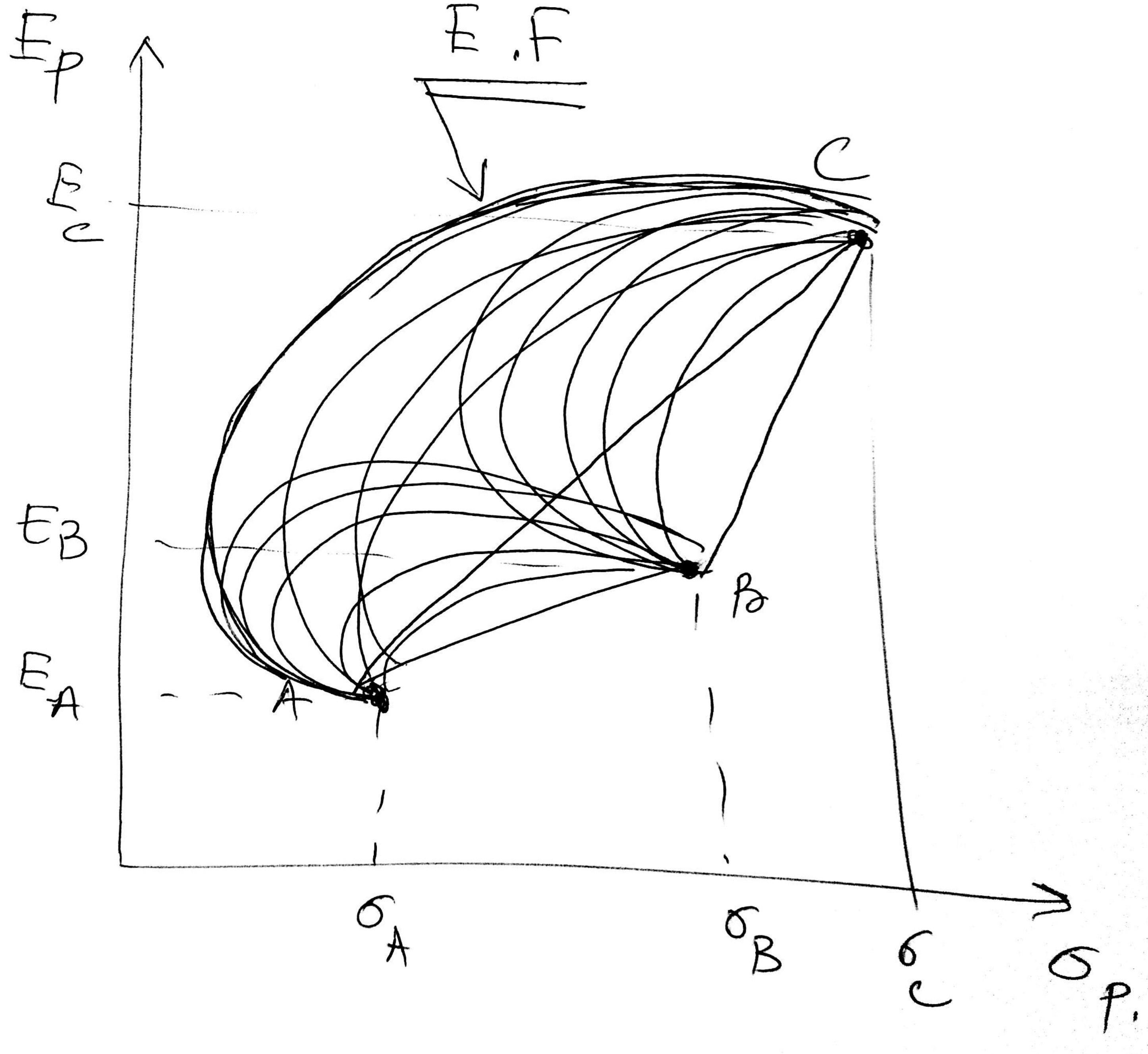
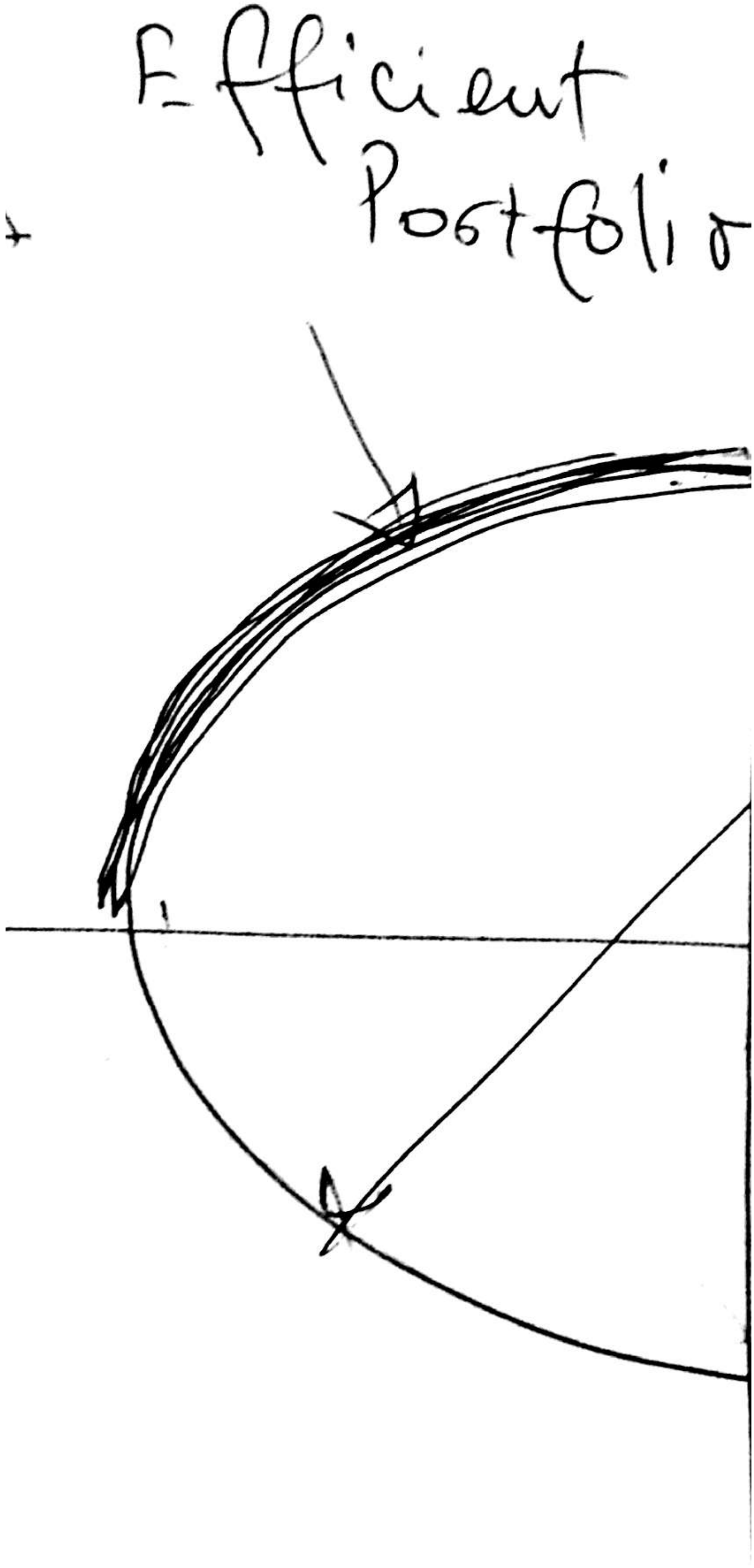
1 Extra-Materials Class - of . April 27, 2017

