

## Endurance Limit Modifying Factors

We have seen that the rotating-beam specimen used in the laboratory to determine endurance limits is prepared very carefully and tested under closely controlled conditions. It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory. Some differences include

- *Material:* composition, basis of failure, variability
- *Manufacturing:* method, heat treatment, fretting corrosion, surface condition, stress concentration
- *Environment:* corrosion, temperature, stress state, relaxation times
- *Design:* size, shape, life, stress state, stress concentration, speed, fretting, galling

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \quad (6-18)$$

where  $k_a$  = surface condition modification factor

$k_b$  = size modification factor

$k_c$  = load modification factor

$k_d$  = temperature modification factor

$k_e$  = reliability factor<sup>13</sup>

$k_f$  = miscellaneous-effects modification factor

$S'_e$  = rotary-beam test specimen endurance limit

$S_e$  = endurance limit at the critical location of a machine part in the geometry and condition of use

### Surface Factor $k_a$

The surface of a rotating-beam specimen is highly polished, with a final polishing in the axial direction to smooth out any circumferential scratches. The surface modification factor depends on the quality of the finish of the actual part surface and on the tensile strength of the part material. To find quantitative expressions for common finishes of machine parts (ground, machined, or cold-drawn, hot-rolled, and as-forged), the coordinates of data points were recaptured from a plot of endurance limit versus ultimate tensile strength of data gathered by Lipson and Noll and reproduced by Horger.<sup>14</sup> The data can be represented by

$$k_a = aS_{ut}^b \quad (6-19)$$

where  $S_{ut}$  is the minimum tensile strength and  $a$  and  $b$  are to be found in Table 6–2.

| Surface Finish         | Factor $a$      |                | Exponent $b$ |
|------------------------|-----------------|----------------|--------------|
|                        | $S_{ut}$ , kpsi | $S_{ut}$ , MPa |              |
| Ground                 | 1.34            | 1.58           | −0.085       |
| Machined or cold-drawn | 2.70            | 4.51           | −0.265       |
| Hot-rolled             | 14.4            | 57.7           | −0.718       |
| As-forged              | 39.9            | 272.           | −0.995       |

### Size Factor $k_b$

The size factor has been evaluated using 133 sets of data points.<sup>15</sup> The results for bending and torsion may be expressed as

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases} \quad (6-20)$$

For axial loading there is no size effect, so

$$k_b = 1 \quad (6-21)$$

#### EXAMPLE 6-4

A steel shaft loaded in bending is 52 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor  $k_b$  if the shaft is used in

- (a) A rotating mode.
- (b) A nonrotating mode.

**Solution** (a) From Eq. (6-20)

**Answer**  $k_b = 1.51d^{-0.157} = 1.51(52)^{-0.157} = 0.812$

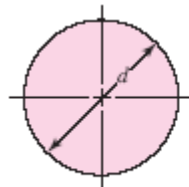
(b) From Table 6-3,

$$d_e = 0.37d = 0.37(52) = 19.24 \text{ mm}$$

From Eq. (6-20),

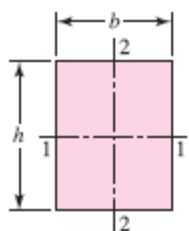
**Answer**  $k_b = \left(\frac{19.24}{7.62}\right)^{-0.107} = 0.906$

# Common Nonrotating Structural Shapes



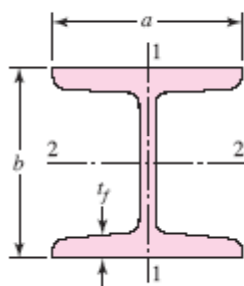
$$A_{0.95\sigma} = 0.01046d^2$$

$$d_e = 0.370d$$

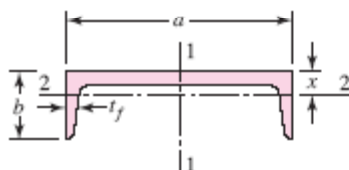


$$A_{0.95\sigma} = 0.05hb$$

$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a \quad \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b-x) & \text{axis 2-2} \end{cases}$$

## Loading Factor $k_c$

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with  $S_{ut}$ . This is discussed further in Sec. 6–17. Here, we will specify average values of the load factor as

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \quad (6-26)$$

## Temperature Factor $k_d$

Effect of Operating Temperature on the Tensile Strength of Steel.\* ( $S_T$  = tensile strength at operating temperature;  $S_{RT}$  = tensile strength at room temperature;  $0.099 \leq \hat{\sigma} \leq 0.110$ )

| Temperature, °C | $S_T/S_{RT}$ | Temperature, °F | $S_T/S_{RT}$ |
|-----------------|--------------|-----------------|--------------|
| 20              | 1.000        | 70              | 1.000        |
| 50              | 1.010        | 100             | 1.008        |
| 100             | 1.020        | 200             | 1.020        |
| 150             | 1.025        | 300             | 1.024        |
| 200             | 1.020        | 400             | 1.018        |
| 250             | 1.000        | 500             | 0.995        |
| 300             | 0.975        | 600             | 0.963        |
| 350             | 0.943        | 700             | 0.927        |
| 400             | 0.900        | 800             | 0.872        |
| 450             | 0.843        | 900             | 0.797        |
| 500             | 0.768        | 1000            | 0.698        |
| 550             | 0.672        | 1100            | 0.567        |
| 600             | 0.549        |                 |              |

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 \\ + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \quad (6-27)$$

where  $70 \leq T_F \leq 1000^\circ\text{F}$ .

Two types of problems arise when temperature is a consideration. If the rotating-beam endurance limit is known at room temperature, then use

$$k_d = \frac{S_T}{S_{RT}} \quad (6-28)$$

from Table 6-4 or Eq. (6-27) and proceed as usual. If the rotating-beam endurance limit is not given, then compute it using Eq. (6-8) and the temperature-corrected tensile strength obtained by using the factor from Table 6-4. Then use  $k_d = 1$ .

