade point, and F=0 grade points, find the student's grade point average.

$$\bar{X}_w = \frac{\sum_{i=1}^{n} X_i W_i}{\sum_{i=1}^{n} W_i} = \frac{4(3)+2(3)+3(4)+1(2)}{3+3+4+2} = 2.67$$

Mean for grouped data

1- The Discrete data

$$\bar{X} = \frac{\sum_{i=1}^{k} f_i X_i}{\sum_{i=1}^{k} f_i}$$

X: represents any particular value .

f : is the frequency in each value.

Example (6)

Table shows the distribution of households by the number of children

Number of children(X)	0	1	2	3	4	5
Number of households(F)	5	7	8	5	3	4

Calculate the arithmetic mean of the number of children

Solution:

							Total
Number of children(X)	0	1	2	3	4	5	
Number of households(F)	5	7	8	5	3	4	32
FX	0	7	16	15	12	20	70

$$\bar{X} = \frac{\sum_{i=1}^{k} f_i X_i}{\sum_{i=1}^{k} f_i} = \frac{70}{32} = 2.19 = 3$$

2-The Arithmetic Mean of Grouped Data (**Continuous data)**

Example (7)

The following table gives the marks of a sample of 30 students.

marks	(2 -4)	(4-6)	[6-8)	(8-10)	(10-12)	(12-14)
frequency	3	6	8	7	4	2

Find the mean

Solution:

marks	Frequency (f)	Midpoint (m)	fm
[2-4)	3	3	9
[4-6)	6	5	30
[6-8)	8	7	56
[8-10)	7	9	63
[10-12)	4	11	44
[12-14]	2	13	26
Total	$\sum f = 30$		$\sum mf$ =228

$$\bar{X} = \frac{\sum fm}{n} = \frac{228}{30} = 7.6$$

The Median

The median is the midpoint of the values after they have been ordered from the smallest to the largest or from the largest to the smallest.

Median for ungrouped data

How do you find the median when (n) is odd?

1- Arrange all values (N) from smallest to largest

2- Find it by counting {(n+1) /2} observations up from the bottom

3- The median is the center of the list

Example (8)

Find the median for the sample values:

3, 5, 1, 6, 7, 4, 8, 7, 5, 10, 4, 7, 8, 9, 2

Solution: Arranging the data in ascending order gives:

1, 2 , 3 , 4 , 4 , 5 , 5 , 6 , 7 , 7 , 7 , 8 , 8 , 9 ,10

n =15 "odd"

The rank of median = (15+1)/2 =8

Median : the value of the observation of order 8 = 6

How do you find the median when (n) is even?

1- Arrange all values (N) from smallest to largest

2- Find it by counting {$R_1 = (n/2)$ $R_2 = (n/2) + 1$)} observations up from the bottom

3- The median is the average of the center two values

Example (9)

Find the median for the sample values:

3, 5, 1, 6, 7, 4, 8, 7, 5, 4, 7, 8, 9, 2

Solution: Arranging the data in ascending order gives:

1, 2 , 3 , 4 , 4 , 5 , 5 , 6 , 7 , 7 , 7 , 8 , 8 , 9

n =14   "even"

$R_1 = n/2 = 14/2 = 7$, $R_2 = (n/2) + 1 = 7+1=8$

The value of the observation of ordered 7 is 5

The value of the observation of ordered 8 is 6

Then the median is the average of these two values:

Median= = (5+6)/2 = 5.5

The Mode

- The mode is the value of the observation that appears most frequently.
- If all values are different or have the same frequency, there is no mode.
- A set of values may have more than one mode.

