Class of April 2017, 17

1 Let's consider a PF with 2 Stocks.
A and B, with ~~E_A~~, ~~E~~

Stock A $\left(E_A, \sigma_A, w_A\right)$ $w_A + w_B = 1$.
Stock B $\left(E_B, \sigma_B, w_B\right)$ $Cov(A,B)$.

Find the Optimal Weights w_A^* and w_B^*.
to have a PF with minimum risk.

Minimizing Risk:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \cdot Cov(A,B)$$

$$\text{Min. } \sigma_p^2 / w_A + w_B = 1. \Rightarrow w_B = 1 - w_A.$$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \cdot \sigma_B^2 + 2 w_A (1 - w_A) \cdot Cov(A,B)$$

$$= w_A^2 \sigma_A^2 + \left(1 - 2w_A + w_A^2\right) \cdot \sigma_B^2 + 2w_A \cdot Cov(A,B)$$

$$- 2 w_A^2 \cdot Cov(A,B)$$

$$= w_A^2 \sigma_A^2 + \sigma_B^2 - 2 w_A \sigma_B^2 + w_A^2 \sigma_B^2 + 2 w_A Cov(A,B)$$

$$- 2 w_A^2 \cdot Cov(A,B)$$

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$$w_A^2\left(\sigma_A^2+\sigma_B^2-2\operatorname{cov}(A,B)\right)+$$

$$A x^2 = 2Ax$$

$$Ax = A$$

$$w_A\left(-2\sigma_B^2+2\operatorname{cov}(A,B)\right)+\sigma_B^2=\sigma_p^2.$$

$$\operatorname{Min}\ \sigma_p^2 \Rightarrow \frac{\partial \sigma_p^2}{\partial w_A}=0.$$

$$\Rightarrow 2w_A\left(\sigma_A^2+\sigma_B^2-2\operatorname{cov}(A,B)\right)+\left(-2\sigma_B^2+2\operatorname{cov}(A,B)\right)=0$$

$$\Rightarrow \cancel{2}w_A\left(\sigma_A^2+\sigma_B^2-2\operatorname{cov}(A,B)\right)=\cancel{2}\left(\sigma_B^2-\operatorname{cov}(A,B)\right)$$

$$\boxed{w_A^*=\frac{\sigma_B^2-\operatorname{Cov}(A,B)}{\sigma_A^2+\sigma_B^2-2\operatorname{cov}(A,B)}}$$

$$\boxed{w_B^*=1-w_A^*}$$

PF $\left\{\begin{array}{l} \text{Stock A}, \; E_A = 12\%, \; \sigma_A = 3\% \\ \\ \text{Stock B}, \; E_B = 15\%, \; \sigma_B = 5\% \\ \text{Cov}(A, B) = 0{,}2 \end{array}\right.$

find the Optimal Weights for a Minimum Risk portfolio.

PF $\Rightarrow$ Min Risk $\Rightarrow$

$$W_A^* = \frac{\sigma_B^2 - \text{Cov}(A, B)}{\sigma_A^2 + \sigma_B^2 - 2\,\text{Cov}(A, B)}$$

$$= \frac{(5\%)^2 - 0{,}2}{(3\%)^2 + (5\%)^2 - 2 \times (0{,}2)}$$

$\boxed{W_A^* = 49.8\%} \Rightarrow \boxed{W_A^* \simeq 0{,}5}$

For an Equally Weighted Portfolio of Stocks A, and B, we can have a portfolio with zero Risk.

2°/ Calculate Risk and E_p for this optimal portfolio.

$$w_A^* = 0{,}5 , \quad w_B^* = 0{,}5$$

$$E_{p^*} = w_A^* \cdot E_A + w_B^* \cdot E_B$$

$$= 0{,}5 \times 12\% + 0{,}5 \times 15\%$$

$$\boxed{E_p^* = 13{,}5\%}$$

$$\sigma_{p^*}^2 = (0{,}5)^2 (3\%)^2 + (0.5)^2 \cdot (5\%)^2 + 2(0{,}5)(0.5) \cdot (0{,}2) = \_\_\_\_$$

PF^* { Long on 0,5 Stock A. Long on 0.5 Stock B.

$$\sigma_{p^*}^2 = w_A^{*2} \sigma_A^2 + w_B^{*2} \sigma_B^2 + 2 w_A^* w_B^* \cdot Cov(A,B).$$

