Exercise 1-stat331

Bayan Almukhlef

#Simple random sample

Example 3.1

Appendix C gives data related to the number of tractors in **30** serially numbered villages of Doraha development block in Punjab (India). Select (1) **WR** and (2) **WOR** simple random sample of **10** villages. Then Calculate

(a) population mean $\bar{Y}$

(b) population variance σ^2

(c) standard deviation of population.

(d) sample mean

(E) Sampling Variance of mean

(i) confidence interval for the mean.

Appendix C Number of tractors, tube wells, and net irrigated area (in hectares) for 69 villages of Doraha development block of Punjab, India

Village No.	Village name	Tractors	Tube wells	Irrigated area
1	Ajnaud	20	102	281
2	Aracha	19	126	337
3	Afjullapur	5	31	77
4	Alampur	10	39	108
5	Bishanpur	12	85	191
6	Shahpur	21	130	208
7	Sirthala	44	222	698
8	Sultanpur	12	70	180
9	Sihora	20	219	458
10	Hole	10	115	288
11	Kotli Afgana	11	41	161
12	Katari	30	133	512
13	Kartarpur	15	55	123
14	Katana Sahib	3	50	100
15	Kaddon	36	249	675
16	Gobindpura	24	119	258
17	Gurditpura	17	95	143
18	Gidrhi	15	100	267
19	Ghadani Kalan	76	551	1178
20	Ghadani Khurd	35	209	523
21	Ghalouti	46	380	583
22	Ghanrash	16	160	310
23	Chankoian Kalan	7	33	129
24	Chankoian Khurd	21	142	184
25	Chapran	7	64	91
26	Jaipura	22	130	240
27	Jahangir	5	27	80
28	Jandali	21	118	429
29	Jargarhi	38	193	583
30	Jarag	59	360	888

Solution

Here village is the sampling unit. The villages in the population are already serially numbered which, otherwise, is the first step involved in the sample selection. Refer to **Appendix B**, and use first column by dropping the last two digits of each four digit number. Then we see that the first random number thus formed is 11. Similarly, the subsequent random numbers are seen to be 5,26,11 ... ,3

(1) By selecting the first 10 random numbers from 1 to 30, without discarding Repetitions (**WR**), we obtain the serial numbers of villages in the sample. These are given below along with their variable values (number of tractors).

village	11	5	26	11	11	24	12	22	9	3
Tractors y_i	11	12	22	11	11	21	30	16	20	5

One can see that 11th village has been selected twice in the with replacement simple random sample where repeated selection of units is permitted.

$$N = 30\,, n = 10$$

$$\mu = \frac{\sum_{i=1}^{N} Y_i}{N} = 22.5667$$

$$\sigma^2 = \frac{\sum_{i=1}^{N} (Y_i - \mu)^2}{N} = 273.3789$$

$$\sigma = \sqrt{\sigma^2} = 16.5342$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{159}{10} = 15.9 \simeq 16$$

$$Sampling\ variance: V(\bar{y}) = \frac{\sigma^2}{n} = \frac{273.3789}{10} = 27.3379$$

stander error of the sample mean

$$se(\bar{y}) = \sqrt{V(\bar{y})} = \sqrt{27.3379} = 5.2286$$

The confidence interval for population mean $\bar{Y}$ *are given by*

$$\bar{y} \mp Z_{1-\frac{\propto}{2}}\, se(\bar{y})$$

$$15.9 \mp (1.96)5.2286$$

$$\bar{Y} \in [5.6519 \quad , \quad 26.1481]$$

NOTE: If all the population units are not known, the actual value of $V(\bar{y})$ can not be obtained. Therefore, we can use an unbiased estimator of $V(\bar{y})$ >> $\hat{V}(\bar{y}) = \frac{s^2}{n}$.