Efficient Frontier for PFs of 3 Stocks A, B, and C.




Efficient
Portfolio

Slide 32:

Single Index Market Model:

= Market Model

= One-factor Model

$$\tilde{R}_i = \alpha_i + \beta_i \tilde{R}_M + \tilde{\varepsilon}_t$$

⇒ It a linear relationship between R_i for a given Stock and the Market Return ($\tilde{R}_M$)

⇒ OLS: Regression

$$\tilde{R}_i = f(\tilde{R}_M)$$

⇒ It is Called one-factor Model, because we are using the $\tilde{R}_M$ as only one-factor explaining the Stock Return.

$$\tilde{R}_i = \alpha_i + \beta_i \tilde{R}_M + \tilde{\varepsilon}_t .$$

$\tilde{\varepsilon}_t$ is containing all the other factor affecting the stock return.

It includes for example:

- CO prices
- Economic Growth.
- Currency Prices.
- Monetary Policy variables ....

Statistically,

- $E(\tilde{\varepsilon}_t) = 0$
- $cov(\tilde{\varepsilon}_t, \tilde{\varepsilon}_{t-1}) = 0$
- $\sigma^2(\tilde{\varepsilon}_t) = cst$

Model:

Implications From the Market Model:

$$\tilde{R}_{i,t} = \alpha_i + \beta_i \tilde{R}_{M_t} + \tilde{\varepsilon}_{i,t} \quad \text{OLS}$$

$$\tilde{y} = \hat{a} + \hat{b}\tilde{x} + \tilde{\varepsilon}$$

$$\boxed{\hat{b} = \frac{\sum(\tilde{x} - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}}$$

$$\hat{b} = \beta = \frac{\sum(R_M - E(R_M))(R_i - E(R_i))/n}{\sum(R_M - E(R_M))^2/n}$$

$$\boxed{\beta_i = \frac{\operatorname{cov}(\tilde{R}_i, \tilde{R}_M)}{\sigma^2_M}}$$

$\beta_M = 1$?

$\cancel{\beta_M} =$

$$\beta_M = \frac{Cov(\tilde{R}_M, \tilde{R}_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1.$$

$$Cov(\tilde{R}_M, \tilde{R}_M) = \frac{\sum (R_M - E(R_M))(R_M - E(R_M))}{n}$$

$$= \frac{\sum (R_M - E(R_M))^2}{n}$$

$$= \sigma_M^2.$$

Implication 1: from the Market Model.

→ Derivation of Beta for Stock **i**.

$$Beta_i = \frac{Cov(\tilde{R}_i, \tilde{R}_M)}{\sigma_M^2}.$$

(Proof of Beta)

Derivation of Systematic and unsystematic Risk:

$$\tilde{R}_i = \alpha_i + \beta_i \tilde{R}_M + \tilde{\varepsilon}_t$$

$$\underbrace{\sigma^2(\tilde{R}_i)}_{\boxed{\text{Total Risk}}} = \sigma^2(\alpha_i + \beta_i \tilde{R}_M + \tilde{\varepsilon}_t)$$

$$= \boxed{\sigma^2(\alpha_i)}^{0} + \sigma^2(\beta_i \tilde{R}_M) + \sigma^2(\tilde{\varepsilon}_t) + \underbrace{2\,\mathrm{cov}(\alpha_i, \tilde{\varepsilon}_t)}_{0} + \underbrace{2\,\mathrm{cov}(\alpha_i, \beta_i \tilde{R}_M)}_{0} + \underbrace{2\,\mathrm{cov}(\beta_i \tilde{R}_M, \tilde{\varepsilon}_t)}_{0}$$

α_i is Constant $\Rightarrow \sigma^2(\alpha_i) = 0$

$\mathrm{cov}(\alpha_i, \tilde{\varepsilon}_i) = 0$

$\mathrm{cov}(\beta_i \tilde{R}_M, \alpha_i) = 0$

$\mathrm{cov}(\beta_i \tilde{R}_M, \tilde{\varepsilon}_t) = 0$

$$\boxed{Var(a\tilde{x}) = a^2 Var(\tilde{x})}$$

$$\underbrace{\sigma^2(\tilde{R}_i)}_{\text{Total Risk}} = \underbrace{\sigma^2(\beta_i \tilde{R}_M)}_{\boxed{\text{Market Risk, Syst. Risk}}} + \underbrace{\sigma^2(\tilde{\varepsilon}_t)}_{\text{Unsystematic Risk}}$$

Unsystematic Risk = Specific Risk = Residual Risk

$$10\% = 8\% + 2\%$$

$$\sigma^2(\tilde{R}_i) = \underbrace{\beta_i^2 \sigma_M^2}_{\text{Market Risk}} + \underbrace{\sigma^2(\tilde{\varepsilon}_t)}_{\text{Residual Risk}}$$

Using the Market Model we can derive the two parts of total Risk for a given asset. (Stock).

% of Market Risk: $\dfrac{\beta_i^2 \sigma_M^2}{\sigma_i^2}$

% of Unsyst Risk := $\dfrac{\sigma^2(\tilde{\varepsilon}_t)}{\sigma_i^2}$

(% of Market Risk) + (% of Residual) = 100%

—

Because, it includes common - unknown variables, the error ($\varepsilon_{i,t}$) term is unknown. Therefore, we cannot estimate $\sigma^2(\varepsilon_{i,t})$ directly.

Comment!

Total Risk = 10%

Market Risk = 8%

Residual Risk = 2%

$$10\% = \underbrace{8\%}_{S.R} + \underbrace{2\%}_{\text{UNSYST RISK}}$$

- Technical Analysis.
- Fundamental Analysis

The Most part of total Risk is generated by the Market. (% of Syst. Risk $\sum$ = 80% = 8/10),

So, it is more useful for investors/Traders to use Technical Analysis to:

- predict Stock Return.
- Allocate Assets in PFs
- Take suitable investment Decisions.

