EXAMPLE 6–10 A 40-mm-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 70 kN. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Solution We begin with some preliminaries. From Table A–20, $S_{ut} = 690$ MPa and $S_y = 580$ MPa. Note that $F_a = F_m = 35$ kN. The Marin factors are, deterministically,

$k_a = 4.51(690)^{-0.265} = 0.798$: Eq. (6–19), Table 6–2, p. 288

$k_b = 1$ (axial loading, see k_c)

$k_c = 0.85$: Eq. (6–26), p. 290

$k_d = k_e = k_f = 1$

$S_e = 0.798(1)0.850(1)(1)(1)0.5(690) = 234$ MPa: Eqs. (6–8), (6–18), p. 282, p. 287

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(35000)}{\pi\ 0.04^2} = 27.9 \text{ <Pa} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(35000)}{\pi\ 0.04^2} = 27.9 \text{ <Pa}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(27.9) = 51.6 \text{ MPa} = \sigma_m$$

(*a*) Let us calculate the factors of safety first. From the bottom panel from Table 6–7 the factor of safety for fatigue is

Answer

$$n_f = \frac{1}{2}\left(\frac{690}{51.5}\right)^2\left(\frac{51.9}{234}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(51.6)234}{690(51.6)}\right]^2}\right\} = 4.13$$

From Eq. (6–49) the factor of safety guarding against first-cycle yield is

Answer

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{580}{51.6 + 51.6} = 5.62$$

Thus, we see that fatigue will occur first and the factor of safety is 4.13. This can be seen in Fig. 6–28 where the load line intersects the Gerber fatigue curve first at point *B*. If the plots are created to true scale it would be seen that $n_f = OB/OA$.

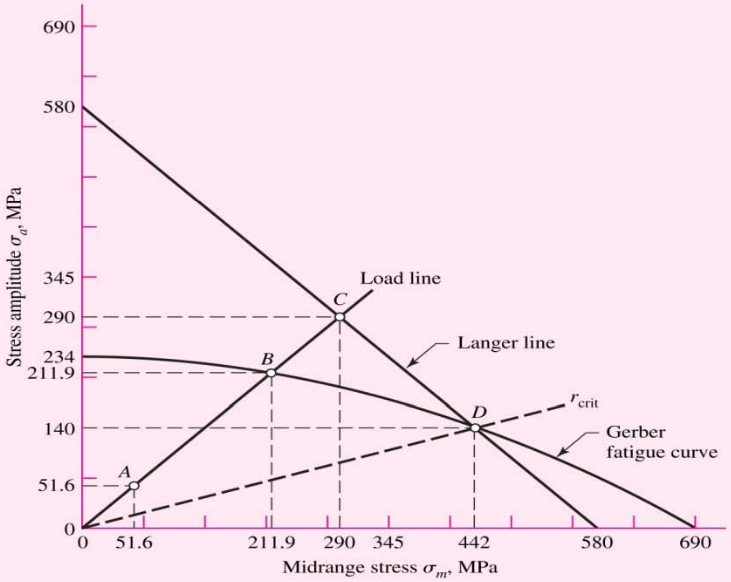
From the first panel of Table 6–7, $r = \sigma_a/\sigma_m = 1$,

Answer

$$S_a = \frac{(1)^2\, 690^2}{2(234)}\left\{-1 + \sqrt{1 + \left[\frac{2(234)}{(1)690}\right]^2}\right\} = 211.9 \text{ MPa}$$

Figure 6–28

Principal points A, B, C, and D on the designer's diagram drawn for Gerber, Langer, and load line.



Answer

$$S_m = \frac{S_a}{r} = \frac{211.9}{1} = 211.9 \text{ MPa}$$

As a check on the previous result, $n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 211.9/51.6 = 4.12$ and we see total agreement.