#### EXAMPLE 6-5

A 1035 steel has a tensile strength of 490 MPa and is to be used for a part that sees  $230^\circ\text{C}$  in service. Estimate the Marin temperature modification factor and  $(S_e)_{230^\circ}$  if

- (a) The room-temperature endurance limit by test is  $(S'_e)_{37^\circ} = 270$  MPa.  
 (b) Only the tensile strength at room temperature is known.

#### Solution

(a) First, from Eq. (6-27),

$$k_d = 0.9877 + 0.6507(10^{-3})(230) - 0.3414(10^{-5})(230^2) \\ + 0.5621(10^{-8})(230^3) - 6.246(10^{-12})(230^4) = 1.00767$$

Thus,

#### Answer

$$(S_e)_{230^\circ} = k_d (S'_e)_{37^\circ} = 1.00767(270) = 272.07 \text{ MPa}$$

(b) Interpolating from Table 6-4 gives

$$(S_T) S_{RT})_{230^\circ} = 1.02 + (1.0 - 1.02) \frac{230 - 200}{250 - 200} = 1.0197$$

Thus, the tensile strength at  $230^\circ\text{C}$  is estimated as

$$(S_{ut})_{230^\circ} = (S_T / S_{RT})_{230^\circ} (S_{ut})_{37^\circ} = 1.0197(490) = 499.7 \text{ MPa}$$

From Eq. (6-8) then,

#### Answer

$$(S_e)_{230^\circ} = 0.5(S_{ut})_{230^\circ} = 0.5(499.7) = 249.9 \text{ MPa}$$

Part a gives the better estimate due to actual testing of the particular material.

**Reliability Factor  $k_e$** 

$$k_e = 1 - 0.08 z_a \quad (6-29)$$

where  $z_a$  is defined by Eq. (20-16) and values for any desired reliability can be determined from Table A-10. Table 6-5 gives reliability factors for some standard specified reliabilities.

| Reliability, % | Transformation Variate $z_a$ | Reliability Factor $k_e$ |
|----------------|------------------------------|--------------------------|
| 50             | 0                            | 1.000                    |
| 90             | 1.288                        | 0.897                    |
| 95             | 1.645                        | 0.868                    |
| 99             | 2.326                        | 0.814                    |
| 99.9           | 3.091                        | 0.753                    |
| 99.99          | 3.719                        | 0.702                    |
| 99.999         | 4.265                        | 0.659                    |
| 99.9999        | 4.753                        | 0.620                    |

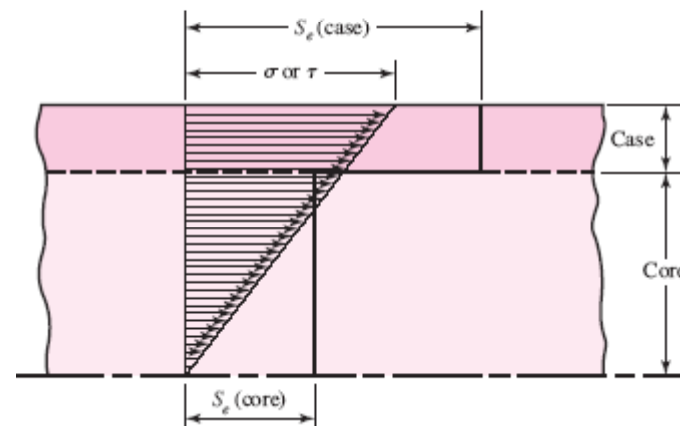
### Miscellaneous-Effects Factor $k_f$

Though the factor  $k_f$  is intended to account for the reduction in endurance limit due to all other effects, it is really intended as a reminder that these must be accounted for, because actual values of  $k_f$  are not always available.

*Residual stresses* may either improve the endurance limit or affect it adversely. Generally, if the residual stress in the surface of the part is compression, the endurance limit is improved. Fatigue failures appear to be tensile failures, or at least to be caused by tensile stress, and so anything that reduces tensile stress will also reduce the possibility of a fatigue failure. Operations such as shot peening, hammering, and cold rolling build compressive stresses into the surface of the part and improve the endurance limit significantly. Of course, the material must not be worked to exhaustion.

The endurance limits of parts that are made from rolled or drawn sheets or bars, as well as parts that are forged, may be affected by the so-called *directional characteristics* of the operation. Rolled or drawn parts, for example, have an endurance limit in the transverse direction that may be 10 to 20 percent less than the endurance limit in the longitudinal direction.

Parts that are case-hardened may fail at the surface or at the maximum core radius, depending upon the stress gradient. Figure 6–19 shows the typical triangular stress distribution of a bar under bending or torsion. Also plotted as a heavy line in this figure are the endurance limits  $S_e$  for the case and core. For this example the endurance limit of the core rules the design because the figure shows that the stress  $\sigma$  or  $\tau$ , whichever applies, at the outer core radius, is appreciably larger than the core endurance limit.





## Corrosion

It is to be expected that parts that operate in a corrosive atmosphere will have a lowered fatigue resistance. This is, of course, true, and it is due to the roughening or pitting of the surface by the corrosive material. But the problem is not so simple as the one of finding the endurance limit of a specimen that has been corroded. The reason for this is that the corrosion and the stressing occur at the same time. Basically, this means that in time any part will fail when subjected to repeated stressing in a corrosive atmosphere. There is no fatigue limit. Thus the designer's problem is to attempt to minimize the factors that affect the fatigue life; these are:

- Mean or static stress
- Alternating stress
- Electrolyte concentration
- Dissolved oxygen in electrolyte
- Material properties and composition
- Temperature
- Cyclic frequency
- Fluid flow rate around specimen

### Electrolytic Plating

Metallic coatings, such as chromium plating, nickel plating, or cadmium plating, reduce the endurance limit by as much as 50 percent. In some cases the reduction by coatings has been so severe that it has been necessary to eliminate the plating process. Zinc plating does not affect the fatigue strength. Anodic oxidation of light alloys reduces bending endurance limits by as much as 39 percent but has no effect on the torsional endurance limit.