(2) In without replacement sample, any repetition (11th village in the present case) is omitted, and another random number is selected as its replacement. Next random numbers from 1 to 30 are 7 and 25. Thus the WOR simple random sample of 10 villages from the population under study is with the following serial numbers:

village	11	5	26	24	12	22	9	3	7	25
Tractors y_i	11	12	22	21	30	16	20	5	44	7

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{188}{10} = 18.8$$

$$S^2 = \frac{\sum_{i=1}^{N} (Y_i - \mu)^2}{N-1} = 282.8057$$

$$V(\bar{y}) = \frac{N-n}{N}\frac{S^2}{n} = \frac{30-10}{30}\frac{282.8057}{10} = 18.8537$$

Stander error of the sample mean

$$se(\bar{y}) = \sqrt{V(\bar{y})} = \sqrt{18.8537} = 4.3421$$

The confidence interval for population mean $\bar{Y}$ are given by

$$\bar{y} \mp Z_{1-\frac{\propto}{2}}\, se(\bar{y})$$

$$18.8 \mp (1.96)4.3421$$

$$\bar{Y} \in [8.5105 \quad , \quad 27.3105]$$

NOTE: If population Variance are not known, we can use an unbiased estimator of $V(\bar{y})$ >> $\hat{V}(\bar{y}) = \frac{N-n}{N}\frac{s^2}{n}$.

 # Simple random sample

Example 3.2

The height (in cm) of 6 students of M.Sc., majoring in statistics, from Punjab Agricultural University, Ludhiana was recorded during 1985. The data, so obtained, are given below:

Table 3.1 Heights of M.Sc. students

Student	Name	Height
1	Sarjinder Singh	168
2	Gurmeet Singh	175
3	Varinder Kumar	185
4	Sukhjinder Singh	173
5	Devinder Kumar	171
6	Gulshan Kumar	172

1. Calculate (a) population mean $\bar{Y}$, and (b) population variance σ^2
2. Enumerate all possible SRS with replacement samples of size n =2. Obtain sampling distribution of mean, and hence show that

$a)\ \ E(\bar{y}) = \bar{Y} = \mu$ $\qquad b)\ V(\bar{y}) = \frac{\sigma^2}{n}$ $\qquad c)\ E(S^2) = \sigma^2$

Solution:

(1) Here N=6, and the study variable height is denoted by y.

population mean $\bar{Y} = \mu = \frac{\sum_{i=1}^{n} Y_i}{N} = \frac{1044}{6} = 174$

population variance $\sigma^2 = \frac{\sum_{i=1}^{n} (Y_i - \mu)^2}{N} = 28.667$

(2) Number of all possible samples of size 2, in case of WR sampling, will be $N^n = 6^2 = 36$.

Table 3.3 All possible WR samples and other related statistics

Sample	Sample units	Height of students	$\bar{y}$	s^2
1	(1, 1)	(168, 168)	168.0	0
2	(1, 2)	(168, 175)	171.5	24.5
3	(1, 3)	(168, 185)	176.5	144.5
4	(1, 4)	(168, 173)	170.5	12.5
5	(1, 5)	(168, 171)	169.5	4.5
6	(1, 6)	(168, 172)	170.0	8.0
7	(2, 1)	(175, 168)	171.5	24.5
8	(2, 2)	(175, 175)	175.0	0
9	(2, 3)	(175, 185)	180.0	50.0
10	(2, 4)	(175, 173)	174.0	2.0
11	(2, 5)	(175, 171)	173.0	8.0
12	(2, 6)	(175, 172)	173.5	4.5
13	(3, 1)	(185, 168)	176.5	144.5
14	(3, 2)	(185, 175)	180.0	50.0
15	(3, 3)	(185, 185)	185.0	0
16	(3, 4)	(185, 173)	179.0	72.0
17	(3, 5)	(185, 171)	178.0	98.0
18	(3, 6)	(185, 172)	178.5	84.5
19	(4, 1)	(173, 168)	170.5	12.5
20	(4, 2)	(173, 175)	174.0	2.0
21	(4, 3)	(173, 185)	179.0	72.0
22	(4, 4)	(173, 173)	173.0	0
23	(4, 5)	(173, 171)	172.0	2.0
24	(4, 6)	(173, 172)	172.5	.5
25	(5, 1)	(171, 168)	169.5	4.5
26	(5, 2)	(171, 175)	173.0	8.0
27	(5, 3)	(171, 185)	178.0	98.0
28	(5, 4)	(171, 173)	172.0	2.0
29	(5, 5)	(171, 171)	171.0	0
30	(5, 6)	(171, 172)	171.5	.5
31	(6, 1)	(172, 168)	170.0	8.0
32	(6, 2)	(172, 175)	173.5	4.5
33	(6, 3)	(172, 185)	178.5	84.5
34	(6, 4)	(172, 173)	172.5	.5
35	(6, 5)	(172, 171)	171.5	.5
36	(6, 6)	(172, 172)	172.0	0
Total			6264	1032

