

"CERTAIN ASPECTS OF THE THEORY OF ABSTRACT HOMOGENEOUS POLYNOMIALS"

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An MSC Thesis

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The results of this Thesis have been published in the following two articles:

- F. B. Jamjoom and N. Zaheer, "*A Coincidence Theorem For Symmetric Multilinear Forms*", J. Math. Anal. Appl. 129(1988), 171-190.
- F. B. Jamjoom and N. Zaheer, "*On Hormander Ratio Theorem*", Proc. Amer. Math. Soc., 102(1988), 311-316.

Summary of the Thesis:

The thesis deals with various aspects of the abstract homogeneous polynomial theory, comprehensively and comparatively, and interrelates these seemingly diverse aspects through a completely new approach, materialized via a new result proved in Chapter II. This result generalizes to vector spaces, a very important and well-known classical result due to Walsh, commonly known as Walsh's coincidence, and serves as a storehouse of many applications and deduces a variety of results, some of which are either new or they are improvements upon known results.

Chapter I on preliminaries explain the basic material needed in succeeding chapters.

In Chapter II, the main theorem (a new result) of the thesis is established. This theorem generalizes Walsh's coincidence theorem to symmetric n -linear forms in vector spaces, and turns out to be equivalent to a known theorem of Zaheer on apolar polynomials.

In chapter III some applications of the new theorem are discussed, and many results are deduced. Some of these results are either improvements upon or equivalent to the corresponding known results due to Walsh, Zervos, Marden, Hörmander, Laguerre, Szego and Zaheer.

In Chapter IV, another application of the main theorem generalizes Hörmander's Range coincidence theorem, and also deduces an improved version of another theorem of Hörmander on rational functions in the complex plane.

In Chapter V, the vector-valued version of the main theorem is proved, and then the relationships between this version and other known results for vector-valued cases are discussed. The last section of this chapter takes up a number of interesting examples and remarks concerning the validity of hypotheses and the degree of generality of the main theorem. In fact, the main theorem of the thesis is a single result in the abstract homogeneous polynomial theory, as Walsh's theorem is in classical polynomial theory, which has many applications in different adjoining areas.