

## Exercises:

- I. Suppose we want to test that the true mean of the age for elementary school students is 10. If we take a sample of 50 students and found that the sample mean was 8.5. Assume that the distribution is normal with standard deviation  $\sigma=2$ . Test whether the true average is less than 10. Use  $\alpha = 0.05$ .

**1. Hypotheses:**

$$H_0: \mu = 10$$

$$H_a: \mu < 10$$

**2. Assumption:**

$\sigma^2$  is known

**3. Test Statistic:**

$$Z = \frac{8.5 - 10}{2/\sqrt{50}} = -5.30$$

**4. Rejection region:**

Reject  $H_0$  if:  $Z < -Z_\alpha = -Z_{0.05} = -1.645$

**5. Decision:**

Since  $Z = -5.3 < -1.645 = -Z_{0.05}$ , we reject  $H_0$ . at  $\alpha = 0.05$

**6. Compute the P\_value and compare your conclusions with the previous one.**

$$P_{value} = P(Z < Z_{ts}) = P(Z < -5.3) = 0 < 0.05 = \alpha$$

We reject  $H_0$  at  $\alpha = 0.05$

- II. Suppose that we are interested in studying the effectiveness of a certain diet program. Let the random variable  $X$  and  $Y$  are as follows:

$X$ : The weight of the individual before the diet program

$Y$ : The weight of the individual after the diet program

We assume that the distributions of these random variables are normal with mean  $\mu_1$  &  $\mu_2$ , respectively. The data is given below

Individual(i)	1	2	3	4	5	6	7	8	9	10
$X_i$	86.6	80.2	91.5	80.6	82.3	81.9	88.4	85.3	83.1	82.1
$Y_i$	79.7	85.9	81.7	82.5	77.9	85.8	81.3	74.7	68.3	69.7

Does this data provide sufficient evidence to allow us to conclude that the diet program is effective? Use  $\alpha = 0.05$

**1. Hypotheses:**

$$\begin{array}{ll} H_0: \mu_1 = \mu_2 & H_0: \mu_D = 0 \\ H_a: \mu_1 \neq \mu_2 & \text{or} \quad H_a: \mu_D \neq 0 \end{array}$$

**2. Test Statistic:**

$$\bar{D} = 5.45, s_D = 7.09$$

$$t = \frac{\bar{D}}{s_D/\sqrt{n}} = \frac{5.45}{7.09/\sqrt{10}} = 2.43$$

**3. Rejection region:**

$$\text{Reject } H_0 \text{ if: } t < -t_{\frac{\alpha}{2}} \text{ or } t > t_{\frac{\alpha}{2}}$$

$$t_{\frac{\alpha}{2}, n-1} = t_{\frac{0.05}{2}, 9} = t_{0.025, 9} = 2.262$$

**4. Decision:**

Since  $t = 2.43 > 2.262$ , we reject  $H_0$ . at  $\alpha = 0.05$