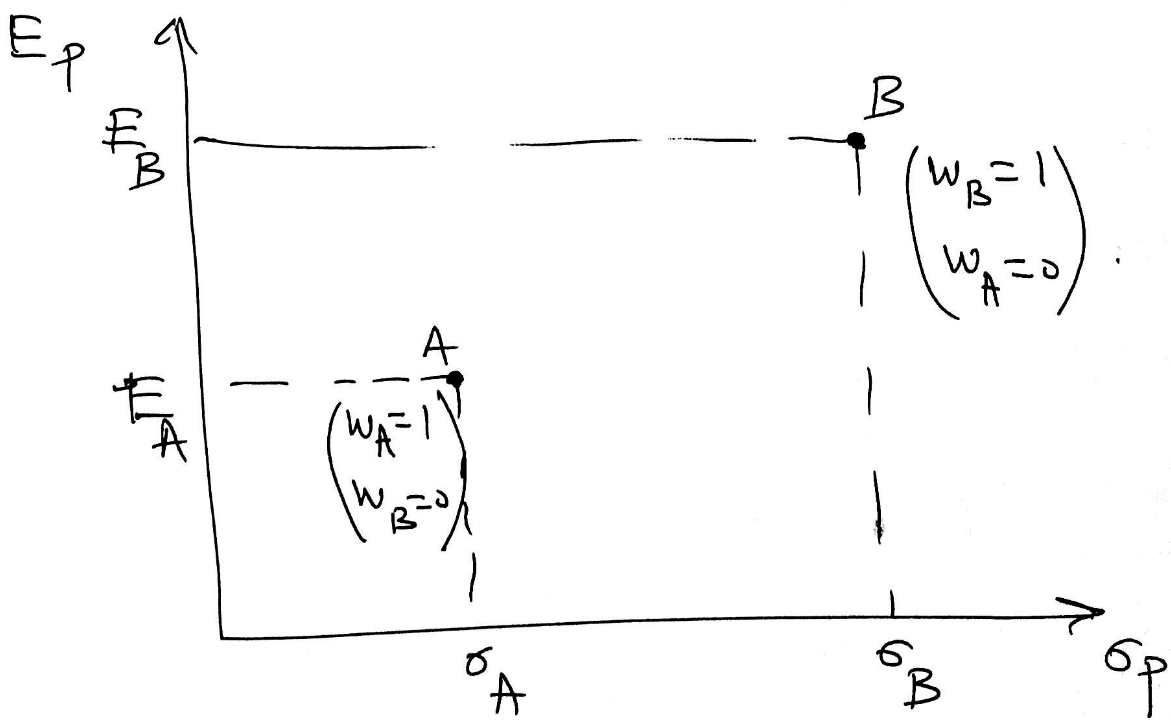
Min.

THE EFFICIENT FRONTIER

let's consider a PF of 2 Stocks. A and B. with.

$E_A < E_B$, w_A,
$\sigma_A < \sigma_B$, w_B,
Corr (A, B) is given.

Where we can plot all the unlimited portfolio in the Risk-Return graph.



Case 1: $\text{CORR}(A,B) = +1$

A, B Are perfectly and positively CORRELATED.

$$E_P = w_A E_A + w_B E_B \quad \boxed{\text{Eq. 1}}$$

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \underbrace{\text{CORR}(A,B)}_{=+1} \cdot \sigma_A \sigma_B$$

$\text{CORR}(A,B) = +1$.

$$\sigma_P^2 = \underline{w_A^2 \sigma_A^2} + \underline{w_B^2 \sigma_B^2} + \underline{2 w_A w_B \sigma_A \sigma_B}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow \sigma_P^2 = \left[w_A \sigma_A + w_B \sigma_B \right]^2$$

$$\Rightarrow \sigma_P = w_A \sigma_A + w_B \sigma_B \quad \boxed{\text{Eq 2}}$$

We know that:

$$w_A + w_B = 1 \Rightarrow w_B = 1 - w_A$$

$$\sigma_P = w_A \sigma_A + (1 - w_A) \cdot \sigma_B$$

$$\boxed{\sigma_P = w_A \left[\sigma_A - \sigma_B \right] + \sigma_B}$$

We Replace w_A from Eq. 2 in Eq. 1.

