Sheet-5

Q.1 Evaluate the following integrals:
(i) $\int_0^2 \int_y^2 e^{x^2} dxdy$, (ii) $\int_0^1 \int_{\sqrt{y}}^1 \cos x^3 dxdy$, (iii) $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dxdy$, (iv) $\int_0^1 \int_x^1 \sec^2\left(\frac{\pi x}{4y}\right) dydx$.

Answers: (i) $\frac{1}{2}(e^4-1)$, (ii) $\frac{1}{3}\sin 1$, (iii) $\frac{1}{3}(e-1)$, (iv) $\frac{2}{\pi}$.

Q.2 Evaluate $\iint_{\mathcal{R}} (x+y)dA$, where $\mathcal{R}$ is the region bounded by the graphs of the equations $y^2 = x$ and $y = x^2$.
Answer: $\frac{3}{10}$.

Q.3 Evaluate $\iint_{\mathcal{R}} xydA$, where $\mathcal{R}$ is the triangular region with vertices $(-2,2)$, $(1,1)$, $(1,0)$.
Answer: $-\frac{9}{24}$.

Q.4 Find the area of the region bounded by the graphs of the equations $y^2 = x$ and $y^2 = 2 - x$.
Answer: $\frac{8}{3}$.

Q.5 Find the volume of the solid in the first octant bounded by the graphs of the equations $y^2 = z$ and $x + 2y = 2$.
Answer: $\frac{1}{6}$.

Q.6 Find the volume of the solid bounded by the graphs of the equations $z = x^2 + y^2$, $x^2 + y^2 = 1$ and $z = 0$.
Answer: $\frac{\pi}{2}$.

Q.7 Find the area of the region bounded by the graphs of the equations $r = 2\cos\theta$, $r = 2\sin\theta$.
Answer: $\frac{\pi}{2} + 1$.

Q.8 Evaluate the integrals:
(i) $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \left(x^2+y^2\right)^{\frac{3}{2}} dydx$, (ii) $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{1}{\sqrt{x^2+y^2}} dydx$, (iii) $\int_{-\sqrt{3}}^{\sqrt{3}} \int_1^{\sqrt{4-x^2}} \left(x^2+y^2\right)^{\frac{3}{2}} dydx$.
Answers: (i) $\frac{1024}{75}$, (ii) $\frac{\sqrt{2}\pi}{4}$, (iii) $\frac{\pi}{8}$.