$$S_i^2 = \frac{(\sum y_i - \bar{y}_i)^2}{n-1}$$

Table 3.4 Sampling distribution of mean

Serial No.	Sample mean ($\bar{y}$)	Frequency (f)	Probability (p)
1	168.0	1	1/36
2	169.5	2	2/36
3	170.0	2	2/36
4	170.5	2	2/36
5	171.0	1	1/36
6	171.5	4	4/36
7	172.0	3	3/36
8	172.5	2	2/36
9	173.0	3	3/36
10	173.5	2	2/36
11	174.0	2	2/36
12	175.0	1	1/36
13	176.5	2	2/36
14	178.0	2	2/36
15	178.5	2	2/36
16	179.0	2	2/36
17	180.0	2	2/36
18	185.0	1	1/36
Total		36	1

(a) Average of all possible sample means, denoted by $E(\bar{y})$, is obtained from column (4) of table 3.3 as

$$E(\bar{y}) = \frac{1}{36}(168 + 171.5 + \cdots + 172) = \frac{6264}{36} = 174 \quad , \quad therfore \;\; E(\bar{y}) = \bar{Y}$$

Hence $\bar{y}$ is an unbiased estimator of $\bar{Y}$

(b) The variance of all possible sample means is given by

$$V(\bar{y}) = \frac{1}{36}\sum_{i=1}^{36}(\bar{y} - E(\bar{y}))^2 = 14.333$$

Now from part (b) of (1), we have

$$V(\bar{y}) = \frac{\sigma^2}{n} = \frac{28.667}{2} = 14.333 \quad , therfore\ the\ relation\ (b)\ is\ \ verified.$$

(c) From table 3.3, we find that the average of all possible sample mean squares is

$$E(S^2) = \frac{1}{36}(0 + 24 + \cdots + 0) = \frac{1032}{36} = 28.667$$

which equals population variance $E(S^2) = \sigma^2$, therefore the relation is verified.

Hence S^2 is an unbiased estimator of σ^2

Example 3.3

From the data given in example 3.2, enumerate all the SRS without replacement samples of size n=2 , then solve the following part

1- write down sampling distribution of mean, Using this distribution show that

$a)\ E(\bar{y}) = \bar{Y} = \mu$ $\quad b)\ V(\bar{y}) = \frac{N-n}{N}\frac{S^2}{n}$ $\quad c)\ E(s^2) = S^2$

height (in cm) of 6 students

2- From each sample, find the 95% confidence limits for the population mean of the student height with the **known** population variance **and** its estimates; use the normal deviate Z =1.96 in both cases.

3- For both the procedures in part (2), find the proportion of the confidence intervals enclosing the actual population mean, that is, the coverage probability.

Solution:

Number of possible **WOR** samples of size n=2 will be $C_2^6 = \binom{6}{2} = \frac{6!}{2!(6-2)!} = 15.$

The heights of students included in various possible samples, along with sample means ($\bar{y}$) and sample mean squares (S^2), are given in table 3.5.

Table 3.5 All possible WOR samples and other related statistics

Serial No.	Sample units	Height of students	$\bar{y}$	s^2
1	(1 , 2)	(168 , 175)	171.5	24.5
2	(1 , 3)	(168 , 185)	176.5	144.5
3	(1 , 4)	(168 , 173)	170.5	12.5
4	(1 , 5)	(168 , 171)	169.5	4.5
5	(1 , 6)	(168 , 172)	170.0	8.0
6	(2 , 3)	(175 , 185)	180.0	50.0
7	(2 , 4)	(175 , 173)	174.0	2.0
8	(2 , 5)	(175 , 171)	173.0	8.0
9	(2 , 6)	(175, 172)	173.5	4.5
10	(3 , 4)	(185 , 173)	179.0	72.0
11	(3 , 5)	(185 , 171)	178.0	98.0
12	(3 , 6)	(185 , 172)	178.5	84.5
13	(4, 5)	(173 , 171)	172.0	2.0
14	(4 , 6)	(173 , 172)	172.5	.5
15	(5 , 6)	(171, 172)	171.5	.5
Total			2610	516

Column (4) in table 3.5 lists mean values for all possible samples. It can also be written in the form of a *sampling distribution* as shown in table 3.6. Probability (p) values, in

this case also, are calculated in the same way as in table 3.4.

