Related topics

First and second law of thermodynamics, reversible cycles, isochoric and isothermal changes, gas laws, efficiency, Stirling engine, conversion of heat, heat pump.

Principle

The Stirling engine is submitted to a load by means of an adjustable torque meter. Rotation frequency and temperature changes of the Stirling engine are observed. Effective mechanical energy and power, as well as effective electrical power, are assessed as a function of rotation frequency. The amount of energy converted to work per cycle can be determined with the assistance of the pV diagram. The efficiency of the Stirling engine can be estimated.

Equipment

Stirling engine, transparent	04372.00	1
Torque meter	04372.02	1
Chimney for stirling engine	04372.04	1
Meter for stirling engine, $pVnT$	04371.97	1
Sensor unit pVn for stirl.eng.	04371.00	1
Screened cable, BNC, l = 750 mm	07542.11	2
Thermocouple NiCr-Ni, sheathed	13615.01	2
Raw alcohol for burning,1000 ml	31150.70	1
Adapter, BNC-socket/4 mm plug pair	07542.27	2
Cobra3 Basic Unit, USB	12150.50	1
Power supply, 12 V-	12151.99	1
Cobra3 Universal Writer	14504.61	1

Tasks

1. Determination of the burner 's thermal power.
2. Calibration of the sensor unit.
3. Calculation of the total energy produced by the engine through determination of the cycle area in the p-V diagram recorded with the Cobra3 unit.
4. Assessment of the mechanical work per revolution, and calculation of the mechanical power output as a function of the rotation frequency with the assistance of the torque meter.
5. Efficiency assessment.

Set-up and procedure

Experimental set up should be carried out as shown in Fig.1. The base plate (mounting plate) of the Stirling engine must be removed, so that the latter can be fixed on the corresponding mounting plate of the pVn sensor unit. The incremental transmitter of the pVn sensor unit is to be firmly connected to the axle of the Stirling engine. The latter is then fixed upon the large base plate.
Before switching on the $pVnT$ meter, make sure it is connected to the pVn sensor. Connect the p exit to the "Analog In 2 / S2" and the V exit to the "Analog In 1 / S1" input of the Cobra3 Basic Unit. Connect the Cobra3 Basic Unit to the USB port. After switching on, the $pVnT$ meter display shows "cal". Both thermocouples should have the same temperature. By pressing the "Calibration T"-button they are calibrated (i.e. setting them to the same temperature value). This calibration of the temperature sensors sets the temperature difference display to zero and does not effect the absolute temperature display. The upper display now shows "OT", which means

Fig. 1: Experimental set-up: Stirling engine.

"upper dead centre point ". At this position, the engine is at its minimum volume - i.e. the working piston should be at its lowest position. Bring the piston down by turning the engine axle by hand and press the "Calibration V " button. Wrong calibration will cause a phase shift in the volume output voltage, and thus lead to a distortion of the pV diagram. The three displays should now be on, showing 0 revs/min, and the actual temperatures for T_1 and T_2 .

1. Thermal output of the burner

The amount of alcohol in the burner is measured before and after the experiment with a measuring glass or the used amount is derermined by weighing the burner before and after the experiment noting down the weight difference. The corresponding duration of the experiment is recorded with a watch or clock.

2. Calibration of the pressure sensor

The pressure sensor must be calibrated so that the pV diagram can be evaluated quantitatively. Start the "measure" program on your computer and select the gauge "Cobra3 Universal Writer". For pressure calibration you may use the "Normal Measurement" with the settings seen in Fig. 2.

