Q.5 A random variable V has the distribution function

$$F(v) = \begin{cases} 0 & \text{for } v < 0, \\ 1 - (1 - v)^A & \text{for } 0 \le v \le 1, \\ 1 & \text{for } v > 1, \end{cases}$$

where $A > 0$ is a parameter. Determine the density function, mean, and variance.

$$\int_a^b f = -\int_b^a f$$

$$f(v) = A(1 - v)^{A-1}$$

$$E(v) = \int_0^1 vA\,(1 - v)^{A-1} dv$$

بالتعويض بـ u
u=1-v   v=1-u
dv=-du

$$= A\int_0^1 (1 - u)u^{A-1} du \quad =$$

$$> \; A\int_0^1 u^{A-1} - u^A du \; => \; A\left[\frac{u^A}{A} - \frac{u^{A+1}}{A+1}\right]_0^1$$

$$E(v) = 1 - \frac{A}{A+1}$$

$$E(v^2) = \int_0^1 v^2 A\,(1 - v)^{A-1} dv$$

بالتعويض بـ u
u=1-v   v=1-u
dv=-du

$$= A\int_0^1 (1 - u)^2 u^{A-1} du \; => \; A\int_0^1 (1 - 2u + u^2)u^{A-1} du$$

$$=> \; A\int_0^1 (u^{A-1} - 2u^A + u^{A+1}) du$$

$$=> \; A\left[\frac{u^A}{A} - \frac{2u^{A+1}}{A+1} + \frac{u^{A+2}}{A+2}\right]_0^1$$

$$E(v^2) = 1 - \frac{2A}{A+1} + \frac{A}{A+2}$$

$$V(v) = E(v^2) - \left(E(v)\right)^2 => 1 - \frac{2A}{A+1} + \frac{A}{A+2} - (1 - \frac{A}{A+1})^2$$

Q.6 Determine the distribution function, mean, and variance corresponding to the triangular density.

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1, \\ 2 - x & \text{for } 1 \le x \le 2, \\ 0 & \text{elsewhere.} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x \le 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x \le 2 \\ 1 & 2 < x \end{cases}$$

$$E(x) = \int_0^1 x^2 dx + \int_1^2 x(2-x)dx = \frac{1}{3} + \frac{2}{3} = 1$$

$$E(x^2) = \int_0^1 x^3 dx + \int_1^2 x^2(2-x)dx = \frac{1}{4} + \frac{11}{12} = \frac{7}{6}$$

$$V(x) = E(x^2) - \left(E(x)\right)^2 = \frac{1}{6}$$

OR

$$V(x) = \int_0^1 (x-1)^2 x\, dx + \int_1^2 (x-1)^2(2-x)dx = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

Q.7 Suppose X is a random variable with finite mean μ and variance σ^2, and $Y = a + bX$ for certain constants $a, b \neq 0$. Determine the mean and variance for Y.

$$E(Y) = E(a + bX) = a + bE(X)$$
$$V(Y) = V(a + bX) = b^2V(X)$$

Q.9 Random variables X and Y are independent and have the probability mass functions

$$p_X(0) = \tfrac{1}{2}, \quad p_Y(1) = \tfrac{1}{6},$$
$$p_X(3) = \tfrac{1}{2}, \quad p_Y(2) = \tfrac{1}{3},$$
$$p_Y(3) = \tfrac{1}{2}.$$

Determine the probability mass function of the sum $Z = X + Y$.

X	Y	Z	P(Z)=P(X)+P(Y)
0	1	1	$\frac{1}{2} * \frac{1}{6} = \frac{1}{12}$
	2	2	$\frac{1}{2} * \frac{1}{3} = \frac{1}{6}$
	3	3	$\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$
3	1	4	$\frac{1}{2} * \frac{1}{6} = \frac{1}{12}$
	2	5	$\frac{1}{2} * \frac{1}{3} = \frac{1}{6}$
	3	6	$\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

Z	1	2	3	4	5	6
P(Z)	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$

Q.10 Random variables U and V are independent and have the probability mass functions

$$p_U(0) = \tfrac{1}{3}, \quad p_V(1) = \tfrac{1}{2},$$
$$p_U(1) = \tfrac{1}{3}, \quad p_V(2) = \tfrac{1}{2}.$$
$$p_U(2) = \tfrac{1}{3},$$

Determine the probability mass function of the sum $W = U + V$.