$$E_p = w_A E_A + w_B E_B .$$

$$= w_A E_A + (1 - w_A) E_B$$

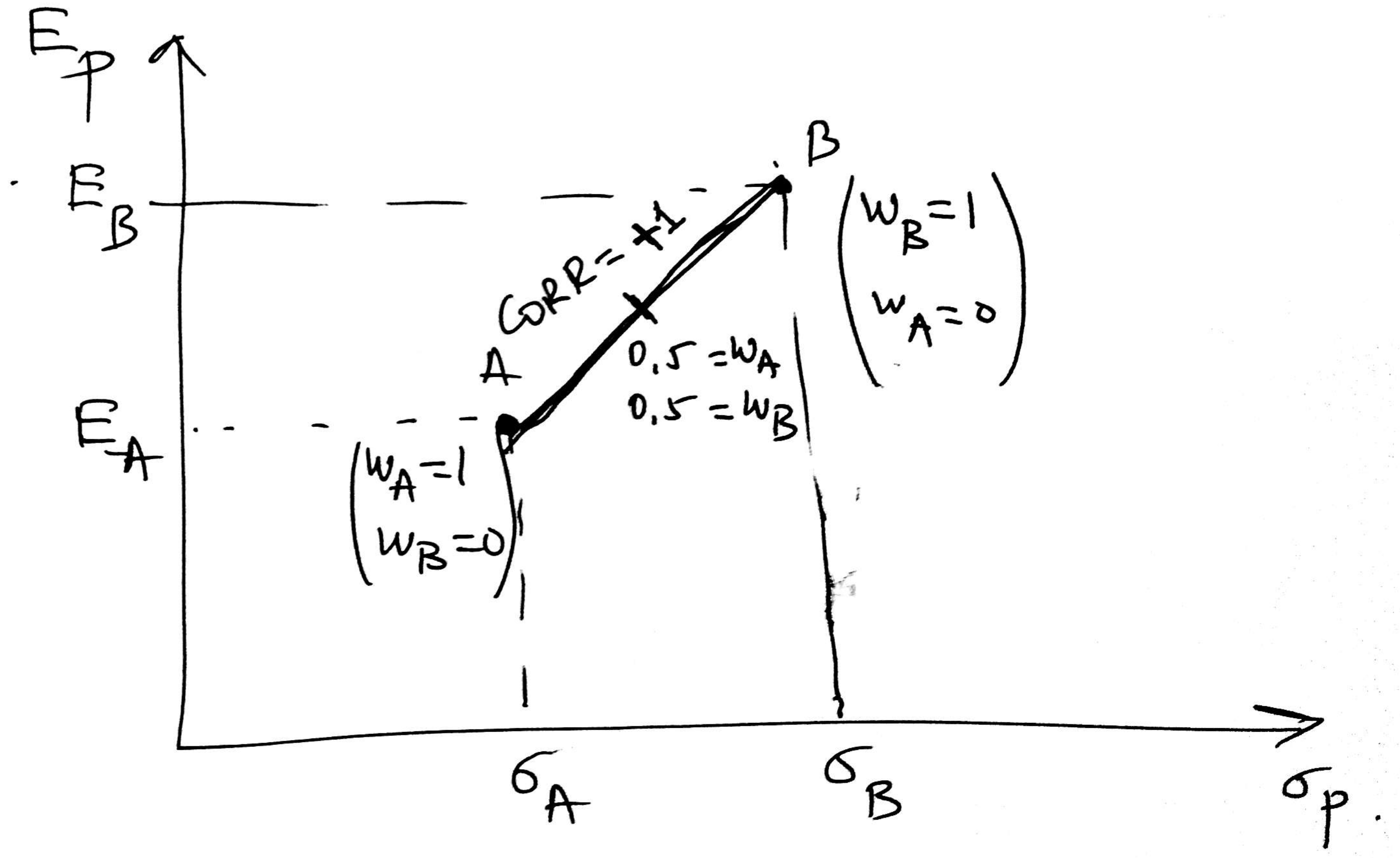
$$\boxed{E_p = w_A \left[E_A - E_B\right] + E_B}$$

$$E_p = \left[\frac{\sigma_p - \sigma_B}{\sigma_A - \sigma_B}\right] \left[E_A - E_B\right] + E_B .$$

$$E_p = \sigma_p \cdot \underbrace{\left(\frac{E_A - E_B}{\sigma_A - \sigma_B}\right)}_{A} - \underbrace{\sigma_B \left(\frac{E_A - E_B}{\sigma_A - \sigma_B}\right)}_{B}$$