We could have detected that fatigue failure would occur first without drawing Fig. 6–28 by calculating r_{crit}. From the third row third column panel of Table 6–7, the intersection point between fatigue and first-cycle yield is

$$S_m = \frac{690^2}{2(234)}\left[1 - \sqrt{1 + \left(\frac{2(234)}{690}\right)^2\left(1 - \frac{580}{234}\right)}\right] = 442 \text{ MPa}$$

$$S_a = S_y - S_m = 580 - 442 = 138 \text{ MPa}$$

The critical slope is thus

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{138}{442} = 0.312$$

which is less than the actual load line of $r = 1$. This indicates that fatigue occurs before first-cycle-yield.

(b) Repeating the same procedure for the ASME-elliptic line, for fatigue

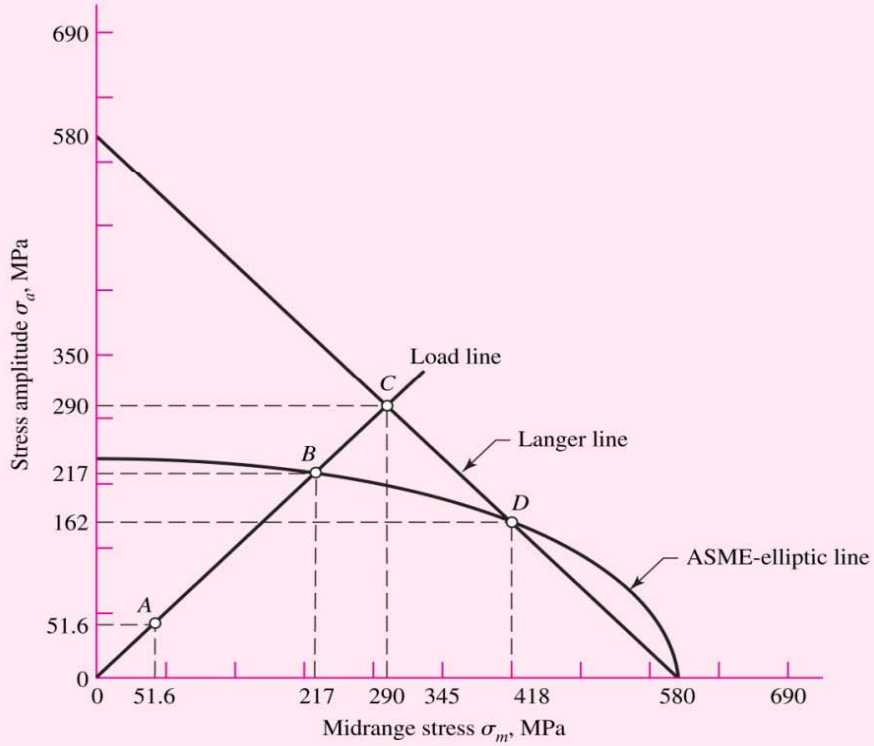
Answer

$$n_f = \sqrt{\frac{1}{(51.6/234)^2 + (51.6/580)^2}} = 4.21$$

Again, this is less than $n_y = 5.62$ and fatigue is predicted to occur first. From the first row second column panel of Table 6–8, with $r = 1$, we obtain the coordinates S_a and S_m of point B in Fig. 6–29 as

Figure 6–29

Principal points A, B, C, and D on the designer's diagram drawn for ASME-elliptic, Langer, and load lines.



Answer

$$S_a = \sqrt{\frac{(1)^2 234^2 (580)^2}{234^2 + (1)^2 580^2}} = 217 \text{ MPa}, \quad S_m = \frac{S_a}{r} = \frac{217}{1} = 217 \text{ MPa}$$

To verify the fatigue factor of safety, $n_f = S_a/\sigma_a = 217/51.6 = 4.21$.

As before, let us calculate r_{crit}. From the third row second column panel of Table 6–8,

$$S_a = \frac{2(580)\,234^2}{234^2 + 580^2} = 162 \text{ MPa}, \ S_m = S_y - S_a = 580 - 162 = 418 \text{ MPa}$$

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{162}{418} = 0.388$$

which again is less than $r = 1$, verifying that fatigue occurs first with $n_f = 4.21$.

