Sheet-9

Q.1 Use Green's theorem to evaluate the line integral

(i) $\oint_C y^2dx + 3xydy$, where C is the closed curve which is the boundary of the region bounded by the graphs of the equations $y = x^{\frac{3}{2}}$ and $y^2 = x$.

Answer: $\frac{1}{8}$.

Q.2 Evaluate the surface integreal

$$\iint_S (x^2 + z^2)dS,$$

where S is the surface of the graph of $x^2 + y^2 - z^2 = 0$ with $1 \leq z \leq 4$.

Answer: $\frac{45\sqrt{2}}{4}\pi$.

Q.3 If $\overrightarrow{F} = -xi - yj + zk$ and S is the portion of the graph $2z = x^2 + y^2$ cut off by the planes $z = 1$ and $z = 2$, find the flux of $\overrightarrow{F}$ through the surface S.

Answer: 18π.

Q.4 If $\overrightarrow{F} = 2xi - yj - zk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$, verify the Divergence theorem.

Q.5 Use Diveregence theorem to find the flux of the force $\overrightarrow{F} = yi - xj + zk$ through the surface S of the region bounded by the graphs of $3z = 4 - x^2 - y^2$ and $z = \sqrt{x^2 + y^2}$.

Answer: $\frac{\pi}{2}$.

Q.6 Let $\overrightarrow{F} = -yi + xj - zk$ and S be the surface of the paraboloid $z = 1 + x^2 + y^2$ inside the cylinder $x^2 + y^2 = 1$. Evaluate the surface integral

$$\iint_S (\nabla \times \overrightarrow{F}).\overrightarrow{n}\, dS.$$

Answer: 2π .

Q.7 Let $\overrightarrow{F} = yi - xj + zk$ and S be the surface of the graph of $z = 6 - x^2 - y^2$ inside the cylinder $x^2 + y^2 = 2$. Verify the stokes theorem.

Answer: Each side is -8π .