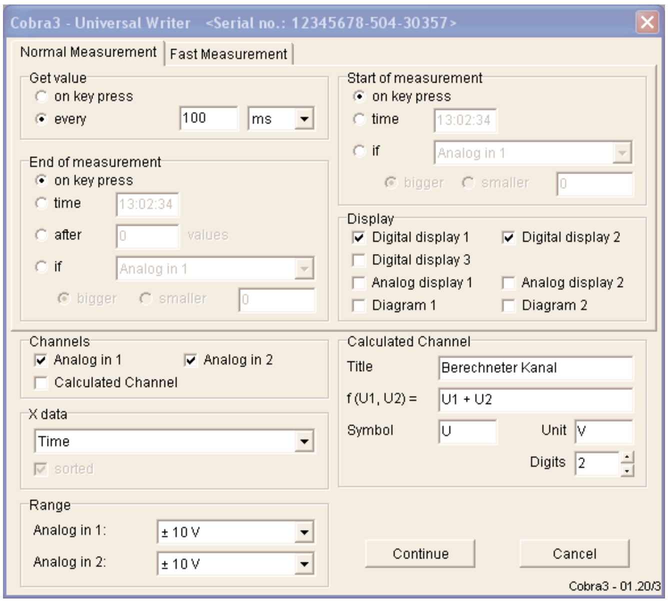


Fig. 2.: Settings for pressure calibration

Loosen the little flexible tube leading to the pressure sensor with the Stirling engine in a well defined piston position, e.g. the "upper dead centre point", where the working piston is all the way down and the volume in the engine minimal. Fix the tube on again. Now the pressure in the engine is equal to the outer pressure of appr. 1013 hPa and the volume in the engine is 32 cm^3. (with the piston all the way up, the volume is 44 cm^3). Start the measurement with the "Continue" button and turn the engine slowly and steadily by hand some cycles to have nothing but isothermal modification with the air inside the engine so that $p \cdot V$ = const.. There must be no phase displacement between volume and pressure! Stop the measurement and scale the volume axis by selecting "Analysis" > "Channel modification…" (Fig. 3) to the real volume with

f := 32 + U1 * 12/5 (1)

since the voltage U_1 changes between 0 and 5 Volt with the piston displacement from 32 to 44 cm^3 (difference 12 cm^3) engine volume.

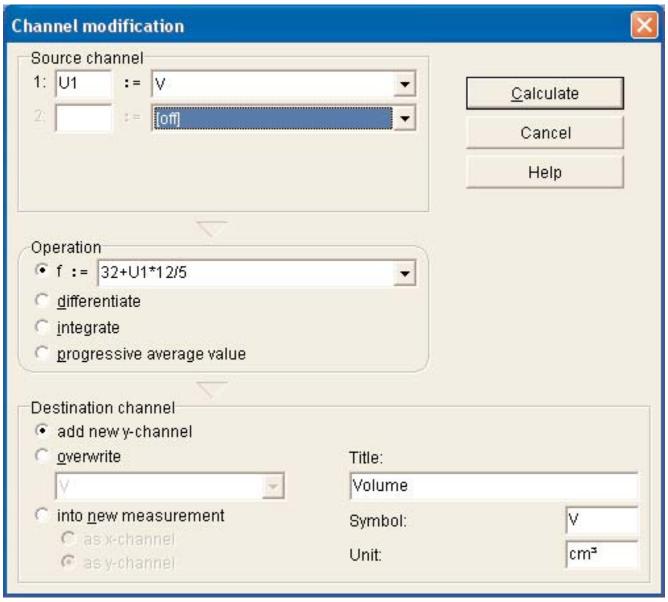


Fig. 3: Channel modification

If you take the OT as point with same pressure as surrounding, you can derive the pressure p by $p \cdot V$ = const. = 1013 hPa · 32 cm^3. The values delivered by the channel modification

f := 1013 * 32/V (2),

which you set to p, can you compare then to the U2 voltage by reading out the values with the "Survey" tool (or by using "Measurement" > "Channel manager…" putting the U2 values to the x-axis and the pressure values to the y-axis The slope of the p – U2 diagram yields the calibration factor.) Your calibration factor will be some hundred hPa (or mbar) per Volt and U2 will read something around 2.5 Volt at surrounding pressure, e.g. the suitabe channel modification may look like

f := 1013 + (U2 – 2,25) * 235 (3)

where you put the actual reading of U2 at surrounding pressure instead of 2,25 and your actual calibration factor instead of the 235.

As an alternative, you can calibrate the pressure sensor with a syringe connected to the little flexible tube. For more details refer to the experimental manual for the version without Cobra 3 (LEP 3.6.04-00).

3. Total energy and pV diagram

Start the burner, note down the time, and after some heating you can start the engine with a shove on the fly-wheel. Begin measurements always after rotation frequency and both temperature values have steadied. Select the "Fast Measurement" chart and use settings like the ones seen in Fig. 4.