### Metal Spraying

Metal spraying results in surface imperfections that can initiate cracks. Limited tests show reductions of 14 percent in the fatigue strength.

### Cyclic Frequency

If, for any reason, the fatigue process becomes time-dependent, then it also becomes frequency-dependent. Under normal conditions, fatigue failure is independent of frequency. But when corrosion or high temperatures, or both, are encountered, the cyclic rate becomes important. The slower the frequency and the higher the temperature, the higher the crack propagation rate and the shorter the life at a given stress level.

### Fretage Corrosion

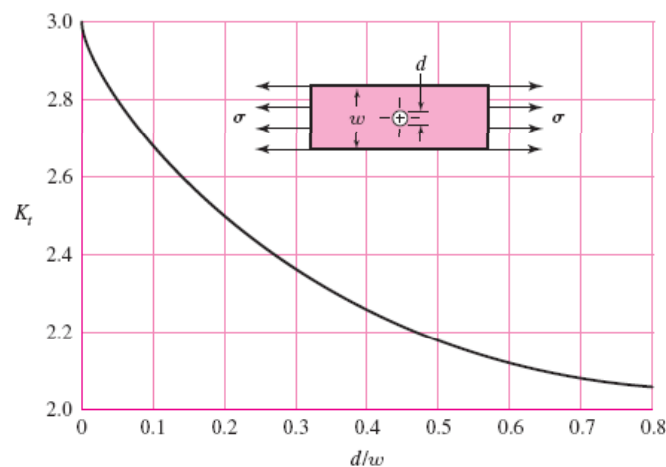
The phenomenon of fretage corrosion is the result of microscopic motions of tightly fitting parts or structures. Bolted joints, bearing-race fits, wheel hubs, and any set of tightly fitted parts are examples. The process involves surface discoloration, pitting, and eventual fatigue. The fretage factor  $k_f$  depends upon the material of the mating pairs and ranges from 0.24 to 0.90.

## Stress Concentration

Any discontinuity in a machine part alters the stress distribution in the neighborhood of the discontinuity so that the elementary stress equations no longer describe the state of stress in the part at these locations. Such discontinuities are called *stress raisers*, and the regions in which they occur are called areas of *stress concentration*.

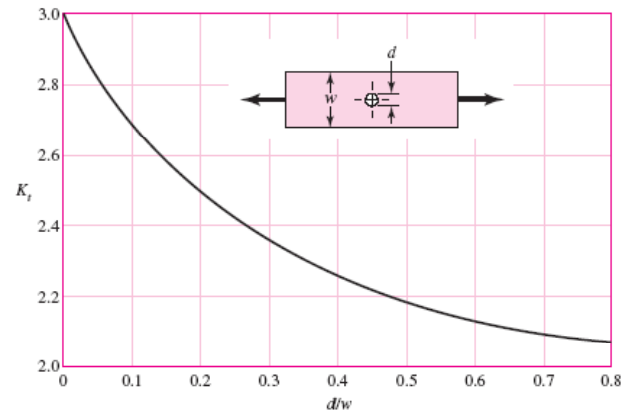
A *theoretical*, or *geometric*, *stress-concentration factor*  $K_t$  or  $K_{ts}$  is used to relate the actual maximum stress at the discontinuity to the nominal stress. The factors are defined by the equations

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \quad K_{ts} = \frac{\tau_{\max}}{\tau_0} \quad (3-48)$$



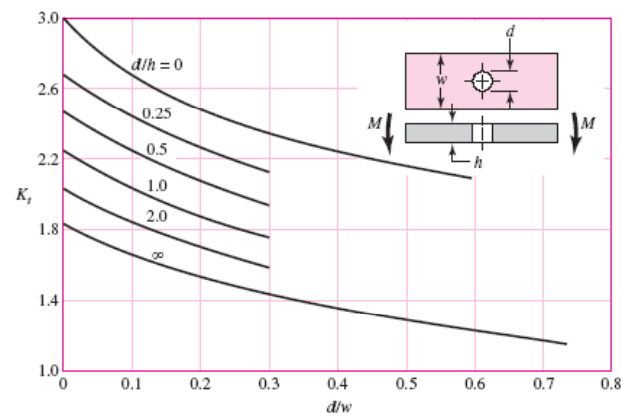
**Figure A-15-1**

Bar in tension or simple compression with a transverse hole.  $\sigma_0 = F/A$ , where  $A = (w - d)t$  and  $t$  is the thickness.



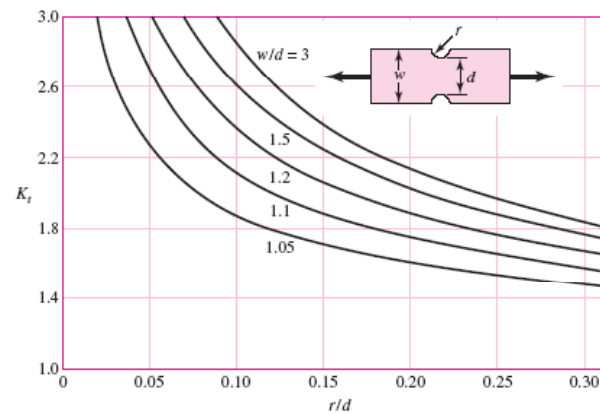
**Figure A-15-2**

Rectangular bar with a transverse hole in bending.  $\sigma_0 = Mc/I$ , where  $I = (w - d)h^3/12$ .



**Figure A-15-3**

Notched rectangular bar in tension or simple compression.  $\sigma_0 = F/A$ , where  $A = dt$  and  $t$  is the thickness.



