

Exercises for chapter 14

Exercises:

- I. In a study of the strength of the egg's shell for a sample of white chicken eggs and obtain the following frequencies:

Weak	Moderate	Strong
37	68	45

Using $\alpha = 0.05$,

- a) Test if the levels of strength of white egg shells occur with equal proportions.

Equal proportions $\Rightarrow p_1 = p_2 = p_3 = \frac{1}{3}$

	Weak	Moderate	Strong	Total
n_i	37	68	45	150
e_i	$n * p_1 = 150 * \frac{1}{3} = 50$	$n * p_2 = 150 * \frac{1}{3} = 50$	$n * p_3 = 150 * \frac{1}{3} = 50$	150

I. Hypotheses:

$H_0: p_1 = p_2 = p_3 = \frac{1}{3}$ vs. $H_a: \text{at least one } p_i \text{ is different}$

II. Test Statistic:

$$\chi^2 = \frac{(37 - 50)^2}{50} + \frac{(68 - 50)^2}{50} + \frac{(45 - 50)^2}{50} = 10.36$$

III. R.R:

Reject H_0 if $\chi^2 \geq \chi_{0.05,2}^2 = 5.992$

IV. Decision:

Since $\chi^2 = 10.36 \geq 5.992 = \chi_{0.05,2}^2$ we reject H_0 .

- b) Test if the proportions of the levels of strength are different from 1/4, 1/2, and 1/4 respectively.

	Weak	Moderate	Strong	Total
n_i	37	68	45	150
e_i	$n * p_1 = 150 * \frac{1}{4} = 37.5$	$n * p_2 = 150 * \frac{1}{2} = 75$	$n * p_3 = 150 * \frac{1}{4} = 37.5$	150

I. Hypotheses:

$$H_0: p_1 = \frac{1}{4}, p_2 = \frac{1}{2}, p_3 = \frac{1}{4} \quad \text{vs.} \quad H_a: \text{at least one } p_i \text{ is different}$$

II. Test Statistic:

$$\chi^2 = \frac{(37 - 37.5)^2}{37.5} + \frac{(68 - 75)^2}{75} + \frac{(45 - 37.5)^2}{37.5} = 2.16$$

III. R.R:

Reject H_0 if $\chi^2 \geq \chi_{0.05,2}^2 = 5.992$

IV. Decision:

Since $\chi^2 = 2.16 \not\geq 5.992 = \chi_{0.05,2}^2$ we accept H_0 .

- II. The use of the internet is known to help student in studying. A random sample of students from KSU University was classified by the usage level of the internet and the degree in final exam of Statistics:

Usage level of internet	A	B	C	D	Total
High	25	46	30	15	116
Moderate	85	25	120	20	250
Low	40	15	15	65	135
Total	150	86	165	100	501

- a) Find the observed frequency of students had degree A and use internet in low level

$$n_{31} = 40$$

- b) Find the expected frequency of students had degree C and use internet in moderate level.

$$\hat{e}_{23} = \frac{n_{2.} \cdot n_{.3}}{n} = \frac{250 \cdot 165}{501} = 82.34$$

- c) Test whether there is a relationship between internet usage and the degree in statistics. use $\alpha = 0.1$.

The expected frequencies:

Usage level of internet	A	B	C	D
High	$\hat{e}_{11} = 34.8$	$\hat{e}_{12} = 19.95$	$\hat{e}_{13} = 38.28$	$\hat{e}_{14} = 23.2$
Moderate	$\hat{e}_{21} = 75$	$\hat{e}_{22} = 43$	$\hat{e}_{23} = 82.5$	$\hat{e}_{24} = 50$
Low	$\hat{e}_{31} = 40.5$	$\hat{e}_{32} = 23.22$	$\hat{e}_{33} = 44.55$	$\hat{e}_{34} = 27$

I. Hypotheses:

H_0 : The internet usage and the degree are independent

H_a : The internet usage and the degree are dependent

II. Test Statistic:

$$\chi^2 = \frac{(25 - 34.8)^2}{34.8} + \frac{(46 - 19.95)^2}{19.95} + \dots + \frac{(65 - 27)^2}{27} = 161.3673$$

III. R.R:

Reject H_0 if $\chi^2 \geq \chi_{0.1,6}^2 = 10.645$

IV. Decision:

Since $\chi^2 = 161.3673 \geq 10.645 = \chi_{0.1,6}^2$ we reject H_0 .