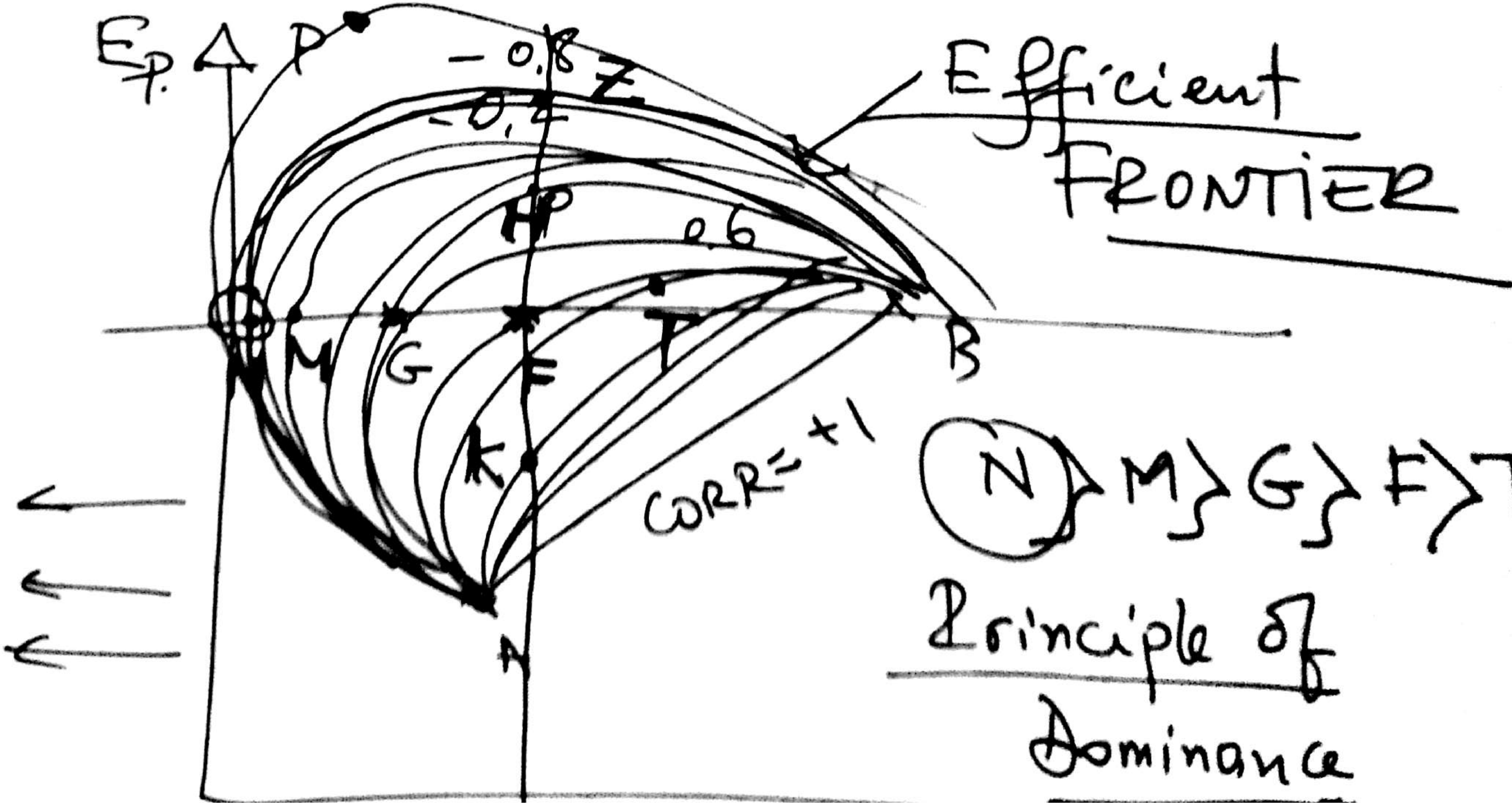
$$+ E_B -$$

For All the Portfolios Combining the 2 Stocks A and B, when $CORR(A, B) = +1$, we have a positive and linear Relationship between their Expected Return and Risk.



σ_p.