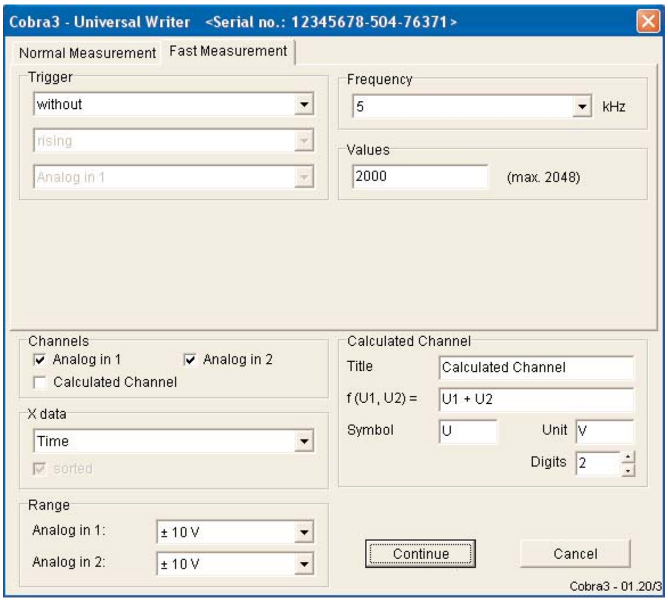


Fig. 4: Measurement settings

Record a curve with the "Continue" button The recorded curves may look like Fig. 5. Scale the volume and pressure curves by "Analysis" > "Channel modification…" with your actual formulas analog to (1) and (3). Save the curves to have a backup for further data manipulation.

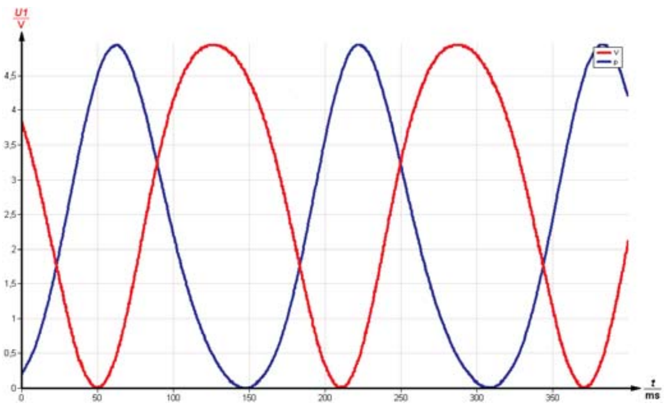


Fig. 5: Measurement example before converting the voltage to the entities to be measured

Cut the curves so that only one complete cycle of the curves is left using the marking tool (the cross symbol next to the magnifying glass) and the cutting tool (scissor symbol). Use "Measurement" > "Channel manager…" and put the the volume to be the x-axis set and put the pressure to be the y-axis set. See Fig. 6. In the then appearing "Convert relation to function" window select "Keep measurement in relation mode".

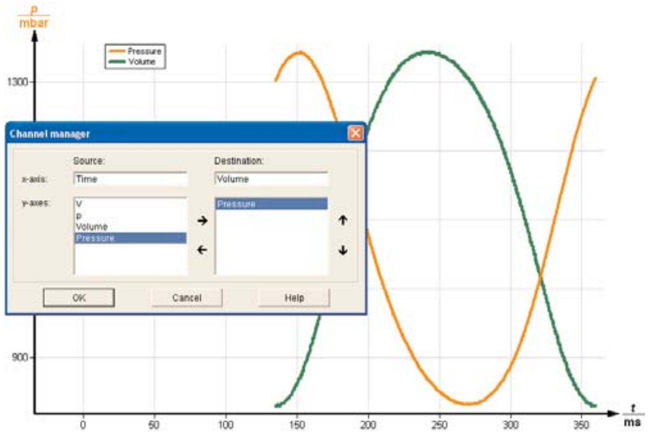


Fig. 6: Using the channel manager on the cut curves

The "Show integral" button can be used for evaluating the cycle surface. The result may look like Fig.7. The cycle has a surface of 2510 mbar · cm^3 = 2510 hPa · cm^3 =

$$= 2510 \cdot 100\ \frac{\text{N}}{\text{m}^2} \cdot 10^{-6}\ \text{m}^3 = 251\ \text{mJ}$$

