Sheet-4

Q.1 Find the interval and radius of convergence for following power series.

1) $\sum_{n=0}^{\infty} \frac{5^n(x-2)^n}{n!}$, 2) $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{n^n}$, 3) $\sum_{n=1}^{\infty} \frac{\ln n(x-1)^n}{n}$,

4) $\sum_{n=1}^{\infty} \frac{n(x-2)^n}{\ln n}$, 5) $\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{3^n}$, 6) $\sum_{n=2}^{\infty} \frac{2^n(x-1)^n}{\ln n}$,

7) $\sum_{n=1}^{\infty} \frac{\ln n(x-3)^n}{3^n}$, 8) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$, 9) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}$

Answers: 1) $(-\infty,\infty)$, 2) $(-\infty,\infty)$, 3) $[0,2)$, Radius 1, 4) $(1,3)$, Radius 1 5) $(-\frac{1}{2},\frac{5}{2})$, Radius $\frac{3}{2}$ 6) $[\frac{1}{2},\frac{3}{2})$, Radius $\frac{1}{2}$ 7) $(0,6)$, Radius 3 8) $(-1,1]$, Radius 1 9) $(-\infty,\infty)$.

Q.2 Find the power series representations for the following functions.

1) $f(x)=\frac{1}{(1-2x)^2}$, $|x|<\frac{1}{2}$. 2) $f(x)=\frac{x-1}{x+1}$, $|x|<1$.
3) $f(x)=\frac{1}{(2+x)^3}$, $|x|<2$.

Answers: 1) $\sum_{n=0}^{\infty}(n+1)(2x)^n$, 2) $-1+2\sum_{n=0}^{\infty}(-1)^n x^{n+1}$, 3) $\frac{1}{8}\sum_{n=0}^{\infty}(-1)^n\frac{(n+1)(n+2)}{2}\left(\frac{x}{2}\right)^n$.