Exercises: for section 5.4

1. A soft-drink vending machine is set so that the amount of drink dispensed is a random variable with a mean of 200 milliliters and a standard deviation of 15 milliliters. **What is the probability that the average (mean) amount dispensed in a random sample of size 36 is at least 204 milliliters?** $\mu = 200, \sigma = 15, n = 36, P(\bar{X} \geq 204) =?$

   i. $E(\bar{X}) = \mu_{\bar{X}} = \mu = 200.$

   ii. $Var(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{15^2}{36} = 6.25 \rightarrow \sigma_{\bar{X}} = \sqrt{6.25} = 2.5.$

   iii. Since $n = 36 > 30$ we can use the CLT. Thus, $\bar{X} \sim N(200, 6.25)$.

   iv. $P(\bar{X} \geq 204) = P\left(Z \geq \frac{204-200}{2.5}\right) = P(Z \geq 1.6)$

   $= 1 - P(Z < 1.6) = 1 - 0.9452 = 0.0548.$

2. Suppose that the time duration of a minor surgery is normally distributed with mean equal to 800 seconds and standard deviation of 40 seconds. **Find the probability that a random sample of 16 surgeries will have average time duration of less than775 seconds.**

   $\mu = 800, \sigma = 40, n = 16, P(\bar{X} < 775) =?$

   v. $E(\bar{X}) = \mu_{\bar{X}} = \mu = 800.$

   vi. $Var(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{40^2}{16} = 100 \rightarrow \sigma_{\bar{X}} = \sqrt{100} = 10.$

   vii. Since the population is normal we have $\bar{X} \sim N(800, 100)$.

   viii. $P(\bar{X} < 775) = P\left(Z < \frac{775-800}{10}\right) = P(Z < -2.5) = 0.0062.$

3. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let $\hat{p}$ be the proportion of smokers in the sample.

$$P = \frac{20}{100} = 0.2, n = 5.$$

(1) Find$\boldsymbol{E(\hat{p}) = \mu_{\hat{p}}}$, the mean $\boldsymbol{\hat{p}}$.

$$E(\hat{p}) = \mu_{\hat{p}} = p = 0.2.$$

(2) Find $\boldsymbol{Var(\hat{p}) = \sigma^2_{\hat{p}}}$, the variance of $\boldsymbol{\hat{p}}$.

$$Var(\hat{p}) = \sigma^2_{\hat{p}} = \frac{p(1-p)}{n} = \frac{0.2 \times 0.8}{5} = 0.032.$$

(3) Find an approximate distribution of $\boldsymbol{\hat{p}}$.

Since $n = 5 < 30$ we cannot use CLT. Thus, we cannot find the distribution for $\hat{p}$.

(4) Find $\boldsymbol{P(\hat{p} > 0.25)}$.

Due to (3), we cannot calculate the probability.