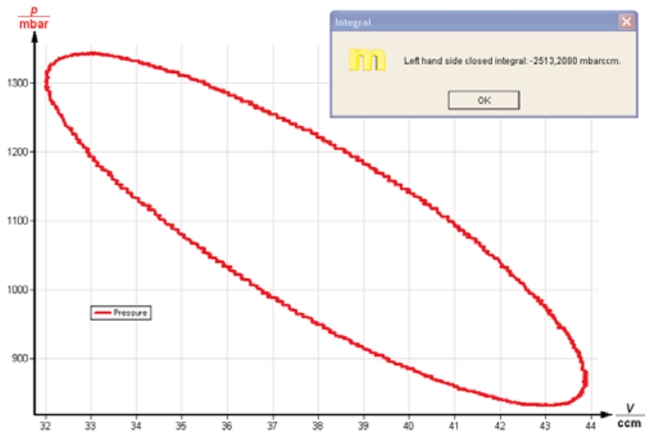


Fig. 7: Cycle in the pV diagam

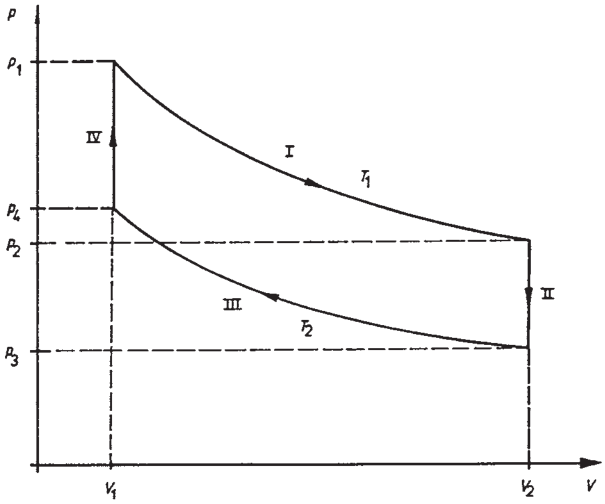
4. Effective mechanical energy

In order to load the engine with a determined torque, the scale of the torque meter is fixed on the large base plate, and the inner metallic piece of the pointer is fixed on the axis before the fly-wheel. Friction between the pointer and the connected metallic piece can be varied by means of the adjusting screw on the pointer. Adjustment must be done carefully, to make sure that the pointer does not begin to oscillate. Start carrying out measurements with a low torque. After each adjustment, wait until torque, rotation frequency and temperatures remain constant. All values and the pV diagrams are to be recorded.

Theory and evaluation

In 1816, Robert Stirling was granted a patent for a hot air engine, which is known today as the Stirling engine. In our times, the Stirling engine is used to study the principle of thermal engines because in this case the conversion process of thermal energy to mechanical energy is particularly clear and relatively easy to understand.
At present, the Stirling engine is undergoing a new phase of further development due to its many advantages.
Thus, for example, it constitutes a closed system, it runs very smoothly, and it can be operated with many different heat sources, which allows to take environmental aspects into consideration, too.

Fig. 8a: pV diagram for the ideal Stirling process.



Theoretically, there are four phases during each engine cycle (see. Fig. 8a and 8b):

1) An isothermal modification when heat is supplied and work produced

$V_1 \rightarrow V_2$ $\quad p_1 \rightarrow p_2$ $\quad$ and T_1 = const.

2) An isochoric modification when the gas is cooled:

$T_1 \rightarrow T_2$ $\quad p_2 \rightarrow p_3$ $\quad$ and V_2 = const.

3) An isothermal modification when heat is produced and work supplied:

$V_2 \rightarrow V_1$ $\quad p_3 \rightarrow p_4$ $\quad$ and T_2 = const.

4) An isochoric modification when heat is supplied to the system:

$T_2 \rightarrow T_1$ $\quad p_4 \rightarrow p_1$ $\quad$ and V_1 = const.

According to the first law of thermodynamics, when thermal energy dQ is supplied to an isolated system, its amount is equal to the sum of the internal energy increase dU of the system and the mechanical work pdV supplied by the latter:

$$dQ = dU + p\,dV$$

It is important for the Stirling cycle that the thermal energy produced during the isochoric cooling phase be stored until it can be used again during the isochoric heating phase (regeneration principle).

