Sheet-8

Q.1 Evaluate the line integrals

(i) $\int\limits_C \sqrt{y}dx - \sqrt{x}dy$, where C is the portion of the graph of $xy = 1$ from $(1,1)$ to $(2, \frac{1}{2})$.

(ii) $\int\limits_C e^x dy + ydx$, where C is the portion of the parabola $y = 1 + x^2$ from the point $(0,1)$ to $(1,2)$.

Answers: (i) $\sqrt{2}$, (ii) $\frac{10}{3}$.

Q.2 If $\overrightarrow{F} = -yi + xj$, find the work done by the force $\overrightarrow{F}$ along the parabola $y = 2x^2$ from the point $(-1,2)$ to $(1,2)$.

Answer: $\frac{4}{3}$.

Q.3 Find the work done by the force $\overrightarrow{F} = -zi + yj + zk$ along the curve $x = t, y = t^2, z = t^3$ from the point $(1,1,1)$ to $(2,4,8)$.

Answer: 15.

Q.4 Check whether the following integrals are independent of pathe.

(i) $\int\limits_C \frac{1}{x-2y}(dx - 2dy)$, (ii) $\int\limits_C (2x-y)dx - 2xdy$, (iii) $\int\limits_C x\cos ydy - y\sin xdx$.

Answers: (i) Yes, (ii) Yes, (iii) No.

Q.5 Evaluate the following integrals.

(i) $\int\limits_{(0,0)}^{(i,\frac{\pi}{2})} y\cos(xy)dx + x\cos(xy)dy$, (ii) $\int\limits_{(0,1)}^{(2,1)} e^{\frac{x}{y}}(\frac{1}{y}dx - \frac{1}{y^2}dy)$.

Answers: (i) 1, (ii) $e^2 - 1$.

Q.6 Use Green's theorem to find the area of the region bounded by the graphs of the equations $y = x^2$ and $y^2 = 8x$.

Answer: $\frac{8}{3}$.