PFC is a portfolio containing A and B when CORR(A,B) = +0.6.



N > M > G > F > T

Principle of Dominance

Efficient PF

For the same level of Expected Return, PF (N) has the lowest level of risk $\Rightarrow$

$\boxed{\text{N is an Efficient Portfolio}}$

However, G, T, M, F Are possible ~~ps~~ PFs, but inefficient

N is a Dominating PF.
G, T, M, F are dominated PFs.

Similarly, for the same level of Risk, PF (Z) has the highest Expected Return. $\Rightarrow$

$Z \succ H \succ E \succ K$

Z is an Efficient Portfolio

$\Rightarrow$ Efficient Frontier is a curve joining All the Efficient Portfolios

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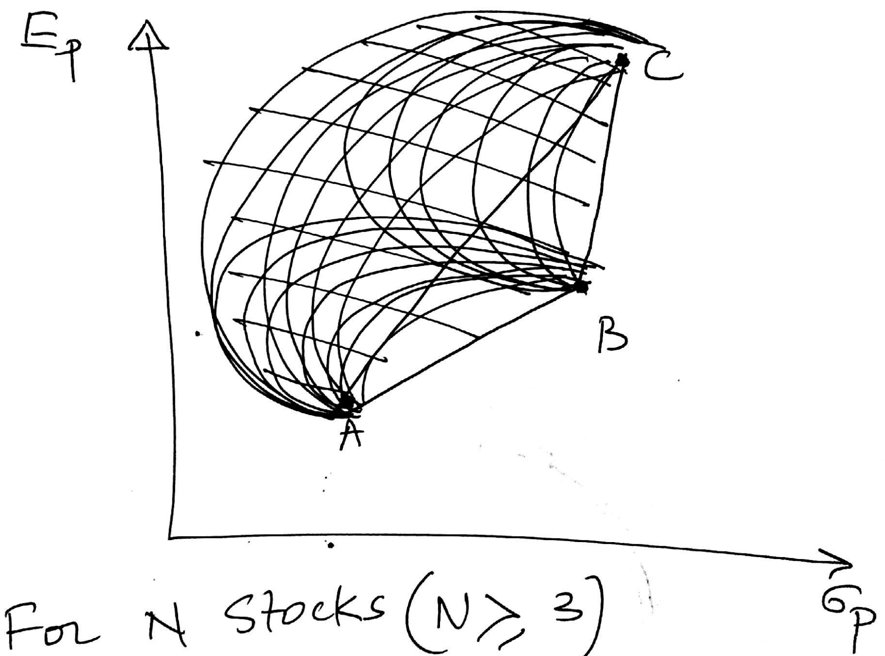
Efficient:

Frontier: ~~because~~ it Makes the separation between possible and impossible PFs.

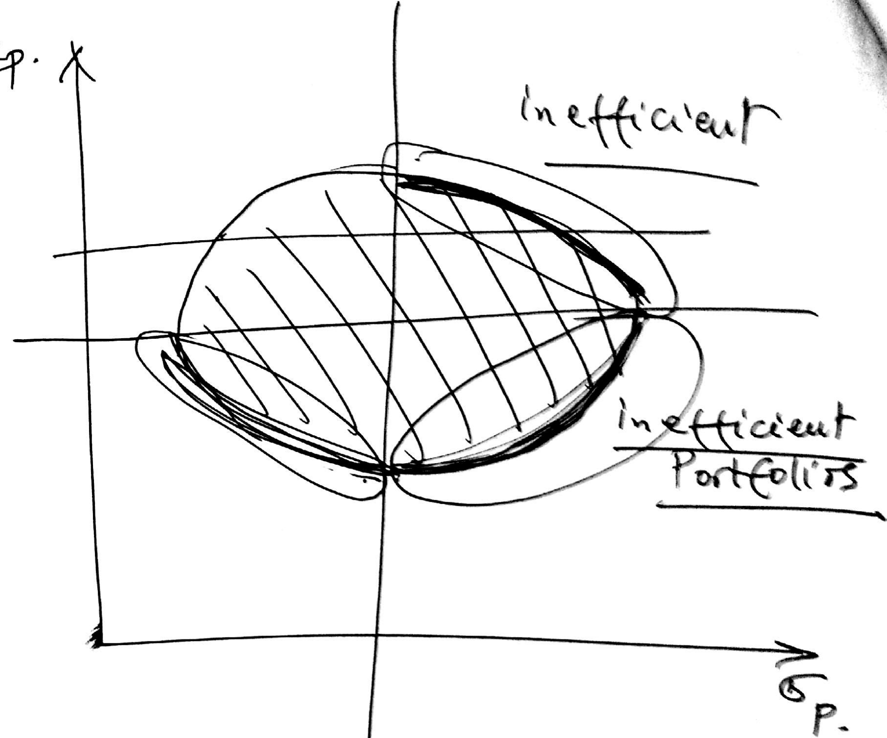
Efficient Frontier $\Rightarrow$ We use only Risky Assets (STOCKS)