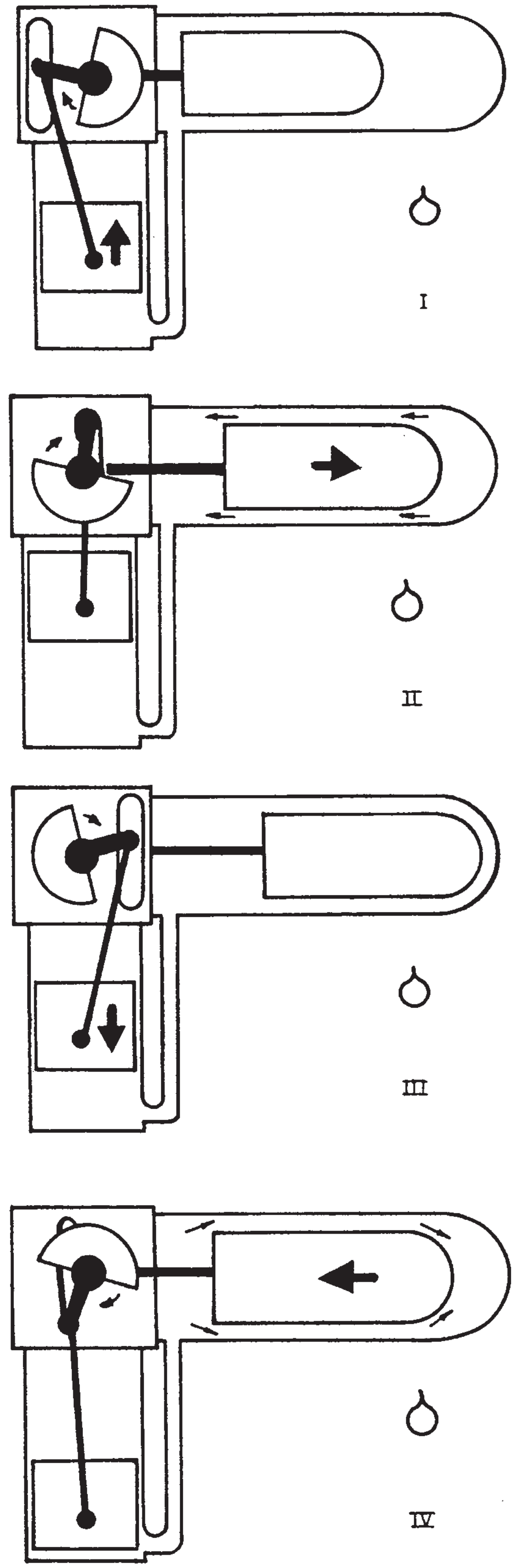


Fig. 8b: Functioning of the transparent Stirling engine.

Thus, during phase IV the amount of thermal energy released during phase II is regeneratively absorbed. This means that only an exchange of thermal energy takes place within the engine. Mechanical work is merely supplied during phases I and III. Due to the fact that internal energy is not modified during isothermal processes, work performed during these phases is respectively equal to the absorbed or released thermal energy.

Since $p \cdot V = \nu \cdot R \cdot T$,

where ν is the number of moles contained in the system, and R = 8.31 J/(mol K) the general gas constant, the amount of work produced during phase I is:

$$W_1 = -\nu \cdot R \cdot T_1 \cdot \ln(V_2/V_1)$$

(it is negative, because this amount of work is supplied).
Consequently, the amount of work supplied during phase III is

$$W_3 = +\nu \cdot R \cdot T_2 \cdot \ln(V_2/V_1)$$

$$|W_1| > W_3 \text{ because } T_1 > T_2$$

The total amount of work is thus given by the sum of W_1 and W_3. This is equal to the area of the pV diagram:

$$W_t = W_1 + W_3$$

$$W_t = -\nu \cdot R \cdot T_1 \cdot \ln(V_2/V_1) + \nu \cdot R \cdot T_2 \cdot \ln(V_2/V_1)$$

$$W_t = -\nu \cdot R \cdot (T_1 - T_2) \cdot \ln(V_2/V_1)$$

Only a part of this total effective energy W_t can be used as effective work W_m through exterior loads applied to the engine. The rest contains losses within the Stirling engine.

The maximum thermal efficiency of a reversible process within a thermal engine is equal to the ratio between the total amount of work $|W_1|$ and the amount of supplied thermal energy $Q_1 = -W_1$

$$\eta_{th} = W_t/W_1$$

$$\eta_{th} = \frac{\nu \cdot R \cdot (T_1 - T_2) \cdot \ln(V_2/V_1)}{\nu \cdot R \cdot T_1 \cdot \ln(V_2/V_1)}$$

$$\eta_{th} = \frac{T_1 - T_2}{T_1}$$

Carnot found this to be the maximum thermal efficiency for any thermal engine, which can only be reached theoretically. One sees that efficiency increases with increasing temperature differences.

1. Thermal power of the burner

Duration	Δt	= 60 min
Amount of alcohol burned	ΔV	= 29 ml
Alcohol density	ρ	= 0.83 g/ml
Specific thermal power	h	= 25 kJ/g

This allows to determine the mass of alcohol burnt per second:

$$\frac{\Delta m}{\Delta t} = 6.69 \cdot 10^{-3} \text{ g/s}$$

as well as the thermal power of the burner: P_H = 167 W.

