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Class April 14th 2019

Risk For A Portfolio

Let's consider a PF of 2 stocks A and B with

Stock A	$E(R_A)$	σ_A	w_A
Stock B	$E(R_B)$	σ_B	w_B

$$\boxed{E(R_P) = \sum_{i=1}^{n} w_i E(R_i)}$$

$$\boxed{\sigma^2(R_P) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \cdot COV(A, B)}$$

We know that:

$$\boxed{CORR(A,B) = \frac{COV(A,B)}{\sigma_A \cdot \sigma_B}}$$

$$\boxed{\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B CORR(A,B) \cdot \sigma_A \cdot \sigma_B}$$

Class April, 10th, 2017

RISK FOR A PORTFOLIO:

Let's consider a PF of 2 Stocks A and B with.

Stock A	$E(R_A)$	σ_A	W_A
Stock B	$E(R_B)$	σ_B	W_B

$$\boxed{E(R_p) = \sum_{i=1} W_i E(R_i)}$$

$$\boxed{\sigma^2(R_p) = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \cdot COV(A, B)}$$

We know that:

$$\boxed{CORR(A, B) = \frac{COV(A, B)}{\sigma_A \cdot \sigma_B}}$$

$$\boxed{\sigma_p^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \, CORR(A, B) \cdot \sigma_A \cdot \sigma_B}$$

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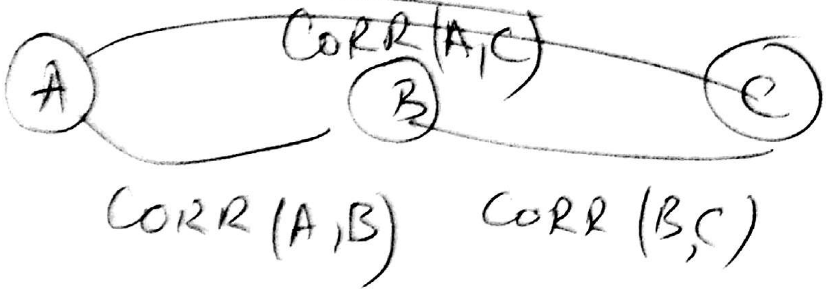
For 3 Stocks: $(n=3)$



$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + \cancel{2}\, w_C^2\sigma_C^2 +$$
$$2 w_A w_B \sigma_A \sigma_B \, CORR(A,B) +$$
$$2 w_A w_C \sigma_A \sigma_C \, CORR(A,C) +$$
$$2 w_B w_C \sigma_B \cdot \sigma_C \, CORR(B,C)$$

Why negatively correlated assets reduce total risk of the portfolios?

It is more useful for investors to include negatively CORRELATED ASSETS IN THEIR PORTFOLIOS. $\Rightarrow$

FOR $\underline{N=2}$ (2 stocks)

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + \overbrace{2 W_A W_B \cdot Cov(A,B)}^{<0}$$

When the 2 Stocks, are negatively correlated $\Rightarrow$ $\underline{Cov(A,B) < 0}$

Conclusion:

$\boxed{\text{lower CORRELATION}} \Rightarrow \boxed{\text{LOWER PORTFOLIO RISK}}$

lower Cov. $\Longrightarrow$

~~Apply~~

$$\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + \overbrace{2 W_A W_B \, CORR(A,B) \cdot \sigma_A \sigma_B}^{<0}$$

$$\boxed{CORR = \frac{COV}{\sigma_A \sigma_B}}$$

if CORR < 0.

Application (1)

PF of 3 Assets, K, L, M

$E(K) = E(L) = E(M)$

$\sigma_K = \sigma_L = \sigma_M$

$$\begin{cases} CORR(K, L) = +0{,}4 \\ CORR(L, M) = 0 \\ CORR(K, M) = -0{,}8 \end{cases}$$

What Are the 2 Stocks included in the Portfolio to Reduce Risk

$\Rightarrow$ It is more useful to include Stocks K and M to Reduce Portfolio Risk, because

$CORR(K, M) = \underline{-0{,}8}$

(Strong and Negative CORR)

App 2:

Find the Optimal Weights W_A^*, and W_B^* to have a portfolio With zero Risk. When the 2 Stocks are negatively and perfectly CORRELATED

$$PF \begin{cases} \text{Stock } A, & E_A, \sigma_A, W_A^* \\ \text{Stock } B, & E_B, \sigma_B, W_B^* \end{cases}$$

$$\boxed{W_A + W_B = 1}$$

$$\sigma_p^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_B \cdot W_B \sigma_A \sigma_B \cdot CORR(A,B)$$

$$CORR(A,B) = -1$$

$$\boxed{\sigma_p^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 - 2W_A W_B \sigma_A \sigma_B} = 0$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(W_A\sigma_A - W_B\sigma_B)^2 =$$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \overbrace{\mathrm{Corr}(A,B)}^{-1} \cdot \sigma_A \sigma_B$$

$$\mathrm{Corr}(A,B) = -1$$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 - 2 w_A w_B \sigma_A \sigma_B = 0$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(\underbrace{w_A \sigma_A - w_B \sigma_B}_{})^2 = 0 \quad , \quad A^2 = 0 \Rightarrow A = 0$$

$$w_A \sigma_A - w_B \sigma_B = 0$$

$$w_B + w_A = 1 \Rightarrow w_B = 1 - w_A$$

$$\Rightarrow \boxed{w_A^* \sigma_A - (1 - w_A) \sigma_B = 0}$$

$$w_A (\sigma_A + \sigma_B) - \sigma_B = 0$$

$$\Rightarrow \boxed{w_A^* = \frac{\sigma_B}{\sigma_A + \sigma_B}} \quad \boxed{w_B^* = 1 - w_A^*}$$

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$$W_A^* = \frac{\sigma_B}{\sigma_A + \sigma_B}, \quad W_B^* = 1 - W_A^*.$$

For a PF of 2 Stocks, A and B.