Firm Fundamentals Cannot help to predict Stock Behavior over time.

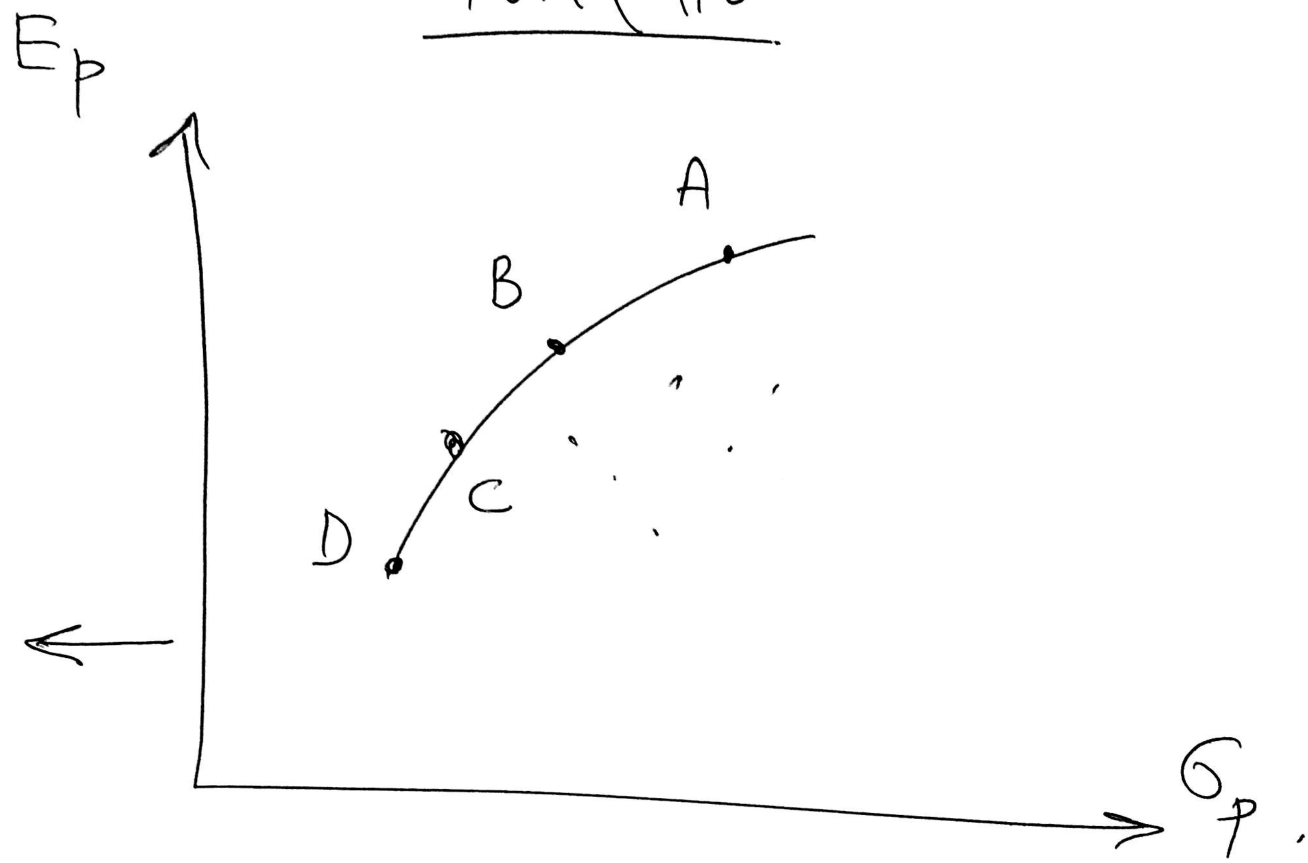
$$\boxed{\beta_i = \frac{\mathrm{Cov}(\tilde{R}_i, \tilde{R}_M)}{\sigma_M^2}}$$

$$\mathrm{Corr}(R_i, R_M) = \frac{\mathrm{Cov}(\tilde{R}_i, \tilde{R}_M)}{\sigma_i \, \sigma_M}$$