## 6-10 Stress Concentration and Notch Sensitivity

In Sec. 3-13 it was pointed out that the existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increases the theoretical stresses significantly in the immediate vicinity of the discontinuity. Equation (3-48) defined a stress concentration factor  $K_t$  (or  $K_{ts}$ ), which is used with the nominal stress to obtain the maximum resulting stress due to the irregularity or defect. It turns out that some materials are not fully sensitive to the presence of notches and hence, for these, a reduced value of  $K_t$  can be used. For these materials, the maximum stress is, in fact,

$$\sigma_{\max} = K_f \sigma_0 \quad \text{or} \quad \tau_{\max} = K_{fs} \tau_0 \quad (6-30)$$

where  $K_f$  is a reduced value of  $K_t$  and  $\sigma_0$  is the nominal stress. The factor  $K_f$  is commonly called a *fatigue stress-concentration factor*, and hence the subscript  $f$ . So it is convenient to think of  $K_f$  as a stress-concentration factor reduced from  $K_t$  because of lessened sensitivity to notches. The resulting factor is defined by the equation

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}} \quad (a)$$

*Notch sensitivity*  $q$  is defined by the equation

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1} \quad (6-31)$$

where  $q$  is usually between zero and unity. Equation (6-31) shows that if  $q = 0$ , then  $K_f = 1$ , and the material has no sensitivity to notches at all. On the other hand, if  $q = 1$ , then  $K_f = K_t$ , and the material has full notch sensitivity. In analysis or design work, find  $K_t$  first, from the geometry of the part. Then specify the material, find  $q$ , and solve for  $K_f$  from the equation

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) \quad (6-32)$$

For steels and 2024 aluminum alloys, use Fig. 6–20 to find  $q$  for bending and axial loading. For shear loading, use Fig. 6–21. In using these charts it is well to know that the actual test results from which the curves were derived exhibit a large amount of

about the true value of  $q$ . Also, note that  $q$  is not far from unity for large notch radii.

The notch sensitivity of the cast irons is very low, varying from 0 to about 0.20, depending upon the tensile strength. To be on the conservative side, it is recommended that the value  $q = 0.20$  be used for all grades of cast iron.

Figure 6–20 has as its basis the *Neuber equation*, which is given by

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \quad (6-33)$$

where  $\sqrt{a}$  is defined as the *Neuber constant* and is a material constant. Equating Eqs. (6–31) and (6–33) yields the notch sensitivity equation

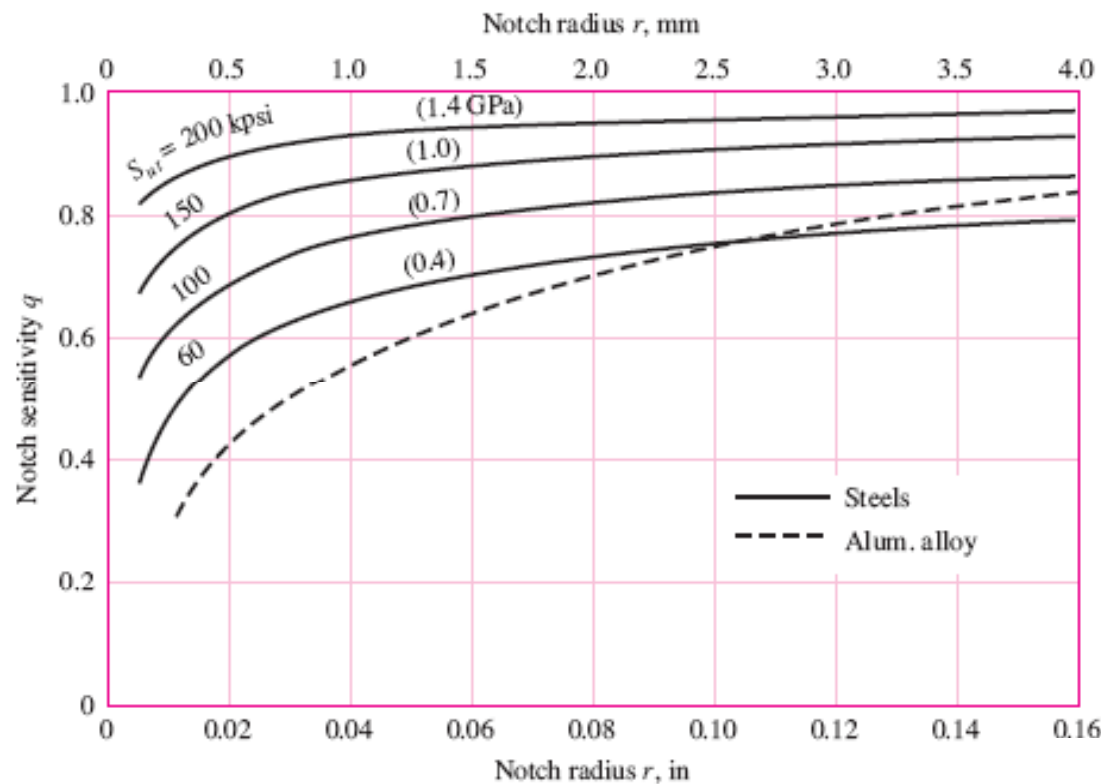
$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \quad (6-34)$$

For steel, with  $S_{ut}$  in *kpsi*, the Neuber constant can be approximated by a third-order polynomial fit of data as

$$\text{Bending or axial: } \sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad (6-35a)$$

$$\text{Torsion: } \sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad (6-35b)$$

To be conservative, it is recommended that the value of  $q = 0.2$  for all grades of cast iron.

