Chap 2 :

Options Pricing : the Binomial Option Pricing Model (BOPM)

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Options Prices:

Speculation with Options will usually not involve holding the Option to maturity, but buying and Selling (closing out) over a very short period of time.

Prior to maturity it can be shown that the option premium varies minute by minute as the Stock price, interest rate and the Volatility of the Stock change.

Let us develop the intuitive arguments that help to

explain the determination of option prices.
we consider each factor in turn, holding all the other factors constant.
This will help us to understand the mathematical formulate for option prices that we will present Later.
we consider an European Style option with Long - trading position (call or put).

Factors Affecting the Value of Options

	Long Call	Long PUT
Time to maturity	+	+
Current Stock price	+	−
Exercise price	−	+
Stock volatility	+	+
RFR	+	− !

In, out and at-the Money Options

	CALL	PUT
In the Money (ITM)	$S_0 > K$	$S_0 < K$
At the Money (ATM)	$S_0 = K$	$S_0 = K$
Out of the Money	$S_0 < K$	$S_0 > K$

It would be extremely useful if all the above factors could be included in a single Equation to determine the call C or put premium P.

$$C \text{ or } P = f(S, K, r, \sigma, T)$$

Time Value

The two components of an option premium are the intrinsic Value and time Value of the option.

The Intrinsic Value is the difference between the underlying asset price and the strike price

For a Call: $\boxed{\text{Underlying Asset Price}}$ − Strike Price

$= S_T - K$

$$Max[0, S_T - K]$$

For a PUT: $\max[0, \underbrace{K}_{\text{Strike Price}} - \underbrace{S_T}_{\text{underlying Asset Price}}]$

$\Rightarrow K - S_T$

By definition, the Only Options that have intrinsic Value are those <u>in the money</u>.

For Calls, in the money refers to options where the K is Less than the S_0.

A Put option, is in the money if its K is greater than the S_0.

in the Money Call $\Rightarrow K < S_0$

in the money PUT $\Rightarrow K > S_0$

any Premium that is in Excess of the option intrinsec Value is referred to the time Value.

Ex :

Assume a Call option has a premium of 9\$. If the option has an intrinsec Value of 7\$, it's time Value would be then 2\$ (9\$ - 7\$ = 2\$).