$$\beta_i = \frac{\mathrm{Corr}(R_i, R_M)\, \sigma_i \, \sigma_M}{\sigma_M^2}$$

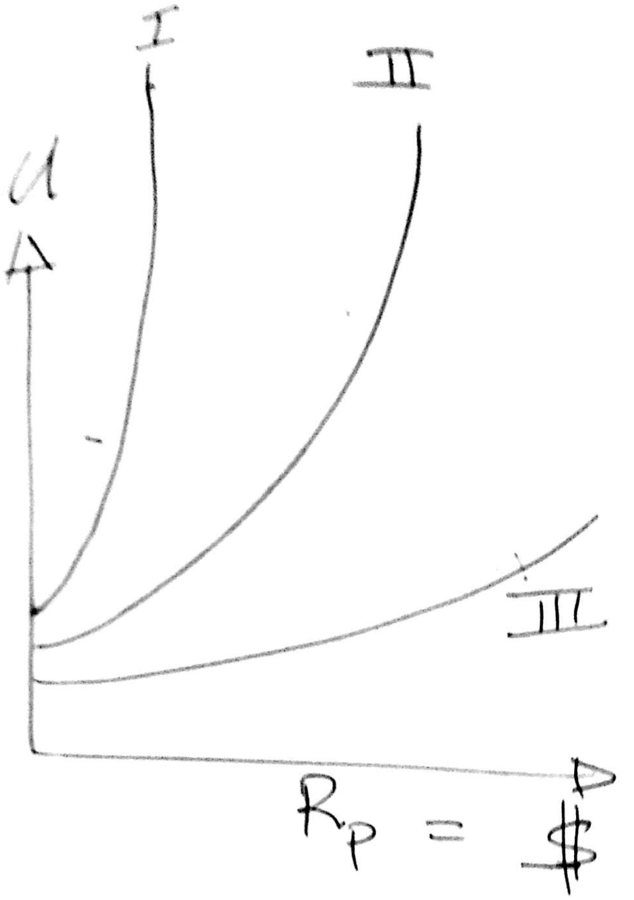
$$\boxed{\beta_i = \frac{\mathrm{Corr}(\tilde{R}_i, \tilde{R}_M) \cdot \sigma_i}{\sigma_M}}$$

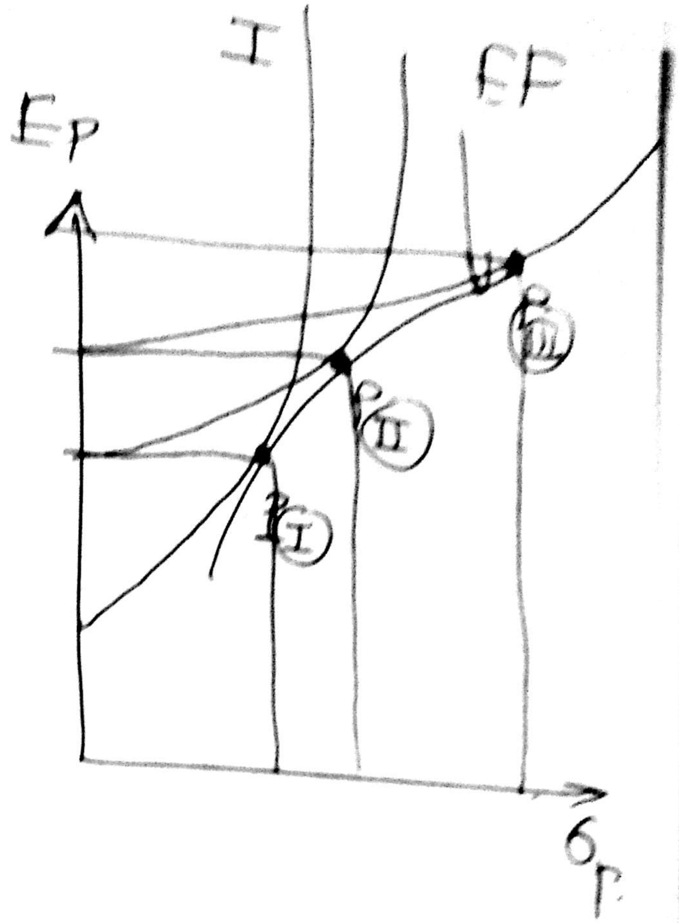
Selection of Optimal Portfolio



- The Selection of Optimal portfolio depends on the degree of Risk Aversion.
- Risk Aversion ⟷ Utility Curve

(المنفعة)





Slope of Utility curve ↑↑ ⟹ Degree of Risk Aversion ↑↑

⟹ Optimal Portfolio ⟹ Combine the 2 <u>curves</u> ⟹ determine the tangency point between the 2 <u>curves</u>.