Sheet-1

Q.1. Check whether the following sequences converge or diverge. If they converge find the limit.

1) $\{\sqrt[n]{n}\}$, 2) $\left\{\left(1-\frac{3}{n}\right)^n\right\}$, 3) $\left\{(-1)^n \frac{\ln n}{n}\right\}$, 4) $\left\{(-1)^n \sin\left(\frac{1}{n}\right)\right\}$, 5) $\left\{\frac{2^n+5^n}{3^n+5^n}\right\}$, 6) $\{n\left(\ln(n+1)-\ln n\right)\}$, 7) $\left\{(2+n)^{\frac{1}{n}}\right\}$, 8) $\left\{(-1)^n \tan^{-1}\left(\frac{1}{n}\right)\right\}$, 9) $\left\{\left(\tan\frac{\pi}{6}\right)^n\right\}$, 10) $\left\{\frac{1-\cos n}{n}\right\}$, 11) $\{1+(-1)^n\}$.

Answers: 1) Converges to 1, 2) Converges to e^{-3}, 3) Converges to 0, 4) Converges to 0, 5) Converges to 1, 6) Converges to 1, 7) Converges to 1, 8) Converges to 0, 9) Converges to 0, 10) Converges to 0, 11) Diverges,

Q.2. Check whether the following series are convergent or divergent. If they are convergent find their sums.

1) $\sum_{n=1}^{\infty}\left(\frac{2n-1}{n+1}\right)$, 2) $\sum_{n=1}^{\infty}\left(\frac{2^n+3^n}{3^n-2^n}\right)$, 3) $\sum_{n=1}^{\infty} 2\,(3)^n$, 4) $\sum_{n=1}^{\infty}\left(\frac{e^n}{4^{n-1}}\right)$, 5) $\sum_{n=1}^{\infty}\frac{e^n}{1+e^n}$, 6) $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$, 7) $\sum_{n=1}^{\infty}\left(\frac{1}{3^n}+\frac{3}{n}\right)$, 8) $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{e^n}{4^n}\right)$, 9) $\sum_{n=1}^{\infty}\frac{4}{n+1}$, 10) $\sum_{n=1}^{\infty}\left(\frac{e^2}{2^n}-\frac{4}{3^n}\right)$, 11) $\sum_{n=1}^{\infty}\frac{2^n}{n}$, 12) $\sum_{n=1}^{\infty}\left(\frac{1}{e^n}+\frac{3^n}{n}\right)$, 13) $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$, 14) $\sum_{n=1}^{\infty} 2^{\frac{3}{n}}$, 15) $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n$, 16) $\sum_{n=1}^{\infty}\frac{3}{n+e^n}$.

Answers: 1) Divergent, 2) Divergent, 3) Divergent, 4) Convergent with sum $\frac{4e}{4-e}$, 5) Divergent, 6) Convergent with sum 1, 7) Divergent, 8) Convergent with sum $\frac{4}{4-e}$, 9) Divergent, 10) Convergent with sum e^2-2, 11) Divergent, 12) Divergent, 13) Divergent, 14) Divergent, 15) Divergent, 16) Convergent