Mode for ungrouped data

Example (10):

Find the mode of the following data:

Group (1): 12, 15, 18, 17, 15, 14, 13, 15

Group (2): 12, 13, 18, 17, 15, 14, 13, 15

Group (3): 12, 10, 18, 17, 11, 14, 13, 15

Group (4): 12, 12, 18, 18, 11, 11, 13, 13

Solution:

The mode of group (1) is **15**

The mode of group (2) is **13 and 15**

There is no mode in group **(3)**

There is no mode in group **(4)**

A comparison of the properties of measures of central tendency

Mode	Median	Mean
It can be computed for open-ended tables	It can be computed for an open-ended table.	It is difficult to compute the mean from an open-ended table.
It can found for both quantitative and qualitative variables.	The median can only found for quantitative and qualitative (ordinal) variables.	The mean can only be found for quantitative variables
A set of values may have more than one mode.	A set of data has a unique median	A set of data has a unique mean
Mode depends on the value of the most frequent.	median in his account depends only on the value that mediates data	All the values are included in computing the mean.
The mode cannot be distorted by extreme values	The median cannot be distorted by extreme values	The mean can be distorted by extreme values

Example (11):

For these situations, state which measure of central tendency—mean, median, or mode—should be used.

A. The most typical case is desired. **Mode**
B. The distribution is open-ended. **Median or Mode**
C. There is an extreme value in the data set. **Median or Mode**
D. The data are Nominal. **Mode**
E. The values are to be divided into two approximately equal groups, one group containing the larger values and one containing the smaller values. **Median**
F. One-half of the factory workers make more than $5.37 per hour, and one-half make less than $5.37 per hour. **Median**
G. The average number of children per family in the Plaza Heights Complex is 1.8. **Mean**
H. Most people prefer red convertibles over any other color. **Mode**
I. The most common fear today is fear of speaking in public. **Mode**
J. The average age of college professors is 42.3 years. **Mean**

The Relative Positions of the Mean, Median and the Mode

Example (12)

The following are the grades a professor gave on the first test in a statistics class: 52,61,74,75,82,83,86,87,88 and 90 . Distribution of grades is :

(A) Negatively skewed	(B) Bimodal
(C) Normally distributed	(D) Positively skewed

Mean = 77.8

Median =82.5

Mode = None

Median > Mean

Geometric Mean

Example (13)

Compute the geometric mean of following percent increases :8,12,14,26,and 5

Solution

$$GM = \sqrt[n]{(X_1)(X_2)(X_3)\dots(X_n)}$$

Geometric Mean = $GM = \sqrt[5]{(1.08)(1.12)(1.14)(1.26)(1.05)} = 1.128$

1.128-1=0.128

12.8 percent increase

Example (15)

In 1985 there were 340,213 cell phone subscribers in the United States. By 2006 the number of cell phone subscribers increased to 233,000,000 .What is the geometric mean annual increase for the period?

Solution

Rate of Increase Over Time

$$GM = \sqrt[n]{\frac{Value\ at\ end\ period}{Value\ at\ start\ of\ period}} - 1$$

$$GM = \sqrt[21]{\frac{233{,}000{,}000}{340{,}213}} - 1 = 1.3646$$

1.3646-1=0.3647

Rate of increase is 36.47 percent per year

Range

The different between the largest and the smallest values

Range = largest value - smallest value

Example (15)

Find the range for the following data:

20, 40, 45, 70, 99, 50, 30, 31, 60, 34

Solution:

20, 30, 31, 34, 40, 45, 50, 60, 70, 99

Largest value = 99

Smallest value = 20

Range = 99 – 20 = 79

Example (16)

Find the range for the following table:

Age (in years)	(15-20)	(20-25)	(25-30)	(30-35)	(35-40)	(40-45)
Frequency	3	6	10	7	6	2

Solution:

The midpoint of the last class =42.5

The midpoint of the first class =17.5

R= 42.5 -17.5=25

Mean Deviation :

The mean of the absolute deviation of a set of data about the data's mean .

$$MD = \frac{\sum|X - \bar{X}|}{n}$$

Example (17)

If the sample is removed from the factory workers of foodstuffs size 5 workers, and record the number of years of experience, and were as follows :

9 5 10 13 8

Calculate the mean deviation

Solution :

$$\bar{X} = 9$$

						Total
X_i	9	5	10	13	8	45
$(X_i - \bar{X})$	0	-4	1	4	-1	0
$\lvert X_i - \bar{X} \rvert$	0	4	1	4	1	10

Mean Deviation : $MD = \frac{\sum\lvert X-\bar{X}\rvert}{n} = \frac{10}{5} = 2$

Variance and Standard Deviation (for ungrouped data)

Variance:The arithmetic mean of the squared deviation from the mean.

