Find a Region of xy-plane so that the following IVP have unique solution on this region.

1.$(x^2-3x+2)\frac{dy}{dx} = \ln(1-y^2)$, $y(\frac{3}{2}) = 0$. Ans: $\mathcal{R} = \{(x, y) : 1 < x < 2, \quad -1 < y < 1\}$.

2. $(16-x^2-y^2)\frac{dy}{dx} = \sqrt{4+y^2}$, $y(2) = -2$. Ans: $\mathcal{R} = \big\{(x, y) : x^2 + y^2 < 16\big\}$.

3. $\sqrt{1-(x+y)}\frac{dy}{dx} = \ln(x-y^2)$, $y(\frac{1}{2}) = \frac{1}{3}$. Ans: $\mathcal{R} = \big\{(x, y) : x + y < 1, \quad y^2 < x\big\}$.

4. $(x^2-xy-x+y)\frac{dy}{dx} = \ln y$, $y(\frac{1}{2}) = \frac{1}{2}$. Ans: $\mathcal{R} = \{(x, y) : 0 < y < x, \quad x < 1\}$.

5. $ydy = \ln(1-x^2)dy$, $y(0) = 1$. Ans: $\mathcal{R} = \{(x, y) : -1 < x < 1, \quad y > 0\}$.

6. $x \tan y dy = y \tan x dx$, $y(1) = \frac{\pi}{4}$ Ans: $\mathcal{R} = \big\{(x, y) : 0 < y < \frac{\pi}{2}, \quad x > 0\big\}$.

7. $\sqrt{(y-x^2)}\frac{dy}{dx} = (4+x^2)$, $y(1) = 2$, Ans: $\mathcal{R} = \big\{(x, y) : y > x^2\big\}$.

8. $\sec^2 x dy = \ln(1-\frac{x^2}{4}-\frac{y^2}{2})dx$, $y(1) = 0$, Ans: $\mathcal{R} = \Big\{(x, y) : \frac{x^2}{4} + \frac{y^2}{2} < 1\Big\}$.

9. Does the existence theorem guarantee unique solution for the IVP

$$(x^2 - 3x + 2)\frac{dy}{dx} = \ln(1 - y^2), \quad y(1) = 1?$$

10. Does the existence theorem guarantee unique solution for the IVP

$$\sqrt{(y - x^2)}\frac{dy}{dx} = (4 + x^2)dy, \quad y(-1) = 1?$$