The Gerber and the ASME-elliptic fatigue failure criteria are very close to each other and are used interchangeably. The ANSI/ASME Standard B106.1M–1985 uses ASME-elliptic for shafting.

EXAMPLE 6–11 A flat-leaf spring is used to retain an oscillating flat-faced follower in contact with a plate cam. The follower range of motion is 50 mm and fixed, so the alternating component of force, bending moment, and stress is fixed, too. The spring is preloaded to adjust to various cam speeds. The preload must be increased to prevent follower float or jump. For lower speeds the preload should be decreased to obtain longer life of cam and follower surfaces. The spring is a steel cantilever 0.8 m long, 50 mm wide, and 6 mm thick, as seen in Fig. 6–30*a*. The spring strengths are $S_{ut} = 1000$ MPa, $S_y = 880$ MPa, and $S_e = 195$ MPa fully corrected. The total cam motion is 50 mm. The designer wishes to preload the spring by deflecting it 50 mm for low speed and 125 mm for high speed.

(*a*) Plot the Gerber-Langer failure lines with the load line.

(*b*) What are the strength factors of safety corresponding to 50 mm and 125 mm preload?

Solution We begin with preliminaries. The second area moment of the cantilever cross section is

$$I = \frac{bh^3}{12} = \frac{0.05(0.006)^3}{12} = 0.9 + 10^{-9}\ \text{m}^4$$

Since, from Table A–9, beam 1, force F and deflection y in a cantilever are related by $F = 3EIy/l^3$, then stress σ and deflection y are related by

$$\sigma = \frac{Mc}{I} = \frac{0.8Fc}{I} = \frac{0.8(3EIy)}{l^3}\frac{c}{I} = \frac{2.4Ecy}{l^3} = Ky$$

where $K = \dfrac{2.4Ec}{l^3} = \dfrac{2.4(210 \times 10^9)(0.003)}{0.8^3} = 2.95\ \text{GPa/m}$

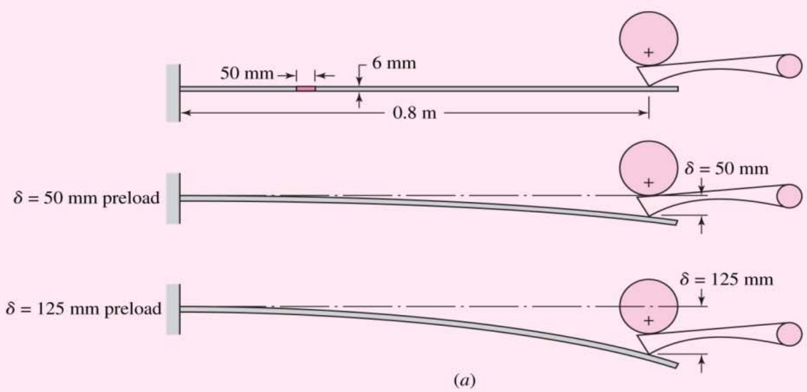
Now the minimums and maximums of y and σ can be defined by

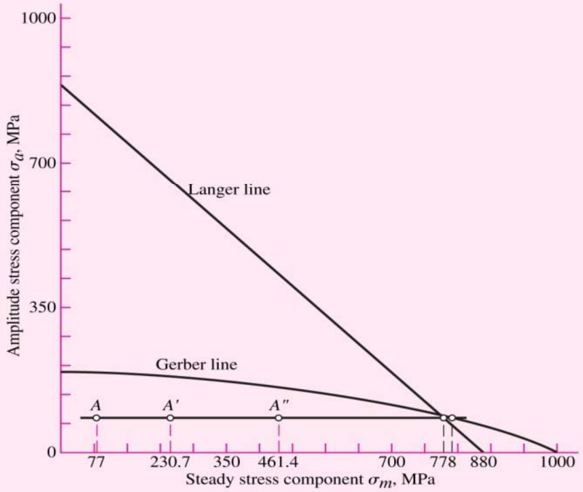
$$y_{\min} = \delta \qquad y_{\max} = 0.05 + \delta$$

$$\sigma_{\min} = K\delta \qquad \sigma_{\max} = K(0.05 + \delta)$$

Figure 6–30

Cam follower retaining spring. (*a*) Geometry; (*b*) designer's fatigue diagram for Ex. 6–11.