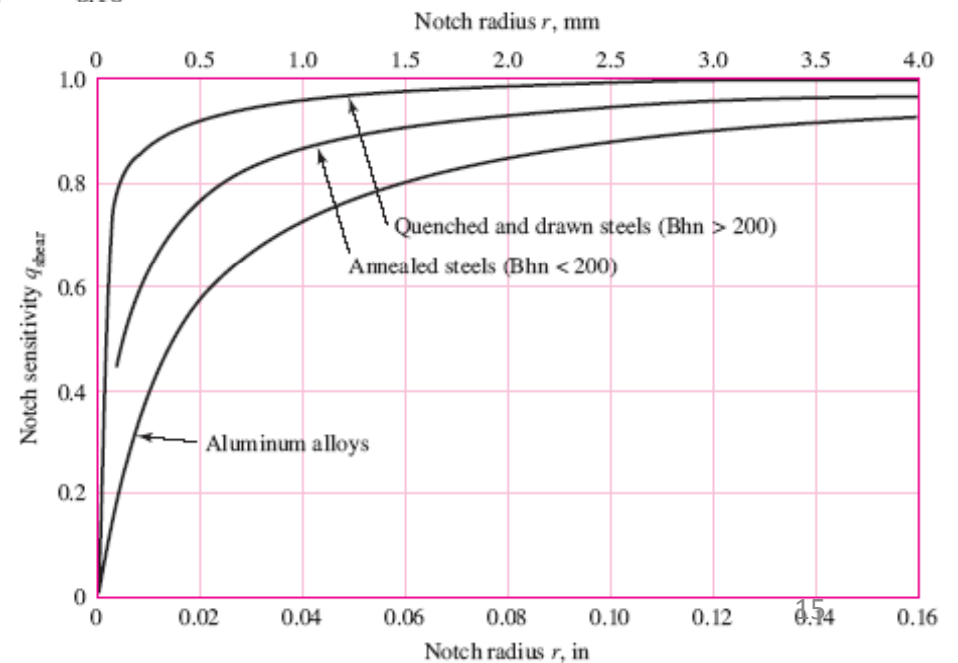


**Figure 6-20**

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of  $q$  corresponding to the  $r = 0.16$ -in (4-mm) ordinate. (From George Sines and J. L. Waisman (eds.), *Metal Fatigue*, McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)

**Figure 6-21**

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of  $q_{\text{shear}}$  corresponding to  $r = 0.16$  in (4 mm).



**EXAMPLE 6-6**

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate  $K_f$  using:

(a) Figure 6-20.

(b) Equations (6-33) and (6-35).

**Solution**

From Fig. A-15-9, using  $D/d = 38/32 = 1.1875$ ,  $r/d = 3/32 = 0.09375$ , we read the graph to find  $K_t \doteq 1.65$ .

(a) From Fig. 6-20, for  $S_{ut} = 690$  MPa and  $r = 3$  mm,  $q \doteq 0.84$ . Thus, from Eq. (6-32)

**Answer**

$$K_f = 1 + q(K_t - 1) \doteq 1 + 0.84(1.65 - 1) = 1.55$$

(b) From Eq. (6-35) with  $S_{ut} = 690$  MPa = 100 kpsi,  $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$ . Substituting this into Eq. (6-33) with  $r = 3$  mm gives

**Answer**

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \doteq 1 + \frac{1.65 - 1}{1 + \frac{0.313}{\sqrt{3}}} = 1.55$$



**EXAMPLE 6-7**

For the step-shaft of Ex. 6-6, it is determined that the fully corrected endurance limit is  $S_e = 280$  MPa. Consider the shaft undergoes a fully reversing nominal stress in the fillet of  $(\sigma_{\text{rev}})_{\text{nom}} = 260$  MPa. Estimate the number of cycles to failure.

**Solution**

From Ex. 6-6,  $K_f = 1.55$ , and the ultimate strength is  $S_{ut} = 690$  MPa = 100 kpsi. The maximum reversing stress is

$$(\sigma_{\text{rev}})_{\text{max}} = K_f (\sigma_{\text{rev}})_{\text{nom}} = 1.55(260) = 403 \text{ MPa}$$

From Fig. 6-18,  $f = 0.845$ . From Eqs. (6-14), (6-15), and (6-16)

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.845(690)]^2}{280} = 1214 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{f S_{ut}}{S_e} = -\frac{1}{3} \log \left[ \frac{0.845(690)}{280} \right] = -0.1062$$

**Answer**

$$N = \left( \frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left( \frac{403}{1214} \right)^{1/-0.1062} = 32.3(10^3) \text{ cycles}$$

**Table A-20**

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ( $\frac{3}{4}$  to  $1\frac{1}{4}$  in). These strengths are suitable for use with the design factor defined in Sec. 1-10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] Source: 1986 SAE Handbook, p. 2.15.