Table 3.6 Sampling distribution of mean

Serial No.	Sample mean ($\bar{y}$)	Frequency (f)	Probability (p)
1	169.5	1	1/15
2	170.0	1	1/15
3	170.5	1	1/15
4	171.5	2	2/15
5	172.0	1	1/15
6	172.5	1	1/15
7	173.0	1	1/15
8	173.5	1	1/15
9	174.0	1	1/15
10	176.5	1	1/15
11	178.0	1	1/15
12	178.5	1	1/15
13	179.0	1	1/15
14	180.0	1	1/15
Total		15	1

(a) The average of all possible 15 sample means is given as

$$E(\bar{y}) = \frac{1}{15}(171.5 + 176.5 + \cdots + 171.5) = \frac{2610}{15} = 174$$

which is same as the population mean worked out in example 3.2. Hence $E(\bar{y}) = \bar{Y} = \mu$

(b) The variance of $\bar{y}$ is given by

$$V(\bar{y}) = \frac{1}{15}\sum_{i=1}^{15}\left(\bar{y}_i - E(\bar{y})\right)^2$$

$$= \frac{1}{15}[(171.5 - 174)^2 + (176.5 - 174)^2 + \cdots + (171.5 - 174)^2]$$

$$= 11.467$$

$$S^2 = \frac{\sum_{i=1}^{n}(Y_i - \mu)^2}{N-1} = 34.4 \quad (population\ mean\ square)$$

Also, $\frac{N-n}{N}\frac{S^2}{n} = \frac{6-2}{6}\frac{34.4}{2} = 11.467$

This verifies the relation $V(\bar{y}) = \frac{N-n}{N}\frac{S^2}{n}$

(c) The average of all the 15 sample mean squares from table 3.5 is

$$E(s^2) = \frac{1}{15}(24.5 + 144.5 + \cdots + 5) = \frac{516}{15} = 34.4$$

Also, $S^2 = 34.4$

This verifies the relation $E(s^2) = S^2$, Which means s^2 is an unbiased estimator of S^2.

By use R Program:

```
population=c("a","b","c","d","e","f")
# select one sample(wor) of size n=2
sample(population,2,replace= FALSE)

[1] "b" "e"

# obtain all the 15 samples(Generate all combinations of size 2).
samples=combn(population,2,simplify = TRUE )
samples

   [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
[1,] "a"  "a"  "a"  "a"  "a"  "b"  "b"  "b"  "b"  "c"   "c"   "c"   "d"   "d"
[2,] "b"  "c"  "d"  "e"  "f"  "c"  "d"  "e"  "f"  "d"   "e"   "f"   "e"   "f"
   [,15]
[1,] "e"
[2,] "f"

# we obtain a matrix of student Height (cm).
y=c(168,175,185,173,171,172)
# to obtain population mean
Ybar=mean(y)
Ybar

[1] 174

# to obtain population standard deviation S.
#The denominator n - 1 is used which gives an unbiased estimator of the
variance.
S=sd(y)
S

[1] 5.865151

var.population=var(y)
var.population

[1] 34.4

# to obtain all the sample values y1,y2 we write
samples=combn(y,2)
samples

   [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
[1,]  168  168  168  168  168  175  175  175  175   185   185   185  173  173
[2,]  175  185  173  171  172  185  173  171  172   173   171   172   171 172
     [,15]
[1,]   171
[2,]   172

# to obtain all the 15 sample means ybar
averages=apply(samples,2,mean) #for a matrix 1 indicates rows,2 indicates
```

```
columns
averages

 [1] 171.5 176.5 170.5 169.5 170.0 180.0 174.0 173.0 173.5 179.0 178.0 178.5
[13] 172.0 172.5 171.5

#to obtain the average of all possible 15 sample means
mean(averages)

[1] 174   #which is same as the population mean

# to obtain all the 15 confidence interval for the population mean
##case 1: we use population standard deviation S to compute standard error

S.pop=sd(y)
se=sqrt((6-2)/(6*2))*S.pop
CI1=averages-1.96*se
CI1

[1] 164.863 169.863 163.863 162.863 163.363 173.363 167.363 166.363 166.863
[10] 172.363 171.363 171.863 165.363 165.863 164.863

CI2=averages+1.96*se
CI2

[1] 178.137 183.137 177.137 176.137 176.637 186.637 180.637 179.637 180.137
[10] 185.637 184.637 185.137 178.637 179.137 178.137

# coverage probability=100%

## case 2: we use sample standard deviation s to compute standard error
s=apply(samples,2,sd)
se=sqrt((6-2)/(6*2))*s
CI1=averages-1.96*se
CI1

[1] 165.8988 162.8972 166.4992 167.0995 166.7993 171.9983 172.3997 169.7993
 [9] 171.0995 169.3980 166.7977 168.0978 170.3997 171.6998 170.6998

CI2=averages+1.96*se
CI2

[1] 177.1012 190.1028 174.5008 171.9005 173.2007 188.0017 175.6003 176.2007
 [9] 175.9005 188.6020 189.2023 188.9022 173.6003 173.3002 172.3002

# coverage probability=10/15=67%
```

