

Solve the following differential equations, also find singular solutions (if any):

1.  $\frac{dy}{dx} = \frac{4xy+3y+4x+3}{2y-3xy-3x+2}$ . Ans:  $y = -\frac{4}{3}x - \frac{17}{9} \ln|2-3x| + c$ , Singular solution  $y = -1$

2.  $(5xy - 10x + 3y - 6)dx = (2 - x)dy$ . Ans:  $y - 2 = c \frac{e^{-5x}}{(2-x)^{13}}$ , Singular solution  $y = 2$

3.  $(x-2y+3)\frac{dy}{dx} = (4y-2x)$ . Ans:  $\frac{1}{5}(x-2y) + \frac{12}{25} \ln(5x - 10y + 3) = x + c$ , Singular solution is  $y = \frac{1}{10}(5x + 3)$

4.  $\frac{dy}{dx} = \frac{2}{\sqrt{y-2x+3}}$ . Ans:  $\frac{1}{2}y + \sqrt{y-2x+3} + \ln(1 - \sqrt{y-2x+3}) = c$ , Singular solution  $y = 2(x-1)$

5.  $(4-x+y)dy = (2-y+x)dx$ . Ans:  $x+y+3\ln(1-x+y) = c$ , Singular solution  $y = x-1$

6.  $\sqrt{x+y}\frac{dy}{dx} = \sqrt{1+x+y}$ .  
 Ans:  $\frac{(x+y)^2}{2} + \frac{1}{2}(x+y+\frac{1}{2})\sqrt{(x+y)^2 - \frac{1}{4}} + \frac{1}{8} \ln\left(x+y+\frac{1}{2} + \sqrt{(x+y)^2 - \frac{1}{4}}\right) = -x + c$

7.  $\frac{dy}{dx} = (y+2x)^{-1} e^{2x+y} - 2$ . Ans:  $-(2x+y+1) = (x+c)e^{(2x+y)}$

8.  $\frac{dy}{dx} = \frac{y-2x}{x+2y}$ . Ans:  $\ln(x^2+y^2) + \tan^{-1}\left(\frac{y}{x}\right) = c$

9.  $xdy = (y + \sqrt{xy})$ . Ans:  $2\sqrt{\frac{y}{x}} = \ln x + c$ , Singular solution  $y = 0$

10.  $y\frac{dy}{dx} = x + \sqrt{y^2 - x^2}$ . Ans:  $(y^2 - x^2) = (x - c)^2$ , Singular solutions  $y = \pm x$

11.  $(y + xe^{\frac{x}{y}})dy = ye^{\frac{x}{y}}dx$ . Ans:  $y = ce^{\frac{x}{y}}$ , Singular solution  $y = 0$

11. Solve the IVP  $xdy + y(\ln y - \ln x - 1)dx = 0$ ,  $y(1) = e$ . Ans:  $y = xe^{\frac{1}{x}}$ , Singular solution  $y = x$