"On The Tensor Products of JC-algebras and JW-algebras"

A PHD Thesis

Fatmah B. Jamjoom

Supervised by
Professor J. D. M. Wright

University of Reading, England, 1990

The results of this Thesis have been published in the following five articles:

- F. B. Jamjoom, *"The Connection Between The Universal Enveloping C*-algebra And The Universal Enveloping Von Neumann Algebra Of A JW-algebra"*, Math. Proc. Camb. Phil. Soc. 112 (1992), 575-579.

- F. B. Jamjoom, *"Nuclear JC-algebras And Tensor Products Of Types"*, Internat. J. Math. & Math. Sci. (16), 4 (1993), 717-724.

- F. B. Jamjoom, *"On The Tensor Products of JC-algebras"*, Quart. J. Math. Oxford (2), 45 (1994), 77-90.

- F. B. Jamjoom, *"On The Tensor Products Of Simple JC-algebras"*, Michigan Math. J. 41 (1994), 289-295.

- F. B. Jamjoom, *"On The Tensor Products Of JW-algebras"*, Can. J. Math., Vol. 47(4), 1995, 786-800.

Summary of the Thesis:

The purpose this thesis is to develop a theory of tensor products of JC-algebras and JW-algebras. The starting point of such theory was given by Hanche-olsen who constructed the so-called universal tensor product of two unital JC-aLgebras A and B which is defined to be the JC-subalgebra of $C^*(A) \underset{\max}{\otimes} C^*(B)$ generated by $A \otimes B$ when A and B are canonically embedded in their respective universal enveloping C*-algebras $C^*(A)$ and $C^*(B)$. The question concerning the definition of a tensor product of two JC-algebras A, B (not necessarily unital) and, when λ is a C*-norm on $C^*(A) \otimes C^*(B)$ then arises naturally.

In Chapter 1, a few pertinent definitions and important earlier results on JC-algebras, JW-algebras and tensor products of C*-algebras are reviewed.

The problem of defining a suitable tensor product of JC-algebras is dealt with in section 2.1 of chapter 2, and it was mainly motivated by Hanche-Olsen's defini tion. Apart from providing a convenient treatment of tensor products of JC-algebras the new definition stress the important contnection between the theory of tensor products of C*-algebras and the theory of tensor products of JC-algebras.

In section 2.3, it is proved that if λ is a C*-norm on $C^*(A) \otimes C^*(B)$, Φ_A, Φ_B are the canonical antiautomorphisms of $C^*(A)$ and $C^*(B)$, respectively, then the *-antiautomorphism $\Phi_A \otimes \Phi_B$ of the *-algebra $C^*(A) \otimes C^*(B)$ need not to be continuous with respec t to the $\lambda-$norm, and then it is shown that $\Phi_A \otimes \Phi_B$

is $\lambda-$continuous if and only if $C^*(A)\underset{\lambda}{\otimes} C^*(B)$ is the universal enveloping C*-algebra of the JC-tensor product $JC(A\underset{\lambda}{\otimes} B)$ of A and B. Further, it is shown that $C^*(JC(A\underset{\lambda}{\otimes} B))$ can be larger than $C^*(A)\underset{\lambda}{\otimes} C^*(B)$, and it is possible for distict C*-norms λ, μ on $C^*(A)\otimes C^*(B)$ to agree on $J(A\otimes B)$ the Jordan algebra generated by $A\otimes B$ in $C^*(A)\otimes C^*(B)$.

In section 2.5, the Jordan analogous of Guichardet's result involving tensor products of homomorphisms is established. In contrast to the theory of tensor produc ts of C*-algebras, it is shown, in section 2.6 that the tensor product of two simple JC-algebras is not simple, in general. A virtually complete structure theory for $JC(A\underset{\min}{\otimes} B)$ when A and B are simple JC-algebras is established in this section.

As in the C*-algebra theory, it is introduced, in chapter III, the class of JC-algebras which behave well with respect to tensor products (the nuclear JC-algebras). The relation between the nuclearity of a JC-algebra A and the nuclearity of its universal enveloping C*-algebra $C^*(A)$ is establised in section 3.1.

Section 3.2 is devoted to the investigation of the tensor products of types of JC-algebras (postl iminal, liminal. antiliminal and dual).

In chapter IV, a generalized theory for tensor products of JW-algebras is constructed by introducing the definition of the JW-tensor product of two JW-algebras M and N, and then studying the relationship between the universal enveloping C*-algebra $C^*(M)$ and the universal enveloping von Neumann algebra $W^*(M)$ of a JW-algebra M. It is proved that $C^*(M)$ can be regarded as the C*-algebra generated by M in $W^*(M)$. An important consequence of this result is that (as in the theory of tensor products of von Neumann algebras) the JW-tensor product $JW(M \bar{\otimes} N)$ of two JW-algebra M and N is the weak-closure of the JC-tensor produc $JC(M \underset{\min}{\otimes} N)$ of M and N in $W^*(M) \bar{\otimes} W^*(N)$. More results on the tensor product of JW-algebras are proved this chapter which, generally, rely on the theory of tensor products of von Neumann algebras.

In section 4.3 the centre of the tensor product of two JW-algebras has been studied, and some counter examples which rules out the establishing of a similar result as in the theory of tensor products of von Neumann algebras are given. In a particular case, an explicit formula for the centre of $JW(M\bar{\otimes}N)$ is established.

The types of the JW-tensor products $JW(M\bar{\otimes}N)$ are investigated in chapter V. A nice feature of the theory is that, it is conceptually simple and remarkably involves the discussion of the types of $W^*(M) \bar{\otimes} W^*(N)$.