(b)

The stress components are thus

$$\sigma_a = \frac{K(0.05 + \delta) - K\delta}{0.05} = K = 76.9 \text{ MPa}$$

$$\sigma_m = \frac{K(0.05 + \delta) + K\delta}{0.05} = K(1 + 40\delta) = 76.9(1 + 40\delta)$$

For $\delta = 0$, $\quad \sigma_a = \sigma_m = 76.9 = 77$ MPa

For $\delta = 50$ mm, $\quad \sigma_a = 77$ MPa, $\sigma_m = 76.9[1 + 40(0.05)] = 230.7$ MPa

For $\delta = 125$ mm, $\quad \sigma_a = 77$ MPa, $\sigma_m = 76.9[1 + 40(0.125)] = 461.4$ MPa

(*a*) A plot of the Gerber and Langer criteria is shown in Fig. 6–30*b*. The three preload deflections of 0, 50, and 125 mm are shown as points A, A', and A''. Note that since σ_a is constant at 77 MPa, the load line is horizontal and does not contain the origin. The intersection between the Gerber line and the load line is found from solving Eq. (6–42) for S_m and substituting 77 MPa for S_a:

$$S_m = S_{ut}\sqrt{1 - \frac{S_a}{S_e}} = 1000\sqrt{1 - \frac{77}{195}} = 778 \text{ MPa}$$

The intersection of the Langer line and the load line is found from solving Eq. (6–44) for S_m and substituting 77 MPa for S_a:

$$S_m = S_y - S_a = 880 - 77 = 803 \text{ MPa}$$

The threats from fatigue and first-cycle yielding are approximately equal.

(*b*) For $\delta = 50$ mm,

Answer $$n_f = \frac{S_m}{\sigma_m} = \frac{778}{230.7} = 3.37 \qquad n_y = \frac{803}{230.7} = 3.48$$

and for $\delta = 125$ mm,

Answer $$n_f = \frac{778}{461.4} = 1.69 \qquad n_y = \frac{803}{461.4} = 1.74$$

EXAMPLE 6–12 Figure 6–31 shows a formed round-wire cantilever spring subjected to a varying force. The hardness tests made on 25 springs gave a minimum hardness of 380 Brinell. It is apparent from the mounting details that there is no stress concentration. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. What number of applications is likely to cause failure? Solve using:
(*a*) Modified Goodman criterion.
(*b*) Gerber criterion.

Solution

$$S_{ut} = 3.41(380) = 1295.8 \text{ MPa}$$

$$S'_e = 0.5(1295.8) = 648 \text{ MPa}$$

$$k_a = 57.7(1295.8)^{-0.718} = 0.336$$

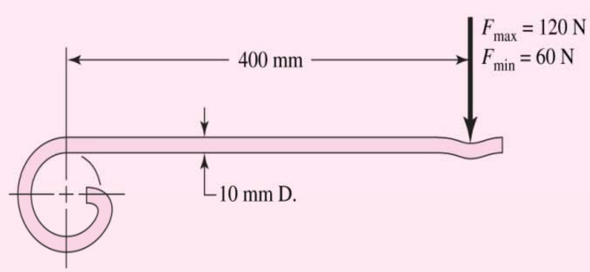
For a non-rotating round bar in bending, Eq. (6–24) gives: $d_e = 0.370d = 0.370(10) =$ 3.7 mm

$$k_b = \left(\frac{3.7}{7.62}\right)^{-0.107} = 1.08$$

$$S_e = 0.336(1.08)(648) = 235 \text{ MPa}$$

$$F_a = \frac{120 - 60}{2} = 30 \text{ N}, \quad F_m = \frac{120 + 60}{2} = 90 \text{ N},$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(90)(400)}{\pi(10^3)} = 366.7 \text{ MPa}$$