Appendix B Random numbers

Column numbers									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3436	6833	5809	9169	5081	5655	6567	8793	6830	1332
6133	4454	2675	3558	7624	5736	2184	4557	0496	8547
9853	3890	5535	3045	9830	5455	8218	9090	7266	4784
5807	5692	6971	6162	6751	5001	5533	2386	0004	2855
6291	0924	1298	7386	5856	2167	8299	9314	0333	8803
4725	9516	8555	0379	7746	9647	2010	0979	7115	6653
7697	6486	3720	6191	3552	1081	6141	7613	5455	3731
3497	2271	9641	0304	4425	6776	1205	2953	5669	1056
8940	4765	1641	0606	4970	7582	7991	6480	2946	5190
1122	6364	5264	1267	4027	4749	0338	8406	1213	5355
4333	0625	3947	1373	6372	9036	7046	4325	3491	8989
7685	1550	0853	4276	1572	9348	6893	2113	8285	9195
0592	8341	4430	0496	9613	2643	6442	0870	5449	8560
3506	0774	0447	7461	4459	0866	1698	0184	4975	5447
8368	2507	3565	4243	6667	8324	3063	8809	4248	1190
2630	1112	6680	4863	6813	4149	8325	2271	1963	9569
3883	3897	1848	8150	8184	1133	6088	3641	6785	0658
1123	3943	5248	0635	9265	4052	1509	1280	0953	9107
1167	9827	4101	4496	1254	6814	2479	5924	5071	1244
7831	0877	3806	9734	3801	1651	7169	3974	1725	9709
2487	9756	9886	6776	9426	0820	3741	5427	5293	3223
1245	3875	9816	8400	2938	2530	0158	5267	4639	5428
5309	4806	3176	8397	5758	2503	1567	5740	2577	8899
7109	0702	4179	0438	5234	9480	9777	2858	4391	0979
8716	7177	3386	7643	6555	8665	0768	4409	3647	9286
9499	5280	5150	2724	6482	6362	1566	2469	9704	8165
3125	4552	6044	0222	7520	1521	8205	0599	5167	1654
3788	6257	0632	0693	2263	5290	0511	0229	5951	6808
2242	2143	8724	1212	9485	3985	7280	0130	7791	6272
0900	4364	6429	8573	9904	2269	6405	9459	3088	6903
7909	4528	8772	1876	2113	4781	8678	4873	2061	1835
0379	2073	2680	8258	6275	7149	6858	4578	5932	9582
0780	6661	0277	0998	0432	8941	8946	9784	6693	2491
8478	8093	6990	2417	0290	5771	1304	3306	8825	5937
2519	7869	9035	4282	0307	7516	2340	1190	8440	6551
2472	0823	6188	3303	0490	9486	2896	0821	5999	3697
8418	5411	9245	0857	3059	6689	6523	8386	6674	7081
8293	5709	4120	5530	8864	0511	5593	1633	4788	1001
9260	1416	2171	0525	6016	9430	2828	6877	2570	4049
6568	1568	4160	0429	3488	3741	3311	3733	7882	6985
6694	5994	7517	1339	6812	4139	6938	8098	6140	2013
2273	6882	2673	6903	4044	3064	6738	7554	7734	7899
6364	5762	0322	2592	3452	9002	0264	6009	1311	5873
6696	1759	0563	8104	5055	4078	2516	1631	5859	1331
3431	2522	2206	3938	7860	1886	1229	7734	3283	8487
4842	3765	3484	2337	0587	9885	8568	3162	3028	7091
8295	9315	5892	6981	4141	1606	1411	3196	9428	3300
4925	4677	8547	5258	7274	2471	4559	6581	8232	7405
5439	0994	3794	8444	1043	4629	5975	3340	3793	6060
2031	0283	3320	1595	7953	2695	0399	9793	6114	2091