For 3 Stocks. (A, B, C)

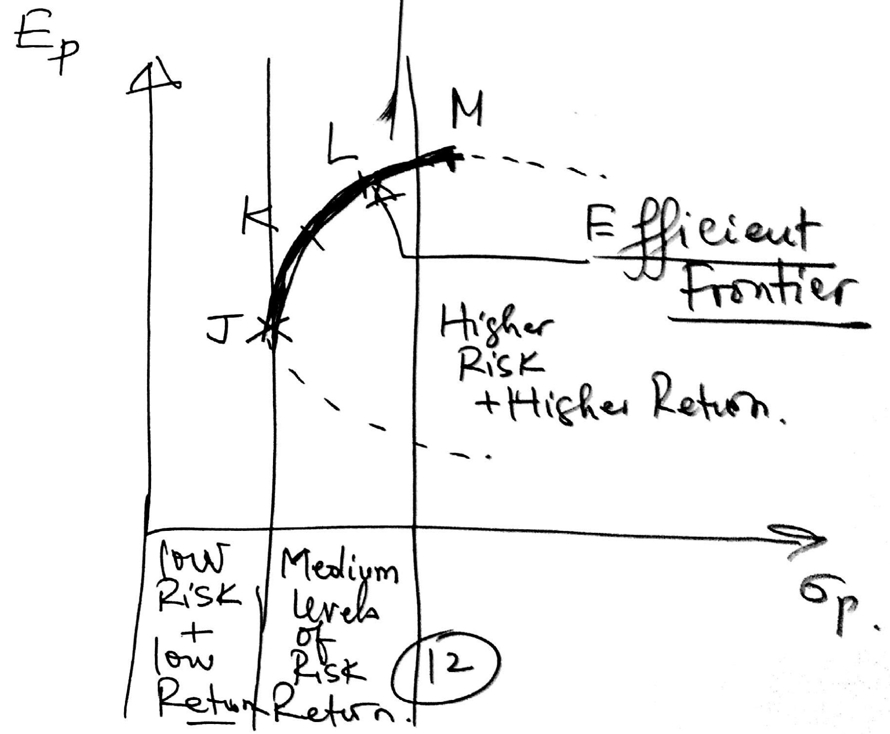
Let's draw the Efficient frontier. For 3 Stocks.



For N Stocks ($N \geq 3$)



Ep.
Inefficient
Inefficient
Portfolios
σp.



Ep
M
L
K
J
Efficient
Frontier
Higher
Risk
+Higher Return.
low
Risk
+
low
Return
Medium
levels
of
Risk
Return.
σp.

Comments:

- All the PFs, J, K, L, M are Efficient.
- The Selection of Best Portfolio (optimal portfolio) will depends on the degree of Risk Aversion.
- Risk Aversion is High. $\Longrightarrow$ J
- Medium. R. Aversion $\Longrightarrow$ K, L
- Low Risk Aversion $\Longrightarrow$ M (Risk Taker)

$\Rightarrow$ Saudi Web. (TADWUL)

- 2 FIRMS (Rajhi + Jarir)
- Data Prices (last 2 months)
- Return. $\left(R_t = \frac{P_{t+1} - P_t}{P_t}\right)$
- Expected Return $= \frac{\sum R_i}{n}$.
- Risk $= \sigma^2$
- Correlation (Rajhi, Jarir)

R_{Paghi}	R_{jariz}	W_R	W_J	E_p	σ_p^2
		1	0		
		0,9	0,1		
		0,8	0.2		
		0,7	0.3		
		0,6	0.4		
		0,5	0,5		
		0,4	0.6		
		0,3	0.7		
		0,2	0,8		
		0,1	0.9		
		0	1		

11 Portfolis

Excel