**Chapter 4: Simulation Models**

**Problem 1:**

In the following table, the distribution function of random variable X is given

|  |  |
| --- | --- |
| X | f(x) |
| 1 | 0.1 |
| 2 | 0.05 |
| 3 | 0.2 |
| 4 | 0.15 |
| 5 | 0.20 |
| 6 | 0.30 |

1. Is f(x) a probability function? Justify your results.
2. Find mean and variance of x
3. Find P(X≤ 4), P(X<4), P(X≤3), P(x<2), P(X>4), P(2<x<4), P(2≤X≤4)

**Solution:**

1. Yes, f(x) is a probability function because ∑ f(x) = 1; (0.1+0.05+0.2+0.15+0.2+0.3=1).
2. Mean of x, μ=(1\*0.1+2\*.05+3\*0.2+4\*0.15+5\*0.2+6\*0.3)=(0.1+0.1+0.6+0.6+1+1.8)=4.2

Variance of X is E(x2)- μ2=(1\*0.1+4\*.05+9\*0.2+16\*0.15+25\*0.2+36\*0.3)-(4.2)2

=(0.1+0.2+1.8+2.4+5+8.1)-17.62=17.8-17.62=0.18

1. Probabilities of
   * P(X≤ 4)=(0.1+0.05+0.2+0.15)=0.5 ; (x in {1,2,3,4})
   * P(X<4)=(0.1+0.05+0.2)=0.35 ; (x in {1,2,3})
   * P(X≤ 2)=(0.1+0.05)=0.15 ; (x in {1,2})
   * P(X<2)=(0.1) ; (x in {1})
   * P(X>4)= (0.2+0.3) or 1- P(X≤ 4)=0.5 ; (x in {5, 6} or {1,2,3,4,5,6}-{1,2,3,4})
   * P(2<x<4)= 0.2 or P(x<4)-P(x≤2) = 0.35 – 0.15 =0.2 ; (x in {3} or {1,2,3}-{1,2})
   * P(2≤X≤4) = (0.05 + 0.2 + 0.15) = 0.4 or P(x≤4)-P(x<2)=0.5 - 0.1=0.4; ; (x in {2,3,4} or {1,2,3,4}-{1})

**Problem 2:**

For an distribution *f(x),* where

1. Is *f(x)* a probability density function?
2. If yes, find P( 1 ≤ *x* ≤ 3)
3. Calculate the mean and variance of *x*

**Solution:**

1. Yes, *f(x)* is a probability density function

OR

1. If yes, find P( 1 ≤ *x* ≤ 3)
2. Calculate the mean and variance of *x*

* Mean *μ = E(x)*
* Variance *V(x)=E(x2)- μ2*

**Problem 3:**

The exponential distribution *f(x),* where

With mean =

The Poisson distribution *P(N(t)=n) is*

Can you explain the relationship between the exponential distribution and the Poisson process?

**Solution:**

In case of having x as a inter-arrival time or service time, and the mean value of X is .

In Poisson distribution, N(t) represents the distribution of arrival rate or service rate. The mean value of n is . Which means n is the inverse of x.