Sources : Rao, C.R., Mitra, S.K., Matthai, A. and Ramamurthy, K.G. (1974). *Formulae and Tables for Statistical Work*. Statistical Publishing Society. Indian Statistical Institute Calcutta.

Appendix B continued...

Column numbers									
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
9787	3792	5241	0556	7070	0786	7431	7157	8539	4118
4479	1397	8435	3542	8435	6169	7996	3314	1299	1935
0191	2800	1056	2735	4816	1979	0042	5824	6636	2332
8710	6903	1347	9332	6962	6786	9875	7565	8683	6490
4656	5960	0812	5144	5355	3335	4784	7573	3841	4255
9974	9230	8049	4971	7555	3935	9405	8545	4329	5358
8493	7128	3654	8976	1901	5496	3453	7539	3255	6742
6135	6954	3436	3841	9009	3768	9256	3631	9066	7153
1217	2748	3864	4752	7407	9975	6372	3308	0000	4734
2623	1282	4389	8889	0764	2328	2140	8843	4986	4413
1144	5336	4426	9003	6956	9406	8464	8827	3143	4754
5854	9981	9079	2908	4755	4620	6455	6793	7539	4031
0615	8188	2812	0270	5733	5339	1175	2919	7343	0477
3624	0853	3128	7952	2678	3011	7710	9734	6386	8400
1185	6832	4918	9236	3026	5795	0352	7533	4435	0306
7391	3210	9540	4085	9234	4892	3962	3883	4538	8286
7195	1986	6146	0946	5421	8430	2128	7602	5609	7064
6137	7286	5283	0609	0941	4935	2521	7937	2153	2629
7401	8099	7482	2210	3662	8253	7507	7809	0094	4401
0192	9452	7189	9552	7498	0105	8295	9762	7434	3518
3621	3037	2274	3803	0946	9874	4911	6797	1227	8494
2661	0047	6628	6199	2526	5631	8334	7668	3994	7439
8072	5085	3576	4939	0352	7386	7690	7108	6668	8246
0839	5224	9768	3839	8495	1668	6957	7031	2032	1468
2354	9266	8034	3813	3648	7825	6156	3605	7796	1645
9050	6800	0490	3261	7748	3609	1050	0591	3799	2827
7174	7703	1540	8001	6230	0387	9553	7447	0240	2511
3465	7017	2278	0357	5800	1048	8382	8800	7608	4325
8805	1265	5202	6872	3282	5331	5398	1426	2805	2110
0250	4100	5263	8506	9848	2451	2031	2026	8661	4163
6088	8366	7751	1577	9534	2458	1886	1522	4161	8726
8833	3449	3499	4223	2854	6855	4042	1294	1728	5494
4675	2535	1915	9783	9754	2790	6856	0352	9628	8342
8990	4993	2922	8842	9904	8442	0105	3308	3320	6361
1790	8590	5792	0983	3494	0945	4966	2194	9823	2599
9276	3967	2486	6242	3276	1884	1847	8922	7356	1528
2965	7991	3777	9303	0536	1517	0570	7212	7593	0566
6620	4234	8407	6890	6904	8599	5876	2608	7320	6117
4706	8319	6252	3177	9108	3069	0910	8241	9842	0895
8395	3882	0259	2092	4885	3434	0879	0000	0790	0735
3991	3406	0151	2594	9137	9924	2393	7699	6116	9655
9644	6763	3512	0139	4119	2722	3219	0070	3830	7997
3658	7813	0207	0357	8225	4497	2435	5121	4776	3611
5728	1882	9120	7893	3503	8579	9070	1952	8390	5517
6221	6366	8192	8429	4387	5484	7553	4053	9458	2292
7635	5248	1750	0868	0173	4989	2300	3916	6732	8284
4368	3113	5887	8439	0026	1902	4114	3127	5140	6684
8635	9723	2550	8216	7531	7732	3963	4014	2099	3030
3304	3254	3936	9361	9771	8255	4592	8808	3803	4010
9336	5666	1349	1932	7326	2151	1573	3045	8746	8059

Appendix B continued...

