

Solve the following differential equations, also find the interval on which the solution is defined:

1. $\frac{dy}{dx} = \frac{(xe^x - 2y)}{x}$. Ans: $x^2y = e^x(x^2 - 2x + 2) + c$, Interval $(-\infty, 0)$ or $(0, \infty)$.

2. $xdy = (4xe^x - y)dx$. Ans: $xy = 4e^x(x - 1) + c$, Interval $(-\infty, 0)$ or $(0, \infty)$.

3. $(x - 2y + 3)\frac{dy}{dx} = 1$. Ans: $(x - 2y + 1) = ce^y$

4. $\frac{dx}{d\theta} + r \csc \theta = \sin \theta$. Ans: $r(\cot \theta - \csc \theta) = \sin \theta - \theta + c$

5. $ydx = 2(x - y^3)dy$. Ans: $x = -2y^3 + cy^2$.

6. $x\frac{dy}{dx} + (2x + 1) = e^{-2x}$. Ans: $xye^{2x} = x + c$

7. Solve the IVP $(x^4 - 3y)dx = x(2 + 3 \ln x)dy$, $y(\sqrt{2}) = 0$, Ans: $y(2 + 3 \ln x) = \frac{x^4}{4} - 1$, Interval $(e^{-\frac{2}{3}}, \infty)$.

8. Solve the IVP $(\frac{x^2}{y} + \frac{2y}{3})dx = (3 - 4 \ln x)dy$, $y(1) = 0$, Ans: $3y^2(3 - 4 \ln x) = 2(x^3 - 1)$.

9. $3xy^2(2 + \ln x)\frac{dy}{dx} = 3x^3 - y^3$, Ans: $y^3(2 + \ln x) = x^3 + c$.

10. $(y^2 + 1)dx = y \sec^2 x dy$, Ans: $2x + \sin 2x = 2 \ln(y^2 + 1) + c$.