2. pV diagram surface

Fig. 9 shows two real pV diagrams for a Stirling engine with and without load (Fig. 9a: no load, Fig. 9b: with a load M of $18.3 \cdot 10^{-3}$ Nm). Assessed surface values are given in table 2.

For other Stirling engines, the pV diagram may have a somewhat different shape. Thus, for example, the surface is a function of supplied thermal power and engine friction at equilibrium rotation frequency.

Comparison of the pV diagrams for an engine submitted or not to an exterior load shows that a higher pressure difference occurs for the load case, corresponding to the larger temperature difference measured at the Stirling engine. If the engine is submitted to a load, the surface of the pV diagram increases merely by 10 %–20 %; it displays a maximum for the total effective energy W_{pV} at medium loads or medium rotation speeds respectively (see Fig.10).

3. Effective mechanical energy and power

Effective mechanical energy during a cycle is calculated with the assistance of the torque M displayed by the torque meter:

$$W_m = 2 \cdot \pi \cdot M$$

The displayed rotation speed n (revolutions per minute) is converted to the frequency f (revolutions per second). This allows to determine the mechanical power:

$$P_m = W_m \cdot f$$

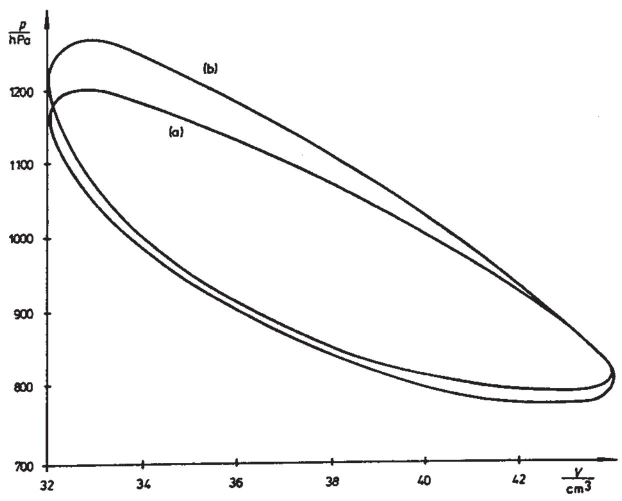


Fig. 9: Real pV diagrams (a) without, and (b) with exterior load.

Table 1 contains measured and calculated values. Fig. 10 displays the total effective energy W_{pV} assessed on the base of the pV diagram, effective mechanical energy W_m as well as friction energy per cycle W_{fr}, as a function of rotation frequency.

$$W_{fr} = W_{pV} - W_m$$

Table 1

M / 10^{-3} Nm	n / min^{-1}	T_1 / °C	T_2 / °C	W_m / mJ	f / Hz	P_m / mW	W_{pV} / mJ	W_{fr} / mJ
0	982	163	74.8	0	16.4	0	198	198
2.5	945	169	77.7	16	15.8	248	201	185
4.0	908	168	78.7	25	15.1	379	205	180
6.5	860	177	77.5	41	14.3	583	210	169
8.2	817	177	77.1	52	13.6	675	216	164
10.5	745	178	76.5	66	12.4	818	221	155
12.2	752	179	76.3	77	12.5	959	230	153
14.0	705	185	76.7	88	11.8	1038	238	150
15.0	650	188	76.9	94	10.8	1017	239	145
16.8	519	190	76.3	106	8.7	919	243	137
18.3	555	192	75.5	115	9.3	1064	245	130
19.5	460	195	74.2	122	7.7	939	246	124
22.0	380	197	72.0	138	6.3	871	247	109
22.4	275	201	70.7	141	4.6	647	235	94

Rotation frequency reaches its maximum value when the engine is not submitted to exterior loads (here: 982 min^{-1}). It is a function of thermal input and friction; in general its values lie within the range 800…1000 min^{-1}. Rotation frequency decreases with increasing exterior loads, until the Stirling engine stops (in general between 150…300 min^{-1}). Temperature T_1 increases strongly with decreasing rotation frequencies; T_2 decreases a little due to the fact that the air in the regenerator (that is on the wall of the displacing piston) is pre heated or respectively cooled to a better extent when rotation frequency is low. Pressure within the Stirling engine also varies with temperatures. This is clearly visible on the pV diagram (see Fig. 9). When adjusting a new torque, load fluctuations and shocks on the axle are unavoidable. Due to this, measurement values may display a large range of scattering. Friction energy per cycle increases with rotation frequencies.

Effective mechanical power displays a marked peak for rotation frequencies within a range of 500…600 min^{-1} (see Fig. 10).