Column numbers									
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
8094	7747	6006	2536	8856	8171	0291	2603	4675	0779
3745	6766	7221	9560	7036	4520	4584	5714	8122	5029
0835	3641	1638	3464	1767	7664	6247	8362	1257	5265
1601	1143	7272	3988	8356	9477	5870	6425	1725	9792
7797	4121	9603	5723	4630	5549	0593	5761	7200	7227
7620	8310	9500	7116	6259	7619	3749	9121	2185	9335
2096	5270	0793	3950	2722	0925	5792	1040	5806	9636
6803	7016	1055	6396	7754	3591	2613	5325	7485	2406
3566	9310	2604	8607	4765	2237	1222	3947	1228	2708
6428	0086	6245	3247	5707	7847	6127	0857	8229	5609
8633	2617	9176	9602	4807	7269	6131	8780	3417	7278
6632	8056	1091	9158	7303	4084	9096	4047	6775	0876
2612	7936	1453	4812	1742	7128	3636	6561	7522	0359
9436	1681	0851	3488	8815	5301	5403	5456	0501	4511
0418	2487	5583	9032	6507	8554	0346	6251	3577	4146
6853	3757	0171	5943	1145	3434	0188	5665	7779	7179
8347	7044	4640	6832	2445	4872	7870	2335	2874	9393
5182	6263	1224	9863	6751	0084	8827	9479	8342	0053
9215	3992	4874	8082	5959	2861	4574	5813	5903	7161
5588	3456	9602	5260	6578	8618	0340	3381	7579	6359
3996	0415	7015	9210	0974	0319	2699	8036	1090	3805
7346	9400	3292	8165	3206	7035	5227	7340	8515	4225
8621	4185	6727	2770	1227	3696	6496	4889	2697	3316
9399	5575	1562	5821	9824	4909	0348	8735	3604	9959
4334	0347	4893	2025	5590	8126	8571	2532	9355	7563
8091	0536	6522	5409	1463	0138	0384	6711	2384	0072
9627	3311	2010	2525	3142	9700	2196	4076	3710	3372
0086	3501	4916	2511	1274	1775	8324	9646	0611	1048
3753	0174	7934	3483	9210	9163	4714	7888	3577	6596
2740	3239	3054	9991	3778	3195	1040	2022	3193	9196
3919	6871	5685	8147	7310	2080	4196	3375	5700	7967
4577	7897	2757	5992	7398	7687	8415	1595	9636	4605
0215	7254	5378	3861	3448	9494	5221	1325	7317	1022
5807	7948	1774	6836	1786	2392	2820	8533	0629	3771
1910	9653	1214	3921	5298	8334	2352	7113	2291	9312
3990	1310	9338	2601	5571	1424	7850	4531	0133	5519
5967	8941	7987	3335	7579	9735	3042	8409	7053	5364
5872	1143	9183	6911	2247	1559	4888	7198	9249	1395
7240	1827	3281	0705	4479	5598	9985	8170	3367	6928
2268	4227	5844	0700	6907	9668	6670	0097	0686	6311
8515	1611	1327	6671	2765	0081	0554	3716	9334	3027
4324	8348	8870	4802	9655	2852	3858	3225	5022	3602
9053	8503	8222	6850	6100	5973	1522	2690	1396	0632
5133	7618	3211	0898	5343	9981	8936	0819	9112	2548
6235	9463	0097	1332	6038	3822	1119	7143	1708	5668
6048	1376	1589	4274	2920	3521	7661	9435	9257	9276
6341	0636	3355	7245	4160	1672	2295	4730	0984	6813
2143	0207	9733	8136	9118	0143	0949	1733	7986	5670
7336	3277	2135	3300	5287	0134	7104	9359	5069	3893
2728	6464	4721	8192	5485	7935	4996	3475	9523	5514

Appendix B continued...