| **Figure 6–31**

$$\sigma_a = \frac{32(30)(400)}{\pi(10^3)} = 122.2 \text{ MPa}$$

$$r = \frac{122.2}{366.7} = 0.333$$

(*a*) Modified Goodman, Table 6–6

$$n_f = \frac{1}{(122.2/235) + (366.7/1295.8)} = 1.25$$

From Fig. 6–18, for $S_{ut} = 1295.8$ MPa, $f = 0.78$

Eq. (6–14): $$a = \frac{[0.78(1295.8)]^2}{235} = 5573 \text{ MPa}$$

Eq. (6–15): $$b = -\frac{1}{3}\log\frac{0.78(1295.8)}{235} = -0.211\ 19$$

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = 1 \quad \Rightarrow \quad S_f = \frac{\sigma_a}{1 - (\sigma_m/S_{ut})} = \frac{122.2}{1 - (366.7/1295.8)} = 170.4 \text{ MPa}$$

Eq. (6–16) with $\sigma_a = S_f$

Answer $$N = \left(\frac{170.4}{5573}\right)^{1/-0.211\ 19} = 14\ 853\ 650 \text{ cycles}$$

(*b*) Gerber, Table 6–7

$$n_f = \frac{1}{2}\left(\frac{1295.8}{366.7}\right)^2\left(\frac{122.2}{235}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(366.7)(235)}{1295.8(122.2)}\right]^2}\right\} = 1.55$$

Answer Thus, infinite life is predicted ($N \geq 10^6$ cycles).

EXAMPLE 6–12 A steel bar undergoes cyclic loading such that $\sigma_{max} = 60$ kpsi and $\sigma_{min} = -20$ kpsi. For the material, $S_{ut} = 80$ kpsi, $S_y = 65$ kpsi, a fully corrected endurance limit of $S_e = 40$ kpsi, and $f = 0.9$. Estimate the number of cycles to a fatigue failure using:

(*a*) Modified Goodman criterion.

(*b*) Gerber criterion.

Solution From the given stresses,

$$\sigma_a = \frac{60 - (-20)}{2} = 40 \text{ kpsi} \qquad \sigma_m = \frac{60 + (-20)}{2} = 20 \text{ kpsi}$$

From the material properties, Eqs. (6–14) to (6–16), p. 277, give

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.9(80)]^2}{40} = 129.6 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(80)}{40}\right] = -0.0851$$

$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{S_f}{129.6}\right)^{-1/0.0851} \tag{1}$$

where S_f replaced σ_a in Eq. (6–16).

(*a*) The modified Goodman line is given by Eq. (6–46), p. 298, where the endurance limit S_e is used for infinite life. For finite life at $S_f > S_e$, replace S_e with S_f in Eq. (6–46) and rearrange giving

$$S_f = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{S_{ut}}} = \frac{40}{1 - \dfrac{20}{80}} = 53.3 \text{ kpsi}$$

Substituting this into Eq. (1) yields

Answer
$$N = \left(\frac{53.3}{129.6}\right)^{-1/0.0851} \doteq 3.4(10^4) \text{ cycles}$$

(*b*) For Gerber, similar to part (*a*), from Eq. (6–47),

$$S_f = \frac{\sigma_a}{1 - \left(\dfrac{\sigma_m}{S_{ut}}\right)^2} = \frac{40}{1 - \left(\dfrac{20}{80}\right)^2} = 42.7 \text{ kpsi}$$

Again, from Eq. (1),

Answer
$$N = \left(\frac{42.7}{129.6}\right)^{-1/0.0851} \doteq 4.6(10^5) \text{ cycles}$$

Comparing the answers, we see a large difference in the results. Again, the modified Goodman criterion is conservative as compared to Gerber for which the moderate difference in S_f is then magnified by a logarithmic S, N relationship.