| 1       | 2                      | 3               | 4                                  | 5                                | 6                        | 7                       | 8                   |
|---------|------------------------|-----------------|------------------------------------|----------------------------------|--------------------------|-------------------------|---------------------|
| UNS No. | SAE and/or<br>AISI No. | Process-<br>ing | Tensile<br>Strength,<br>MPa (kpsi) | Yield<br>Strength,<br>MPa (kpsi) | Elongation in<br>2 in, % | Reduction in<br>Area, % | Brinell<br>Hardness |
| G10060  | 1006                   | HR              | 300 (43)                           | 170 (24)                         | 30                       | 55                      | 86                  |
|         |                        | CD              | 330 (48)                           | 280 (41)                         | 20                       | 45                      | 95                  |
| G10100  | 1010                   | HR              | 320 (47)                           | 180 (26)                         | 28                       | 50                      | 95                  |
|         |                        | CD              | 370 (53)                           | 300 (44)                         | 20                       | 40                      | 105                 |
| G10150  | 1015                   | HR              | 340 (50)                           | 190 (27.5)                       | 28                       | 50                      | 101                 |
|         |                        | CD              | 390 (56)                           | 320 (47)                         | 18                       | 40                      | 111                 |
| G10180  | 1018                   | HR              | 400 (58)                           | 220 (32)                         | 25                       | 50                      | 116                 |
|         |                        | CD              | 440 (64)                           | 370 (54)                         | 15                       | 40                      | 126                 |
| G10200  | 1020                   | HR              | 380 (55)                           | 210 (30)                         | 25                       | 50                      | 111                 |
|         |                        | CD              | 470 (68)                           | 390 (57)                         | 15                       | 40                      | 131                 |
| G10300  | 1030                   | HR              | 470 (68)                           | 260 (37.5)                       | 20                       | 42                      | 137                 |
|         |                        | CD              | 520 (76)                           | 440 (64)                         | 12                       | 35                      | 149                 |
| G10350  | 1035                   | HR              | 500 (72)                           | 270 (39.5)                       | 18                       | 40                      | 143                 |
|         |                        | CD              | 550 (80)                           | 460 (67)                         | 12                       | 35                      | 163                 |
| G10400  | 1040                   | HR              | 520 (76)                           | 290 (42)                         | 18                       | 40                      | 149                 |
|         |                        | CD              | 590 (85)                           | 490 (71)                         | 12                       | 35                      | 170                 |
| G10450  | 1045                   | HR              | 570 (82)                           | 310 (45)                         | 16                       | 40                      | 163                 |
|         |                        | CD              | 630 (91)                           | 530 (77)                         | 12                       | 35                      | 179                 |
| G10500  | 1050                   | HR              | 620 (90)                           | 340 (49.5)                       | 15                       | 35                      | 179                 |
|         |                        | CD              | 690 (100)                          | 580 (84)                         | 10                       | 30                      | 197                 |
| G10600  | 1060                   | HR              | 680 (98)                           | 370 (54)                         | 12                       | 30                      | 201                 |
| G10800  | 1080                   | HR              | 770 (112)                          | 420 (61.5)                       | 10                       | 25                      | 229                 |
| G10950  | 1095                   | HR              | 830 (120)                          | 460 (66)                         | 10                       | 25                      | 248                 |

**EXAMPLE 6–8**

A 1015 hot-rolled steel bar has been machined to a diameter of 25 mm. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 300°C. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

**Solution**

From Table A–20,  $S_{ut} = 340$  MPa at 20°C. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Table 6–4. From Table 6–4,

$$\left( \frac{S_T}{S_{RT}} \right)_{300^\circ} = 0.975$$

The ultimate strength at 300°C is then

$$(S_{ut})_{300^\circ} = (S_T / S_{RT})_{300^\circ} (S_{ut})_{20^\circ} = 0.975(340) = 331.5 \text{ MPa}$$

The rotating-beam specimen endurance limit at 300°C is then estimated from Eq. (6–8) as

$$S'_e = 0.5(331.5) = 165.8 \text{ MPa}$$

Next, we determine the Marin factors. For the machined surface, Eq. (6–19) with Table 6–2 gives

$$k_a = aS_{ut}^b = 4.51(331.5^{-0.265}) = 0.969$$

For axial loading, from Eq. (6–21), the size factor  $k_b = 1$ , and from Eq. (6–26) the loading factor is  $k_c = 0.85$ . The temperature factor  $k_d = 1$ , since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6–5,  $k_e = 0.814$ . Finally, since no other conditions were given, the miscellaneous factor is  $k_f = 1$ . The endurance limit for the part is estimated by Eq. (6–18) as

Answer

$$\begin{aligned} S_e &= k_a k_b k_c k_d k_e k_f S'_e \\ &= 0.969(1)(0.85)(1)(0.814)(1)165.8 = 111 \text{ MPa} \end{aligned}$$

For the fatigue strength at 70 000 cycles we need to construct the  $S$ - $N$  equation. From p. 285, since  $S_{ut} = 331.5 < 490$  MPa, then  $f = 0.9$ . From Eq. (6–14)

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.9(331.5)]^2}{111} = 891 \text{ MPa}$$

and Eq. (6–15)

$$b = -\frac{1}{3} \log \left( \frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left[ \frac{0.9(331.5)}{111} \right] = -0.1431$$

Finally, for the fatigue strength at 70 000 cycles, Eq. (6–13) gives

Answer

$$S_f = a N^b = 891(70\,000)^{-0.1431} = 180.5 \text{ MPa}$$

### EXAMPLE 6-9

Figure 6-22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM “minimum” strengths, estimate the life of the part.

### Solution

From Fig. 6-22*b* we learn that failure will probably occur at *B* rather than at *C* or at the point of maximum moment. Point *B* has a smaller cross section, a higher bending moment, and a higher stress-concentration factor than *C*, and the location of maximum moment has a larger size and no stress-concentration factor.

We shall solve the problem by first estimating the strength at point *B*, since the strength will be different elsewhere, and comparing this strength with the stress at the same point.

From Table A-20 we find  $S_{ut} = 690$  MPa and  $S_y = 580$  MPa. The endurance limit  $S'_e$  is estimated as

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

From Eq. (6-19) and Table 6-2,

$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6-20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

Since  $k_c = k_d = k_e = k_f = 1$ ,

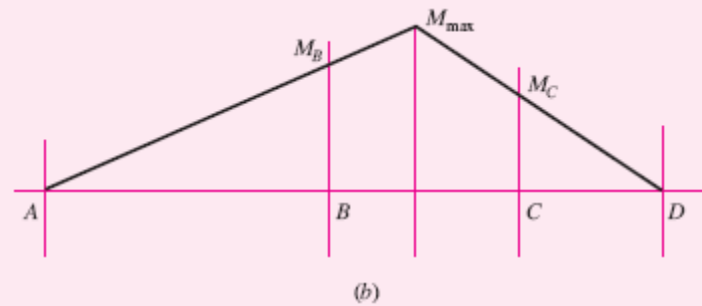
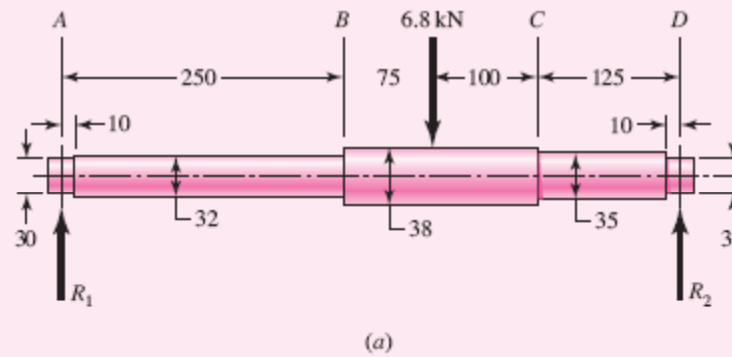
$$S_e = 0.798(0.858)345 = 236 \text{ MPa}$$