U	V	W	P(W)=P(U)+P(V)
0	1	1	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$
1		2	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$
2		3	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$
0	2	2	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$
1		3	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$
2		4	$\frac{1}{2}*\frac{1}{3}=\frac{1}{6}$

Z	1	2	3	4
P(Z)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

Q.12 A fair die is rolled 10 times.

n=10 P=0.5 $\Omega = \{1,2,3,4,5,6\}$ x: An even number will show => {2,4,6}

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

a- What is the probability that the rolled die will not show an even number?

$$P(X = 0) = \binom{10}{0} (0.5)^0 (0.5)^{10-0} =$$

b- What is the probability that the rolled die will show an even number7 times?

$$(X = 7) = \binom{10}{7} (0.5)^7 (0.5)^{10-7} =$$

c-What is the probability that the rolled die will show an odd number3 times?

$$(X = 7) = \binom{10}{7}(0.5)^7(0.5)^{10-7} =$$

d-What is the probability that the rolled die will show an odd number 3,5,7,9 times?
e-What is the probability that the rolled die will show an odd number one times?

$$(X = 9) = \binom{10}{9}(0.5)^9(0.5)^{10-9} =$$

f-What is the probability that the rolled die will show just an even number?

$$(X = 10) = \binom{10}{10}(0.5)^9(0.5)^{10-9} =$$

g-What is the probability that the rolled die will show an even number 5 times?

$$(X = 5) = \binom{10}{5}(0.5)^{10}(0.5)^{10-10} =$$

Q.13. Suppose that five fair coins are tossed independently.

n=5 P=0.5 $\Omega = \{H, T\}$ x: AHead will show => {H}

$$\mathrm{P(X = x)} = \binom{n}{x} p^x q^{n-x}$$

a- What is the probability that exactlyone of the coins will be different from the remaining four?

$$\mathrm{P(X = 1) + P(X = 4)} = \binom{5}{1}(0.5)^1(0.5)^{5-1} + \binom{5}{4}(0.5)^4(0.5)^{5-4}$$

b-What is the probability that Head will show 3 times?

$$\mathrm{P(X = 3)} = \binom{5}{3}(0.5)^3(0.5)^{5-3}$$

c-What is the probability that Head will show at least 3 times?

$$P(X \geq 3) =$$

d-What is the probability that Head will show at most 3 times?

$$P(X \leq 3) =$$

e-What is the probability that Head will show less than 3 times?

$$P(X < 3) =$$

f-What is the probability that Head will show more than 3 times?

$$P(X > 3)$$

Q1. Suppose two dice are tossed (for each die, it is equally likely that 1, 2, 3, 4, 5, or 6 dots will show).

a) What is the probability that the total of the two dice will add up to 7 or 11?

b) What is the probability that the total of the two dice will add up to a number other than 2 or 12?

c) Are the events
E1 = first die shows a 3
E2 = total of the two dice is 6
Independent events?

d) Are the events
E1 = first die shows a 3
E2 = total of the two dice is 7
independent events?

e) Given that the total of the two dice is 5, what is the probability that the first die showed 2 dots?

f) Given that the first die shows 5, what is the probability that the total of the two dice is even?

Q2. A desk contains three drawers. Drawer 1 contains twogold coins. Drawer 2 contains one gold coin and one silvercoin. Drawer 3 contains two silver coins. I randomly choosea drawer and then randomly choose a coin. If a silver coinis chosen, what is the probability that I chose drawer 3?