Population variance

$$\sigma^2 = \frac{\sum(X-\mu)^2}{N}$$

Sample variance

$$S^2 = \frac{\sum(X-\bar{X})^2}{n-1}$$

Standard Deviation :The square root of the variance

Population Standard Deviation :

$$\sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}}$$

Sample Standard Deviation:

$$S = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}}$$

Example (18)

The following are age (in years) of a sample of babies from clinic A:

2, 3, 4, 5, 4, 5, 6, 3

Find the variance and Standard Deviation

Solution:

$$\bar{X} = 4$$

									Total
X_i	2	3	4	5	4	5	6	3	32
$(X_i - \bar{X})$	-2	-1	0	1	0	1	2	-1	0
$(X_i - \bar{X})^2$	4	1	0	1	0	1	4	1	12

Sample variance

$$S^2 = \frac{\sum(X-\bar{X})^2}{n-1} = \frac{12}{7} = 1.71$$

Sample Standard Deviation:

$$S = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}} = \sqrt{1.71} = 1.31$$

Variance and Standard Deviation
for Grouped Data

$$variance\ S^2 = \frac{\sum f(M - \bar{X})^2}{n-1}$$

$$Standard\ deviation \quad S = \sqrt{\frac{\sum f(M - \bar{X})^2}{n-1}}$$

M : is the midpoint of the class.

f : is the class frequency.

n : is the of observations in the sample .

$\bar{X}$: is the designation for the sample mean

Example (19)

The following table gives the marks of a sample of students.

marks	(4-6)	(6-8)	(8-10)	(10-12)	(12-14)	(14-16)
frequency	1	3	7	4	3	2

Find the variance.

Marks	f	Midpoint (mi)	$f_i m_i$	$(m_i - \overline{X})$	$(m_i - \overline{X})^2$	$f(m_i - \overline{X})^2$
(4-6)	1	5	5	-5.1	26.01	26.01
(6-8)	3	7	21	-3.1	9.61	28.83
(8-10)	7	9	63	-1.1	1.21	8.47
(10-12)	4	11	44	0.9	0.81	3.24
(12-14)	3	13	39	2.9	8.41	25.23
(14-16)	2	15	30	4.9	24.01	48.02
Total	20		202			139.8

$$\bar{X} = \frac{202}{20} = 10.1$$

$$variance = S^2 = \frac{\sum f(M - \bar{X})^2}{n - 1} = \frac{139.8}{20 - 1} = 7.36$$

$$Standard\ deviation\ = \ S = \sqrt{\frac{\sum f(M - \bar{X})^2}{n - 1}}$$

$$= \sqrt{\frac{139.8}{20 - 1}} = \sqrt{7.36} = 2.71$$

- **Other solution :**

$$S^2 = \frac{\sum m^2 f - \frac{(\sum mf)^2}{n}}{n - 1}$$

$$S = \sqrt{\frac{\sum m^2 f - \frac{(\sum mf)^2}{n}}{n - 1}}$$

fm^{2}f

Marks	f	Midpoint (m)	fm	F(m^2)
(4-6)	1	5	5	25
(6-8)	3	7	21	147
(8-10)	7	9	63	567
(10-12)	4	11	44	484
(12-14)	3	13	39	507
(14-16)	2	15	30	450
Total	20		202	2180

$$S^2 = \frac{2180 - 2040.2}{19} = 7.36$$

$$S = \sqrt{7.36} = 2.71$$