4. Real and ideal Stirling process, efficiency assessment

The idealised Stirling process runs along isochoric and isothermal lines (see Fig. 8a). The real process can stray considerably from this, due to several reasons:

a. Both pistons run with a constant phase shift of 90°, which causes the diagram to have no sharp angles, as in the case of the idealised process.

b. Gas velocity is too high for an isothermal change of state in the case of an engine running at 1000 revs/ min.

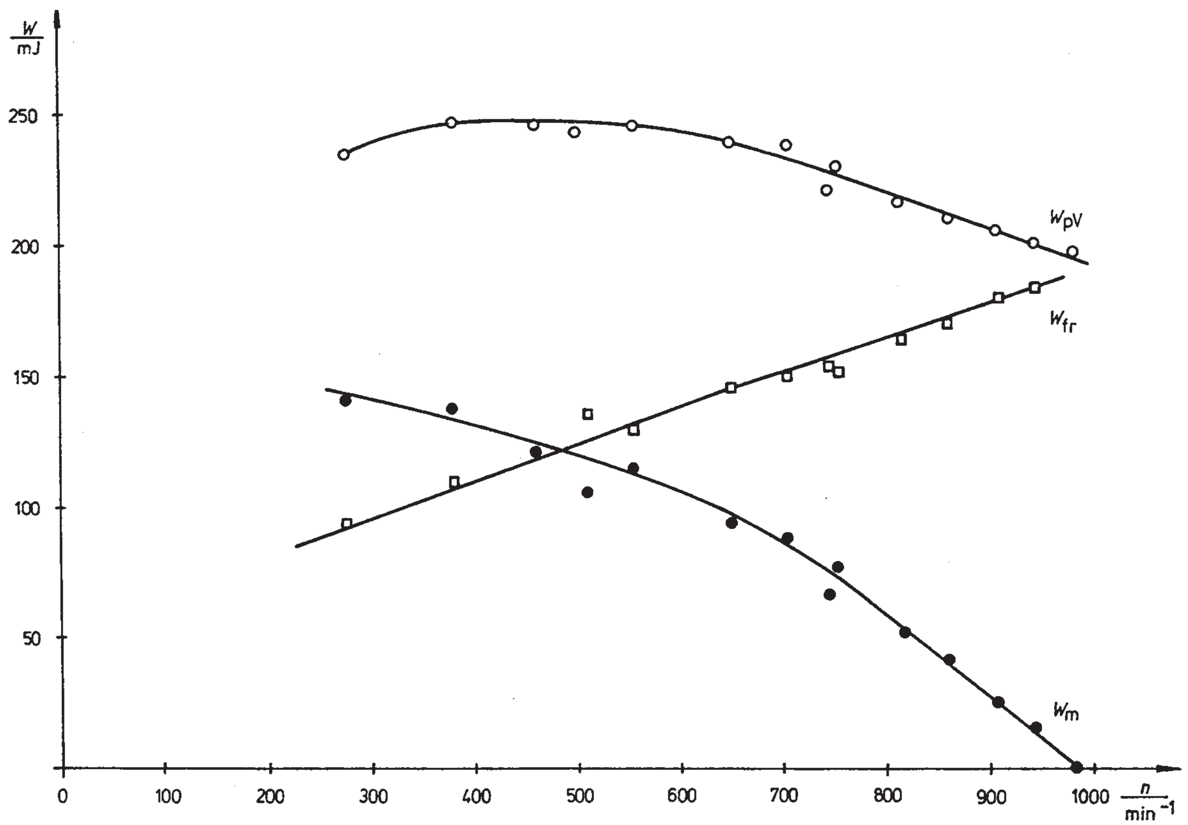


Fig. 10: Mechanical energy as a function of rotation frequency.

c. The regenerator does not work at 100 % efficiency. The air within the Stirling engine reaches the cold zone warmer, and the warm zone colder, as would be the case for the ideal process. Larger thermal input and cooling capacity are required.
d. During the ideal process, the total amount of working medium is forced from the cold zone into the warm zone. In the real process, there is a clearance volume, e.g. in the case of this Stirling engine the regenerator volume (that is the volume next to the displacing piston), and in the working cylinder.
e. There are large losses of pressure, as the working piston is not air tight.
f. Friction losses occur at all friction surfaces and within the streaming gas.

Isotherms can be adapted to a measured pV diagram with the assistance of the measured temperatures T_1 and T_2 This is carried out, using a measurement in the maximum power range of the Stirling engine as an example.