For many *brittle* materials, the first quadrant fatigue failure criteria follows a concave upward Smith-Dolan locus represented by

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \qquad (6\text{–}50)$$

or as a design equation,

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \qquad (6\text{–}51)$$

For a radial load line of slope r, we substitute S_a/r for S_m in Eq. (6–50) and solve for S_a, obtaining

$$S_a = \frac{r S_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4r S_{ut} S_e}{(r S_{ut} + S_e)^2}}\right] \qquad (6\text{–}52)$$

The fatigue diagram for a brittle material differs markedly from that of a ductile material because:

- Yielding is not involved since the material may not have a yield strength.
- Characteristically, the compressive ultimate strength exceeds the ultimate tensile strength severalfold.

- First-quadrant fatigue failure locus is concave-upward (Smith-Dolan), for example, and as flat as Goodman. Brittle materials are more sensitive to midrange stress, being lowered, but compressive midrange stresses are beneficial.
- Not enough work has been done on brittle fatigue to discover insightful generalities, so we stay in the first and a bit of the second quadrant.

The most likely domain of designer use is in the range from $-S_{ut} \le \sigma_m \le S_{ut}$. The locus in the first quadrant is Goodman, Smith-Dolan, or something in between. The portion of the second quadrant that is used is represented by a straight line between the points $-S_{ut}$, S_{ut} and 0, S_e, which has the equation

$$S_a = S_e + \left(\frac{S_e}{S_{ut}} - 1 \right) S_m \qquad -S_{ut} \le S_m \le 0 \quad \text{(for cast iron)} \tag{6–53}$$

Table A–24 gives properties of gray cast iron. The endurance limit stated is really $k_a k_b S'_e$ and only corrections k_c, k_d, k_e, and k_f need be made. The average k_c for axial and torsional loading is 0.9.

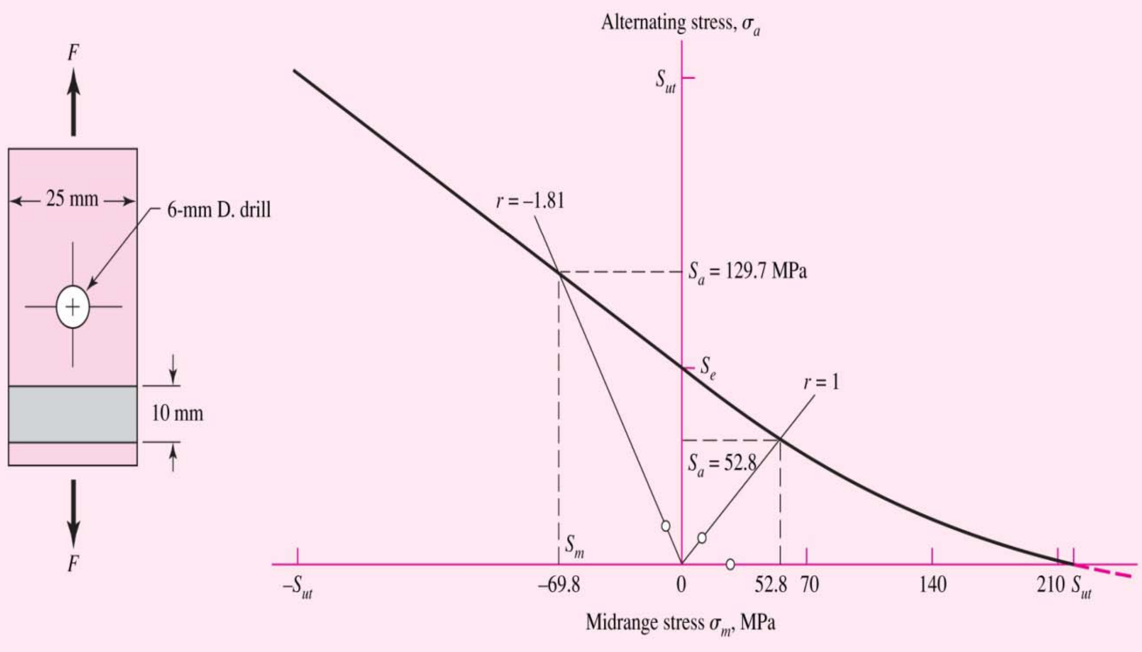
EXAMPLE 6–13 A grade 30 gray cast iron is subjected to a load F applied to a 25 by 10-mm cross-section link with a 6-mm-diameter hole drilled in the center as depicted in Fig. 6–32*a*. The surfaces are machined. In the neighborhood of the hole, what is the factor of safety guarding against failure under the following conditions:

(*a*) The load $F = 4500$ N tensile, steady.