$$\boxed{\text{Time Value} = \text{Premium} - IV}$$

$$\boxed{\text{Premium} = T.V + IV}$$

IV and TV of CALL

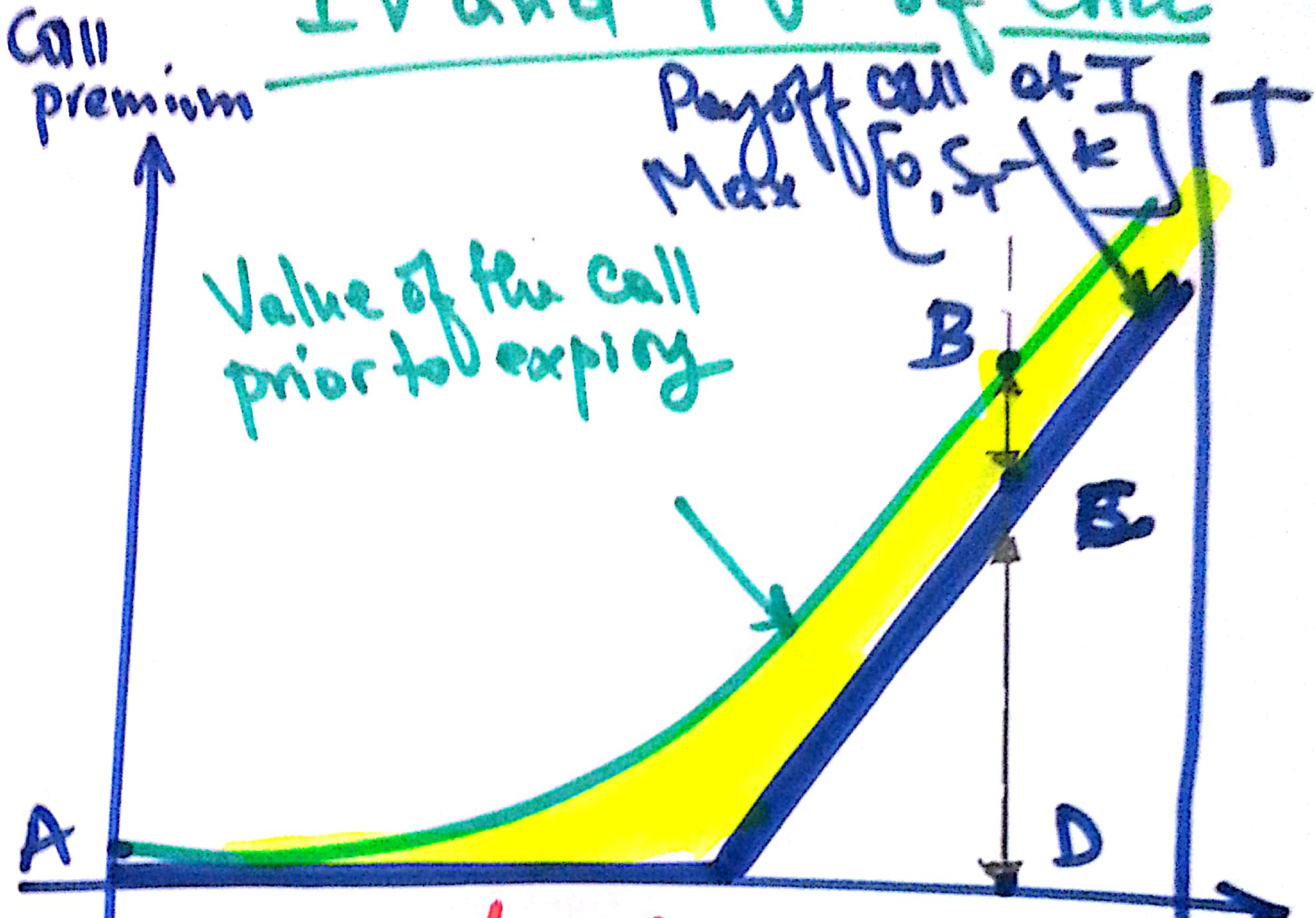

Fig: TV and IV

BC = Time Value of the call

DC = Intrinsic Value of the call

BD = BC + CD

$\boxed{\text{Call premium} = TV + IV}$

In general, the more time to expiration, the greater the time value of the option.

It represents the amount of time the option position has to become profitable due to a favorable move in the Stock Price.

In most cases, investors are willing to pay higher premium for more time (assuming that the options have the same ~~price~~ exercise price, Since time increases the probability that the ~~&~~ option become profitable.

increases.

The time Value decreases over time and decay to zero at the Expiration Date.

This phenomenon is known as **time decay:**

Option Premium $= \cancel{I.V} + TV$

P or C $= \boxed{\text{Intrinsec Value}} + \boxed{\text{time Value}}$

*For Long CALLS:

$$IV = \max[S_T - K, 0]$$

$$TV = C - IV.$$

*For Long PUTS:

$$IV = \max[0, K - S_T]$$

$$TV = P - IV.$$

Introduction:

We Start by Setting out the PUT-CALL parity condition for european options. ~~that~~

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Detailed Presentation of the BOPM (Binomial Option Pricing Model)

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Idea: How to Construct a free-Risk portfolio from 2 Risky assets (CALL + Stock)

$\boxed{\text{Principle of Delta Hedging}}$

Delta Hedging

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Option Pricing

Discrete Model	Continuous Model
BOPM	B-S Model
Cox-Ross-Rubinstein [1979]	Black-Scholes Model [1973]
ch2	ch3

1. CALL - PUT Parity for European options

If we know the price of the call using BOMP or B-S Model, we can use the put-call parity relationship to determine the price of the put. for options having the same strike Price (K) and Expiration date (T).

PUT-CALL is an Arbitrage relationship between European calls and Puts premia. The call-put parity is given as:

$$\boxed{S + P = C + Ke^{-rT}}$$

Long Stock + Long Put = Long Call + cash

$$\boxed{S + P = C + Ke^{-rT}}$$

Ke^{-rT} is holding an amount of cash equal to Ke^{-rT} in a risk free Rate (such as deposit or a T-bill that matures at T)

Demonstration: (Proof)

to demonstrate the put-call parity, we form 2 portfolios and show that they have the same payoff at Time T.

⇒ we conclude that They must worth the same today.

Port A :	1 put + Stock at $t=0$
Port B	1 Call + Ke^{-rT} at $t=0$

Returns from the 2 portfolios A and B. [PUT-CALL parity].

At T:

	$S_T > K$	$S_T < K$
PF(A)	S_T	K
PF(B)	$(S_T - K) + K = S_T$	K

For PF(A):
At the Expiration: if $S_T > K$, the put option expires worthless $P = Max\{0, S_T - K\} = 0$, and the Stock worth S_T.

Alternatively, if $S_T < K$, the PUT payoff is $(K - S_T)$ and the share worth S_T. Subsequently, the PF(A) payoff will be:

$$((K - S_T) + K) = S_T.$$

For PF(B):

* if $S_T > K \Rightarrow$ the call payoff is $(S_T - K)$

$\Rightarrow$ the cash (Ke^{-rT}) held in the risk free rate has a value of K

$\Rightarrow$ the payoff of the PF(B) will be

$$(S_T - K) + K = S_T$$

* if $S_T < K$, the call option expires worthless $C = \text{Max}[0, S_T - K] = 0$

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→ the amount held in the RFR will ~~too~~ equals (K). the payoff of the PF (A) will be K.

⇒ As show in the table, both portfolios (A) and (B) yield identical payoffs at (T).

⇒ Since the optims are european and cannot be exercised prior to the maturity, the two portfolios must also have identical Values today (At t = 0)

$$\text{Long Call option} + Ke^{-rT} = \text{Long Put option} + \text{Long share}.$$

$$\boxed{C + Ke^{-rT} = P + S}$$

$$\boxed{C - P = S - Ke^{-rT}}$$

$$\boxed{P = C + Ke^{-rT} - S}$$

$\Rightarrow$ At $t=0$, we know:

K, S, rfr $\longrightarrow$ we use B-S or BOPM

$\downarrow$

C

$\downarrow$

We use the Call-put parity $\longrightarrow$ Find the premium of the put