**Chapter 8**

**Random-Variate Generation**

**Example1:**

Let the probability density function of random value x be an exponential distribution with mean 1/λ= 5 minutes. Use the uniform random number generator in EXCEL to produce 20 consecutive values of x.

**Solution:**

f(x)= λ e- λx, x≥0

for λ=1/5, f(x)= 1/5 e- x/5

F(x)=1- e- x/5, x≥0

1-e- x/5=R

X=-(5)\*ln(1-R)

We use EXCEL to produce the following consecutive values of R

|  |  |
| --- | --- |
| i | Ri |
| 1 | 0.554931 |
| 2 | 0.095157 |
| 3 | 0.73531 |
| 4 | 0.713431 |
| 5 | 0.152417 |
| 6 | 0.1239 |
| 7 | 0.372779 |
| 8 | 0.865727 |
| 9 | 0.906628 |
| 10 | 0.373264 |
| 11 | 0.243593 |
| 12 | 0.306922 |
| 13 | 0.437154 |
| 14 | 0.662161 |
| 15 | 0.131981 |
| 16 | 0.832104 |
| 17 | 0.836737 |
| 18 | 0.368437 |
| 19 | 0.499496 |
| 20 | 0.663147 |

Using the equation X=-(5)\*ln(1-R), then

|  |  |  |
| --- | --- | --- |
| I | Ri | xi |
| 1 | 0.554931 | 4.047624 |
| 2 | 0.095157 | 0.499967 |
| 3 | 0.73531 | 6.645977 |
| 4 | 0.713431 | 6.24888 |
| 5 | 0.152417 | 0.826831 |
| 6 | 0.1239 | 0.661372 |
| 7 | 0.372779 | 2.332285 |
| 8 | 0.865727 | 10.03939 |
| 9 | 0.906628 | 11.8558 |
| 10 | 0.373264 | 2.336147 |
| 11 | 0.243593 | 1.39588 |
| 12 | 0.306922 | 1.833063 |
| 13 | 0.437154 | 2.873747 |
| 14 | 0.662161 | 5.425931 |
| 15 | 0.131981 | 0.707711 |
| 16 | 0.832104 | 8.922053 |
| 17 | 0.836737 | 9.061975 |
| 18 | 0.368437 | 2.297786 |
| 19 | 0.499496 | 3.460698 |
| 20 | 0.663147 | 5.440537 |

**Example2:**

Let the following probability distribution of random variable x be

|  |  |  |
| --- | --- | --- |
| Interval i | x | probability |
| 1 | 0<x≤2 | 0.12 |
| 2 | 2<x≤3 | 0.18 |
| 3 | 3<x≤5 | 0.21 |
| 4 | 5<x≤6 | 0.08 |
| 5 | 6<x≤9 | 0.19 |
| 6 | 9<x≤10 | 0.06 |
| 7 | 10<x≤12 | 0.05 |
| 8 | 12<x≤16 | 0.11 |

If we know that the distribution of x is continues,

* find the inverse cumulative distribution of X
* generate 5 random values (use EXCEL), and get the corresponding x’s

**Solution**:

First we calculate the cumulative probabilities of x

|  |  |  |  |
| --- | --- | --- | --- |
| Interval i | X | Probability | Cumulative  Probability  p |
| 1 | 0<x≤2 | 0.12 | 0.12 |
| 2 | 2<x≤3 | 0.18 | 0.3 |
| 3 | 3<x≤5 | 0.21 | 0.51 |
| 4 | 5<x≤6 | 0.08 | 0.59 |
| 5 | 6<x≤9 | 0.19 | 0.78 |
| 6 | 9<x≤10 | 0.06 | 0.84 |
| 7 | 10<x≤12 | 0.05 | 0.89 |
| 8 | 12<x≤16 | 0.11 | 1 |

From the above, the relationship between x values and their cumulative probability values should be represented by a group of straight lines with slope

ai= (xi – xi-1)/ (pi – pi-1) for xi-1 <x≤ xi

then for xi-1 <x≤ xi , xi = xi-1+ ai\*( pi – pi-1). For a general value x such that xi-1 <x≤ xi ,

x= xi-1+ ai\*( R – pi-1), where R, pi-1 <R≤ pi , is generated by a uniform random number generator, and ai given as

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Interval i | X | Probability | Cumulative  Probability  Pi | ai |
| 1 | 0<x≤2 | 0.12 | 0.12 | 16.66667 |
| 2 | 2<x≤3 | 0.18 | 0.3 | 5.555556 |
| 3 | 3<x≤5 | 0.21 | 0.51 | 9.52381 |
| 4 | 5<x≤6 | 0.08 | 0.59 | 12.5 |
| 5 | 6<x≤9 | 0.19 | 0.78 | 15.78947 |
| 6 | 9<x≤10 | 0.06 | 0.84 | 16.66667 |
| 7 | 10<x≤12 | 0.05 | 0.89 | 40 |
| 8 | 12<x≤16 | 0.11 | 1 | 36.36364 |