To find the geometric stress-concentration factor  $K_t$  we enter Fig. A-15-9 with  $D/d = 38/32 = 1.1875$  and  $r/d = 3/32 = 0.09375$  and read  $K_t \doteq 1.65$ . Substituting  $S_{ut} = 690/6.89 = 100$  kpsi into Eq. (6-35) yields  $\sqrt{a} = 0.0622 \sqrt{\text{in}} = 0.313 \sqrt{\text{mm}}$ . Substituting this into Eq. (6-33) gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$

**Figure 6-22**

(a) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel. (b) Bending-moment diagram.



The next step is to estimate the bending stress at point *B*. The bending moment is

$$M_B = R_1 x = \frac{225F}{550} 250 = \frac{225(6.8)}{550} 250 = 695.5 \text{ N} \cdot \text{m}$$

Just to the left of *B* the section modulus is  $I/c = \pi d^3/32 = \pi 32^3/32 = 3.217 (10^3) \text{ mm}^3$ . The reversing bending stress is, assuming infinite life,

$$\sigma = K_f \frac{M_B}{I/c} = 1.55 \frac{695.5}{3.217} (10)^{-6} = 335.1 (10^6) \text{ Pa} = 335.1 \text{ MPa}$$

This stress is greater than  $S_e$  and less than  $S_y$ . This means we have both finite life and no yielding on the first cycle.

For finite life, we will need to use Eq. (6-16). The ultimate strength,  $S_{ut} = 690 \text{ MPa} = 100 \text{ kpsi}$ . From Fig. 6-18,  $f = 0.844$ . From Eq. (6-14)

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

and from Eq. (6-15)

$$b = -\frac{1}{3} \log \left( \frac{f S_{ut}}{S_e} \right) = -\frac{1}{3} \log \left[ \frac{0.844(690)}{236} \right] = -0.1308$$

From Eq. (6-16),

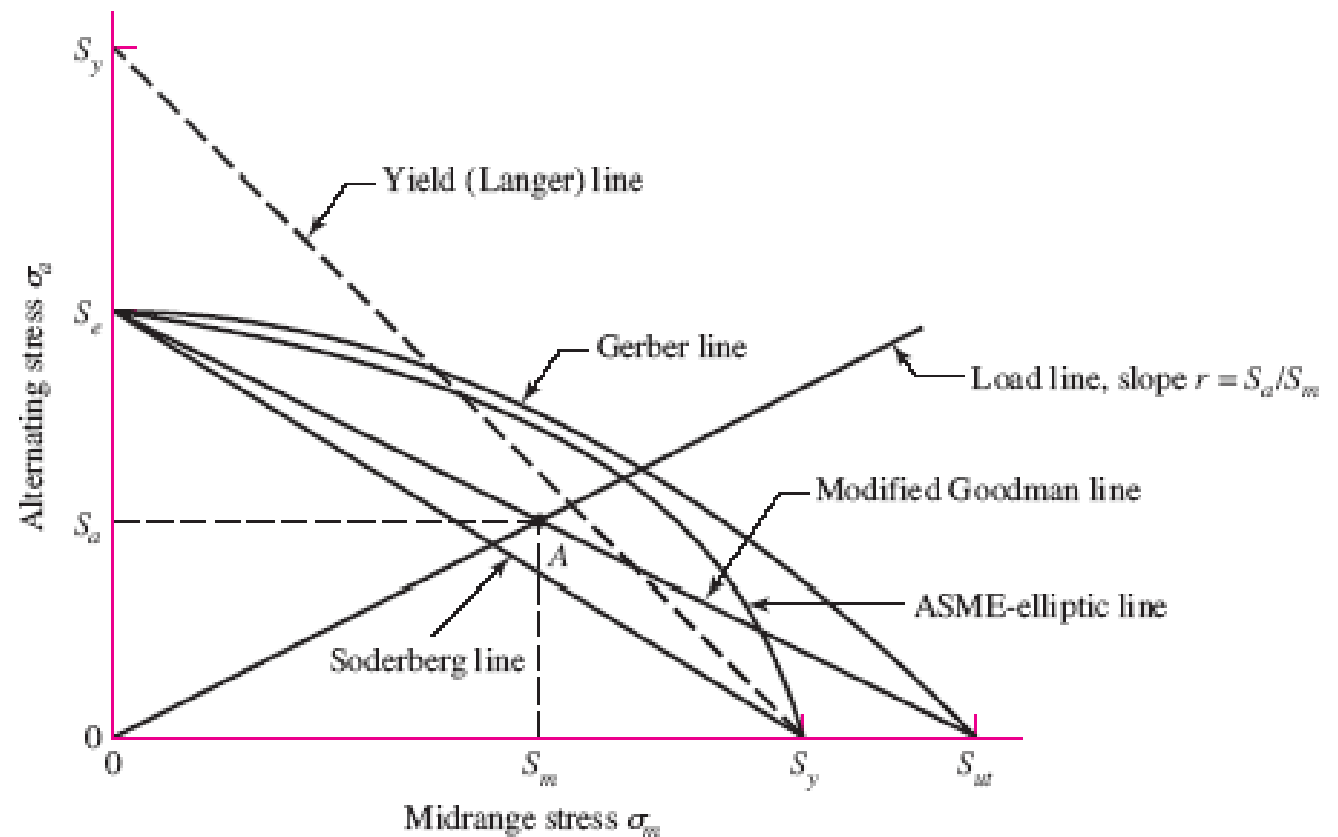
**Answer**

$$N = \left( \frac{\sigma_a}{a} \right)^{1/b} = \left( \frac{335.1}{1437} \right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$

## 6-12 Fatigue Failure Criteria for Fluctuating Stress

**Figure 6-27**

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength  $S_m$  as the limiting value of  $\sigma_m$  corresponding to the strength  $S_a$ , which, paired with  $\sigma_m$ , is the limiting value of  $\sigma_a$ .