Column numbers									
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
6415	5554	3592	8008	9408	2092	9842	3197	1404	1505
4668	3479	4073	6941	8286	3374	3696	7856	8980	0359
7592	3903	7895	1113	7646	9201	9081	2630	1617	1188
2012	1096	2958	4788	4882	1855	8190	9726	6716	1384
7884	8004	7831	8264	0028	8118	5011	5704	9394	7669
5510	8160	6173	5655	4415	0147	1091	4426	2843	5578
4440	0095	4067	9078	6205	7488	1851	3537	7191	0856
8436	4936	3013	6818	1577	0249	5107	5304	3872	4157
3740	3172	2775	5781	0318	8932	9220	3784	0501	8375
1174	3869	9985	4443	1127	7390	1463	8524	2272	4275
8494	5214	9020	4568	3508	1257	9685	6310	9763	1887
8792	6689	3521	4407	2017	8527	2230	1851	4023	2258
0865	4556	4015	0082	1239	7058	1189	3174	0220	1167
7141	0799	4764	5283	4291	4822	3735	1393	2477	6782
7185	3986	7047	9210	2791	7610	7264	4771	0548	5172
3672	8714	8853	9825	5869	6281	2371	1890	9480	2968
7753	9791	3436	4604	7991	5222	9280	1584	7141	0221
9332	5082	8900	4209	4117	8644	8712	7337	1689	8793
0759	2206	4220	2394	4346	8483	6968	2344	1902	0848
8493	6032	3585	2162	6301	4929	7087	2907	2690	5039
6776	2659	7323	9619	7727	6460	6745	1051	7662	7512
5135	7118	4458	1394	0526	5121	2062	0977	7338	5744
7714	3485	5412	0716	6914	8192	6483	1946	4271	0995
9777	1915	1183	3177	6568	6698	4649	3899	2691	4413
7960	4876	8841	3538	4519	0872	5860	8181	5777	0233
1714	4061	6365	7480	9312	1139	0715	0571	2575	5990
7460	0288	1075	3483	1041	5427	6457	0985	1657	8742
0275	8595	0812	9021	4808	8247	0089	7034	8719	5878
7735	0399	3931	3135	1585	7292	8362	4006	1184	9676
8661	9964	9969	2444	6095	2003	9320	2837	4397	0297
1273	7133	4874	1100	7854	4596	6787	8574	6098	5526
7784	9159	6674	3243	2531	6093	8906	8855	8614	2781
0707	0067	6433	6058	4381	0146	1186	9913	3668	6347
9594	8627	5507	2956	6166	7271	9511	5069	1022	9889
6690	2781	1790	9596	6472	8774	9058	7915	3647	3525
3476	7990	0690	0043	1357	9568	1541	3726	9223	4385
9994	1061	7951	3010	6997	4759	0473	2848	7504	6904
8308	8100	7244	4206	7766	6916	6866	4064	6714	1805
7260	8057	8779	6368	0601	1872	3160	8731	3646	2789
4755	3425	1299	7990	8366	1368	3611	8864	1341	9349
7156	7190	6054	3489	8939	9089	2637	9180	3991	7161
1469	1763	1918	2547	7708	1900	1665	1860	3078	7851
1270	4109	9428	0933	1444	7467	1771	3482	1497	6492
5485	7802	3094	7249	3901	2827	8294	1329	7170	1758
7123	0850	6297	5479	1416	1837	9305	3749	8541	5161
2187	4696	2470	7234	4809	5408	3266	6252	5987	5794
7595	1895	6183	2013	4399	5255	6714	1839	6132	2653
3021	1523	2005	2009	9631	1274	9902	4203	8312	9572
3317	8741	2688	9392	0136	9293	7815	1781	1990	4057
6711	3947	5004	2625	5105	0116	1895	6729	3159	6492

How to use calculator

Mode (3:stat) ___ Then ___(1: 1- VAR)

يظهر جدول لإدخال البيانات

Example: 2, 4,6, 9

2 = , 4= , 6 = , 9=

نضغط (AC)

To find mean , standard deviation and variance

نضغط

Shift (1)

وتختار الرقم

5:Var (or 4:Var)

ويظهر في الشاشة

1:n	2: $\overline{X}$
3: σ_n (or σ_x)	4: σ_{n-1} (or S_x)

*** ملاحظة : بين كل خطوة وخطوة نضغط AC *******

For mean : Push 2

For Sample standard deviation :Push 4

For Population standard deviation :Push 3

*To find sample variance = square (Sample standard deviation)

* To find Population variance = square (Population standard deviation)