Let the generated Ri values (by EXCEL) be

|  |  |
| --- | --- |
| I | R |
| 1 | 0.554931 |
| 2 | 0.095157 |
| 3 | 0.73531 |
| 4 | 0.713431 |
| 5 | 0.152417 |

Then the corresponding x values are (corrected)

|  |  |  |
| --- | --- | --- |
| i | R | X |
| 1 | 0.554931 | 5.561638 |
| 2 | 0.095157 | 1.58595 |
| 3 | 0.73531 | 8.421834 |
| 4 | 0.713431 | 8.057184 |
| 5 | 0.152417 | 2.180094 |

**Example 3:**

Assume Y is an Erlang random variable with parameters K equals 10 and ϴ equals 0.2. Use the convolution method to generate an Erlang random value of Y.

**Solution**:

Using the convolution method, we need to generate 10 Exponential random values with mean 1/(10\*0.2). Using the convolution method, we need to generate **10** Exponential random values with mean 1/(10\*0.2).

Assume the uniform random used values are

|  |  |
| --- | --- |
| C | Ri |
| 1 | 0.554931 |
| 2 | 0.095157 |
| 3 | 0.73531 |
| 4 | 0.713431 |
| 5 | 0.152417 |
| 6 | 0.1239 |
| 7 | 0.372779 |
| 8 | 0.865727 |
| 9 | 0.906628 |
| 10 | 0.373264 |

Using the equation that calculate exponential random values from uniform random values

X=-(1/(10\*0.2))\*ln(R).

|  |  |  |
| --- | --- | --- |
| c | Ri | Xi |
| 1 | 0.554931 | 0.294456 |
| 2 | 0.095157 | 1.176114 |
| 3 | 0.73531 | 0.153732 |
| 4 | 0.713431 | 0.168835 |
| 5 | 0.152417 | 0.940568 |
| 6 | 0.1239 | 1.04414 |
| 7 | 0.372779 | 0.493385 |
| 8 | 0.865727 | 0.072093 |
| 9 | 0.906628 | 0.049012 |
| 10 | 0.373264 | 0.492735 |

Then using the equation given in chapter 8, Y= 4.885067

**Example4:**

Assume Y is a poisson random variable with mean 10. Use the Acceptance- Rejection technique to generate a poisson random value of Y.

**Solution**:

Since we don’t know exactly how many random values we are going to generate, we generate the following 20 uniform random values.

|  |  |
| --- | --- |
| I | Ri |
| 1 | 0.055808 |
| 2 | 0.3056 |
| 3 | 0.421779 |
| 4 | 0.166887 |
| 5 | 0.273628 |
| 6 | 0.168742 |
| 7 | 0.172127 |
| 8 | 0.801833 |
| 9 | 0.816693 |
| 10 | 0.745711 |
| 11 | 0.098463 |
| 12 | 0.773789 |
| 13 | 0.066884 |
| 14 | 0.339218 |
| 15 | 0.478064 |
| 16 | 0.108374 |
| 17 | 0.680072 |
| 18 | 0.745385 |
| 19 | 0.846652 |
| 20 | 0.296725 |

Using the equation that calculate exponential random values from uniform random values

X=-1/10\*ln(R).

We need to calculate the sum’s of Xi‘s and compare it with 1.

|  |  |  |  |
| --- | --- | --- | --- |
| c | Ri | Xi | SUM |
| 1 | 0.055808 | 0.288584 | 0.288584 |
| 2 | 0.3056 | 0.118548 | 0.407132 |
| 3 | 0.421779 | 0.086327 | 0.49346 |
| 4 | 0.166887 | 0.179044 | 0.672504 |
| 5 | 0.273628 | 0.129599 | 0.802102 |
| 6 | 0.168742 | 0.177938 | 0.98004 |
| 7 | 0.172127 | 0.175952 | 1.155993 |
| 8 | 0.801833 | 0.022085 | 1.178078 |
| 9 | 0.816693 | 0.020249 | 1.198327 |
| 10 | 0.745711 | 0.029342 | 1.227669 |
| 11 | 0.098463 | 0.231807 | 1.459476 |
| 12 | 0.773789 | 0.025646 | 1.485122 |
| 13 | 0.066884 | 0.270479 | 1.755601 |
| 14 | 0.339218 | 0.108111 | 1.863712 |
| 15 | 0.478064 | 0.073801 | 1.937513 |
| 16 | 0.108374 | 0.222217 | 2.15973 |
| 17 | 0.680072 | 0.038556 | 2.198286 |
| 18 | 0.745385 | 0.029386 | 2.227672 |
| 19 | 0.846652 | 0.016647 | 2.244318 |
| 20 | 0.296725 | 0.121495 | 2.365813 |

From the above table, the sum is greater than 1 when c equals 7. Then the poisson random value is 6