Solve the following differential equations:

1. $((1+y)\cos(x+y)-x\sin(x+y))dx+(y\cos(x+y)+(1-x)\sin(x+y))dy=0.$
   Ans: $x\cos(x+y)+y\sin(x+y)=c$

2. $\left(\frac{x}{1+x^2+y^2}-x\right)dx=(y-\frac{y}{1+x^2+y^2})dy.$ Ans: $x^2+y^2=\tan\left(x^2+y^2+c\right)$

3. $\left(\frac{x}{x^2+y^2}-\frac{y}{\sqrt{x^2+y^2}}\right)dy=\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{y}{x^2+y^2}\right)dx.$ Ans: $y=x\tan\left(\sqrt{x^2+y^2}+c\right)$

4. $\left(e^{\frac{x}{y}}+\frac{y}{x}e^{\frac{y}{x}}-e^{\frac{y}{x}}\right)dx=\left(\frac{x}{y}e^{\frac{x}{y}}+e^{\frac{y}{x}}-e^{\frac{x}{y}}\right)dx.$ Ans: $ye^{\frac{x}{y}}-xe^{\frac{y}{x}}=c$

5. $(xy+y-\sin x)\,dx+\left(\frac{1}{2}x^2+x\right)dy=0.$ Ans: $\frac{1}{2}x^2y+xy+\cos x=c$

6. Show that the differential equaton

$$\left(\cos x+\sin x+\frac{\sin x-\cos x}{y}\right)dx=\left(\frac{\cos x-\sin x}{y}\right)dy$$

is not exact and that $\mu=xy$ is an integrating factor. Hence solve the differential equation. Ans: $xy=\frac{c}{\sin x-\cos x}$.

7. Show that the differential equaton

$$\left(xy^2-3x^3\right)dx+\left(3x^2y-x^4\right)dy=0,\quad y>0$$

is not exact and that $\mu=xy^{-1}$ is an integrating factor. Hence solve the differential equation. Ans: $xy^3-x^3y=c$.

8. Solve the differential equation

$$\left(\cos x-\frac{1}{x}\right)dy=\left(y\sin x-\frac{y}{x}\cos x+\frac{1}{x}\right)dx,\quad x>0$$

Ans: $xy\cos x=x+y+c$.

9. Solve the differential equation

$$\left(e^x+\frac{1-e^y}{y}\right)dx+\frac{1}{y}\left(e^x-xe^y+1\right)dy=0,\quad y>0$$

Ans: $ye^x-xe^y+x+y=c$.

10. Solve the differential equation

$$\left(\frac{e^y-ye^x}{x}\right)dx=\left(\frac{e^x-xe^y}{x}\right)dy,\quad x>0$$

Ans: $xe^y=ye^x+c$.