$M = 18.3 \cdot 10^{-3}$ Nm

$T_1 = 192\,°C = 465$ K

$T_2 = 75.5\,°C = 349$ K

The following relation is valid for an ideal gas:

$$p \cdot V = \nu \cdot R \cdot T$$

Due to the fact that the working piston of the Stirling engine is not air tight, the number of moles ν contained within the engine during operation must be evaluated with the assistance of the pV diagram (see Fig. 9). One or two points are selected in the middle of the diagram surface. They are allocated to the isotherm at the average temperature:

$$T_m = \frac{T_1 + T_2}{2} = 407 \text{ K}.$$

Example:

1st point: $V = 38.0$ cm^3	$p = 969$ hPa corresponds to this
2nd point: $p = 1017$ hPa	$V = 36.8$ cm^3 corresponds to this

With $R = 8.31$ J/(mol K), one obtains, as an average of both assessments:

$\nu = 1.10 \cdot 10^{-3}$ moles

The isotherms for temperatures T_1 and T_2, calculated with the assistance of this value, are represented in Fig. 11, together with the pV diagram. When comparing measured and theoretical curves, it must be taken into account that the displayed temperatures oniy can be average values. In the vicimty of the flame, temperature is higher than T_1 and lower than T_2 within the working cylinder. Volume increase only takes place within the cold working cylinder; for this reason average temperatures are shifted towards lower values than those displayed for a large volume, and the curve of the pV diagram is somewhat steeper than the isotherms. Overlapping may also occur when comparing various pV diagrams with theoretical curves.

Efficiency assessment for this maximum power example:

The effective energy per cycle is (see Table 1):

$W_m = 115$ mJ

During one cycle, the burner supplies the following thermal energy:

$W_H = P_H / f$
$W_H = 18.0$ J

This yields a total efficiency of:

$\eta = W_m / W_H$
$\eta = 115$ mJ/18.0 J
$\eta = 0.6\,\%$

The efficiency of the Stirling engine is constituted by several components:

Efficiency of the heater:

$\eta_H = |W_1| / W_H$
$\eta_H = \nu \cdot R \cdot T_1 \ln (V_2/V_1) / W_H$
$\eta_H = 1.35$ J/18.0 J
$\eta_H = 7.5\,\%$

Thermal efficiency (Carnot):

$\eta_{th} = W_t / W_1$
$\eta_{th} = (T_1 - T_2)/T_1$
$\eta_{th} = (465\text{ K} - 349\text{ K})/465\text{ K} = 25\,\%$

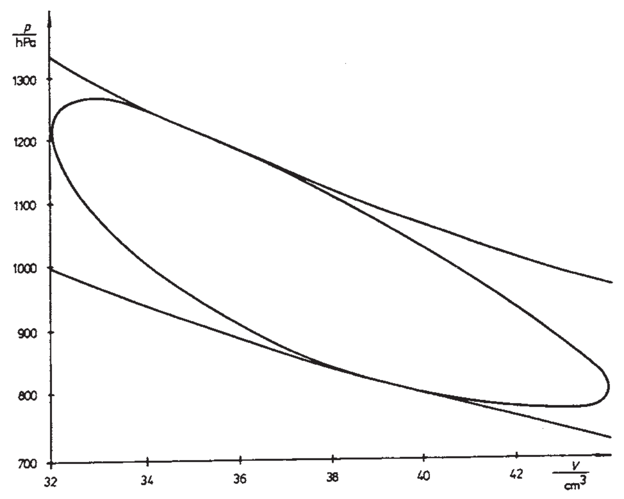


Fig. 11: pV diagram and isotherms.

Interior efficiency:

$\eta_i = W_{pV} / |W_t|$

$\eta_i = W_{pV} / (\nu \cdot R\,(T_1 - T_2) \ln (V_2/V_1))$

η_i = 245 mJ/339 mJ = 72 %

Mechanical efficiency:

$\eta_m = W_m / W_{pV}$

η_m = 115 mJ/245 mJ = 47 %

Fig. 12: Stirling engine with accessories for heating by the sun.

Note

The experiments can also be performed with the help of the sun as heating source. Therefore you need the accessories for solar motor work. The setup is shown in Fig. 12.
For conversion of the energy generated by the Stirling motor to electric energy and to drive the Stirling motor as a heat pump or as a refrigerating machine you can use the motor/generator unit 04372.01. For further details refer to its manual or the experimental manual for the version without Cobra 3 (LEP 3.6.04-00).

Fig. 13: Motor/generator unit