$E(R_A) = 12\%$ $\quad \sigma_A = 3\%$

$E(R_B) = 15\%$ $\quad \sigma_B = 5\%$

· $CORR(A, B) = -1$

· ~~Find the~~

1/ Calculate the Optimal Weights W_A^*, W_B^*, to have a zero-Risk Portfolio.

2°/ Comment Your results.

3°/ Calcule the $E(R_p)$ and σ_p for this portfolio.

1°/ The Optimal Weights

$$W_A^* = \frac{\sigma_B}{\sigma_A + \sigma_B} = \frac{5\%}{3\% + 5\%} = \frac{5}{8} = 62.5\%$$

$$W_B^* = 1 - W_A^* = 37.5\%$$

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2°/ **Comment :**

To have a zero Risk Portfolio.
we invest :

62,5% in Stock A.
37.5% in Stock B.

We are long on Both Stocks A and B. We invest more in Stock A having the lowest Risk

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3°/ $E(R_p^*) = W_A^* \cdot E(R_A) + W_B^* \cdot E(R_B)$

$= (0.625 \times 12\%) + (0.375 \times 15\%) = 13.125\%$

$\sigma_p^{2*} \overset{?}{=} 0 \Rightarrow$ zero Risk Portfolio.

PF | $E(R_p^*) = 13.125\%$ T.Bill $= 5\%$

$\sigma_p = 0$ $\sigma^2 = 0$

Derivation: (Proof)

$\cancel{\sigma_p^2}$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \cdot \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \overset{+1}{\overline{CORR(A,B)}}$$

$CORR(A,B) = +1$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B = 0$$

$$\sigma_p^2 = \left(w_A \sigma_A + w_B \sigma_B\right)^2 = 0$$

$$\Rightarrow w_A \sigma_A + w_B \sigma_B = 0 \qquad \boxed{w_B = 1 - w_A}$$

$$\Rightarrow w_A \sigma_A + (1 - w_A) \cdot \sigma_B = 0$$

$$\Rightarrow w_A \left[\sigma_A - \sigma_B\right] + \sigma_B = 0$$

$$\Rightarrow \boxed{w_A^* = \frac{-\sigma_B}{\sigma_A - \sigma_B}}$$

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1°/ $\boxed{W_A^* = \dfrac{-\sigma_B}{\sigma_A - \sigma_B}}$

$E(R_A) = 12\%$, $\sigma_A = 3\%$

$E(R_B) = 15\%$, $\sigma_B = 5\%$

$$W_A^* = \frac{-5\%}{3\% - 5\%} = \frac{-5}{-2} = \underline{2.5}$$

$$W_B^* = 1 - W_A^* = 1 - 2.5 = -1.5$$

2°/ <u>Comment:</u>

To have a zero risk portfolio ⇒ we take:

- Short position (−1.5) on stock B
- Long position (2.5) in stock A

Because, Corr(A,B) = +1, we have to take opposite trading positions to have a PF with zero risk

3°/ $E(R_P^*) = 2.5 \times (12\%) - 1.5 \times (15\%)$

$= 30\% - 22.5\% = \underline{7.5\%}$

$\sigma_P = 0$

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How to Find Optimal Weights to minimize Portfolio Risk

Let's Consider a PF of 2 stocks. A and B, with:

$E(R_A)$, σ_A, w_A $\quad \boxed{w_A + w_B = 1}$

$E(R_B)$, σ_B, w_B

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \cdot cov(A,B)$$

$$w_A + w_B = 1 \Rightarrow w_B = 1 - w_A$$

$$\Rightarrow \sigma_p^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2 w_A (1 - w_A) Cov(A,B)$$

$$= w_A^2 \sigma_A^2 + (1 - 2w_A + w_A^2) \sigma_B^2 + 2 w_A \, Cov(A,B) - 2 w_A^2 \cdot cov(A,B)$$

$$= w_A^2 \sigma_A^2 + \sigma_B^2 - 2 w_A \sigma_B^2 + w_A^2 \sigma_B^2 + 2 w_A Cov(A,B) - 2 w_A^2 cov(A,B)$$

σ_P^2

$$w_A^2\left[\sigma_A^2+\sigma_B^2-2\,cov(A,B)\right]+$$

$$w_A\left[-2\sigma_B^2+2\,cov(A,B)\right]+\sigma_B^2$$

$$f_{min} \longrightarrow \underline{f'=0}$$

$$2w_A\left[\sigma_A^2+\sigma_B^2-2\,cov(A,B)\right]+$$

$$\left[-2\sigma_B^2+2\,cov(A,B)\right]=0$$

$$\boxed{w_A^*=\frac{2\sigma_B^2-2\,cov(A,B)}{\sigma_A^2+\sigma_B^2-2\,cov(A,B)}}$$

$$\boxed{w_B^*=1-w_A^*}$$

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