Five criteria of failure are diagrammed in Fig. 6-27: the Soderberg, the modified Goodman, the Gerber, the ASME-elliptic, and yielding. The diagram shows that only the Soderberg criterion guards against any yielding, but is biased low.



The criterion equation for the Soderberg line is

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1 \quad (6-40)$$

Similarly, we find the modified Goodman relation to be

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad (6-41)$$

Examination of Fig. 6–25 shows that both a parabola and an ellipse have a better opportunity to pass among the midrange tension data and to permit quantification of the probability of failure. The Gerber failure criterion is written as

$$\frac{S_a}{S_e} + \left( \frac{S_m}{S_{ut}} \right)^2 = 1 \quad (6-42)$$

and the ASME-elliptic is written as

$$\left( \frac{S_a}{S_e} \right)^2 + \left( \frac{S_m}{S_y} \right)^2 = 1 \quad (6-43)$$

The *Langer* first-cycle-yielding criterion is used in connection with the fatigue curve:

$$S_a + S_m = S_y \quad (6-44)$$

The stresses  $n\sigma_a$  and  $n\sigma_m$  can replace  $S_a$  and  $S_m$ , where  $n$  is the design factor or factor of safety. Then, Eq. (6–40), the Soderberg line, becomes

$$\text{Soderberg} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \quad (6-45)$$

Equation (6–41), the modified Goodman line, becomes

$$\text{mod-Goodman} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (6-46)$$

Equation (6–42), the Gerber line, becomes

$$\text{Gerber} \quad \frac{n\sigma_a}{S_e} + \left( \frac{n\sigma_m}{S_{ut}} \right)^2 = 1 \quad (6-47)$$

Equation (6–43), the ASME-elliptic line, becomes

$$\text{ASME-elliptic} \quad \left( \frac{n\sigma_a}{S_e} \right)^2 + \left( \frac{n\sigma_m}{S_y} \right)^2 = 1 \quad (6-48)$$

$$\text{Langer static yield} \quad \sigma_a + \sigma_m = \frac{S_y}{n}$$

The failure criteria are used in conjunction with a load line,  $r = S_a/S_m = \sigma_a/\sigma_m$ . Principal intersections are tabulated in Tables 6–6 to 6–8. Formal expressions for fatigue factor of safety are given in the lower panel of Tables 6–6 to 6–8. The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and fatigue

**Table 6–6**

Amplitude and Steady  
Coordinates of Strength  
and Important  
Intersections in First  
Quadrant for Modified  
Goodman and Langer  
Failure Criteria

| Intersecting Equations   | Intersection Coordinates  |
|--|---|
| $\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ <p>Load line <math>r = \frac{S_a}{S_m}</math></p> | $S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$ $S_m = \frac{S_a}{r}$                     |
| $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ <p>Load line <math>r = \frac{S_a}{S_m}</math></p>    | $S_e = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$                                 |
| $\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$           | $S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$ $S_e = S_y - S_m, r_{crit} = S_a/S_m$ |

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

**Table 6–8**

Amplitude and Steady  
Coordinates of Strength  
and Important  
Intersections in First  
Quadrant for ASME-  
Elliptic and Langer  
Failure Criteria

**Intersecting Equations**

$$\left(\frac{S_o}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\text{Load line } r = S_o/S_m$$

$$\frac{S_o}{S_y} + \frac{S_m}{S_y} = 1$$

$$\text{Load line } r = S_o/S_m$$

$$\left(\frac{S_o}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\frac{S_o}{S_y} + \frac{S_m}{S_y} = 1$$

**Intersection Coordinates**

$$S_o = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$$

$$S_m = \frac{S_e}{r}$$

$$S_o = \frac{r S_y}{1 + r}$$

$$S_m = \frac{S_y}{1 + r}$$

$$S_o = 0, \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$$

$$S_m = S_y - S_o, r_{\text{crit}} = S_o/S_m$$

Fatigue factor of safety

$$n_f = \sqrt{\frac{1}{(\sigma_o/S_e)^2 + (\sigma_m/S_y)^2}}$$

**Table 6-7**

Amplitude and Steady  
Coordinates of Strength  
and Important  
Intersections in First  
Quadrant for Gerber  
and Langer Failure  
Criteria

| Intersecting Equations   | Intersection Coordinates  |
|--|---|
| $\frac{S_o}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ <p>Load line <math>r = \frac{S_o}{S_m}</math></p>  | $S_o = \frac{r^2 S_{ut}^2}{2S_e} \left[ -1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2} \right]$ $S_m = \frac{S_o}{r}$   |
| $\frac{S_o}{S_y} + \frac{S_m}{S_y} = 1$ <p>Load line <math>r = \frac{S_o}{S_m}</math></p>  | $S_o = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$   |
| $\frac{S_o}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ $\frac{S_o}{S_y} + \frac{S_m}{S_y} = 1$  | $S_m = \frac{S_{ut}^2}{2S_e} \left[ 1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2 \left(1 - \frac{S_y}{S_e}\right)} \right]$ $S_o = S_y - S_m, r_{crit} = S_o/S_m$ |
| Fatigue factor of safety   |   |
| $n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_o}{S_e} \left[ -1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut} \sigma_o}\right)^2} \right] \quad \sigma_m > 0$ |   |