(*b*) The load is 4500 N repeatedly applied.

(*c*) The load fluctuates between -4500 N and 1300 N without column action.

Use the Smith-Dolan fatigue locus.



(*a*) (*b*)

Figure 6–32

The grade 30 cast-iron part in axial fatigue with (*a*) its geometry displayed and (*b*) its designer's fatigue diagram for the circumstances of Ex. 6–13.

Solution Some preparatory work is needed. From Table A–24, $S_{ut} = 214$ MPa, $S_{uc} = 752$ MPa, $k_a k_b S'_e = 97$ MPa. Since k_c for axial loading is 0.9, then $S_e = (k_a k_b S'_e)k_c = 97(0.9) = 87.3$ MPa. From Table A–15–1, $A = t(w - d) = 0.01(0.025 - 0.006) = 190 \times 10^{-6}\ \text{m}^2$, $d/w = 6/25 = 0.24$, and $K_t = 2.45$. The notch sensitivity for cast iron is 0.20 (see p. 296), so

$$K_f = 1 + q(K_t - 1) = 1 + 0.20(2.45 - 1) = 1.29$$

(*a*) $$\sigma_a = \frac{K_f F_a}{A} = \frac{1.29(0)}{A} = 0 \qquad \sigma_m = \frac{K_f F_m}{A} = \frac{1.29(4500)}{190 \times 10^{-6}} = 30.6\ \text{MPa}$$

and

Answer

$$n = \frac{S_{ut}}{\sigma_m} = \frac{214}{30.6} = 6.99$$

(*b*) $$F_a = F_m = \frac{F}{2} = \frac{4500}{2} = 2250\ \text{N}$$

$$\sigma_a = \sigma_m = \frac{K_f F_a}{A} = \frac{1.29(2250)}{190 \times 10^{-6}} = 15.3\ \text{MPa}$$

$$r = \frac{\sigma_a}{\sigma_m} = 1$$

From Eq. (6–52),

$$S_a = \frac{(1)31 + 12.6}{2}\left[-1 + \sqrt{1 + \frac{4(1)214(87.3)}{[(1)214 + 87.3]^2}}\right] = 52.8\ \text{MPa}$$

Answer

$$n = \frac{S_a}{\sigma_a} = \frac{52.8}{15.3} = 3.45$$

$$(c) \quad F_a = \frac{1}{2}|1300 - (-4500)| = 2900 \text{ N} \quad \sigma_a = \frac{1.29(2900)}{190 \times 10^{-6}} = 19.7 \text{ MPa}$$

$$F_m = \frac{1}{2}[1300 + (-4500)| = -1600 \text{ N} \quad \sigma_m = \frac{1.29(-1600)}{190 \times 10^{-6}} = -10.9 \text{ MPa}$$

$$r = \frac{\sigma_a}{\sigma_m} = \frac{19.7}{-10.9} = -1.81$$

From Eq. (6–53), $S_a = S_e + (S_e/S_{ut} - 1)S_m$ and $S_m = S_a/r$. It follows that

$$S_a = \frac{S_e}{1 - \dfrac{1}{r}\left(\dfrac{S_e}{S_{ut}} - 1\right)} = \frac{87.3}{1 - \dfrac{1}{-1.81}\left(\dfrac{87.3}{214} - 1\right)} = 129.7 \text{ MPa}$$

Answer

$$n = \frac{S_a}{\sigma_a} = \frac{129.7}{19.7} = 6.58$$

Figure 6–32*b* shows the portion of the designer's fatigue diagram that was constructed.