STAT 324 Final . 1431 - 1432 H sem. 2

Q1 X continuous r.v.

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

1- $\mu = E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_0^1 x \cdot 2(1-x)\, dx$

$= \int_0^1 2x - 2x^2\, dx = x^2 - \frac{2}{3}x^3 \Big|_0^1 = \frac{1}{3}$
$= 0.33$

2- $\sigma^2 = Var(X) = E(x^2) - [E(X)]^2$

$E(x^2) = \int_0^1 2x^2(1-x)\, dx = \int_0^1 2x^2 - 2x^3\, dx$

$= \frac{2}{3}x^3 - \frac{1}{2}x^4 \Big|_0^1 = \frac{1}{6} = 0.167$

$Var(X) = 0.167 - (0.33)^2 = 0.0555$
$= \underline{0.056}$

3- $P(X = 0.5) = 0$

4- $P(0.5 < X < 1) = \int_{0.5}^{1} 2(1-x)\, dx$

$= 2x - x^2 \Big|_{0.5}^{1} = 1 - 0.75 = 0.25$

5- CDF $F(x) = \int_0^x f(t)\, dt = \int_0^x 2(1-t)\, dt$

$F(x) = \underline{2x - x^2}$, $0 < x < 1$

$= \underline{x(2-x)}$

$\lesssim$ $f(x) = \frac{d}{dx} F(x)$

Q2

	M	R	L	
A	150	65	5	220
B	100	125	55	280
	250	190	60	500

6- $P(R \mid A) = \dfrac{P(R \cap A)}{P(A)} = \dfrac{65/500}{220/500} = 0.295$

7- $P(B \cap L) = \dfrac{55}{500} = 0.11$

8- $P(B \cup L) = P(B) + P(L) - P(B \cap L)$

$= \dfrac{280}{500} + \dfrac{60}{500} - \dfrac{55}{500} = \dfrac{285}{500} = 0.57$

Q3

Machine A → 800 defective 0.01

Machine B → 200 defective 0.02

9- $P(D) = P(A)\,P(D \mid A) + P(B)\,P(D \mid B)$

$= 0.8\,(0.01) + 0.2\,(0.02) = 0.012$

10- $P(A \mid D) = \dfrac{P(A)\,P(D \mid A)}{P(D)} = \dfrac{0.8\,(0.01)}{0.012} = 0.667$

Q4

x	0	1	2	3
$f(x)$	0.216	0.432	0.288	0.064

11- $E(X) = \sum x\, f(x) = 1.2$

12- $Var(X) = E(X^2) - [E(X)]^2$

$= \sum x^2 f(x) - [E(X)]^2$

$= 2.16 - [1.2]^2 = 0.72$

13- $P(X < 2)$

$= P(X=0) + P(X=1)$

$= 0.216 + 0.432$

$= 0.648$

Q5 X and Y independent r.v.'s

$\mu_X = 1$ $\mu_Y = -2$

$\sigma^2_X = 2$ $\sigma^2_Y = 1$

14) $E(X-3Y+1) = E(X) - 3E(Y) + 1$

$= 1 - 3(-2) + 1 = 8$

15) $Var(X-3Y+1) = Var(X) + 9\,Var(Y)$

$= 2 + 9(1) = 11$

16) $E(Y^2)$??

$Var(Y) = E(Y^2) - [E(Y)]^2$

$1 = E(Y^2) - (-2)^2$

$E(Y^2) = 5$

17) lower bound $= 1 - \frac{1}{4} = 0.75$

$P(-4 < Y < 0)$

$-4-(-2) < Y-(-2) < 0-(-2)$

$-2 < Y - \mu_Y < 2$

$\geq 1 - \frac{1}{k^2}$

$-2\sigma_Y < Y - \mu_Y < 2\sigma_Y$

$k = 2$

Q6 Population Sample

$p = 0.15 \longrightarrow n = 7$

$f(x) = \binom{n}{x} p^x q^{n-x} = \binom{7}{x} (0.15)^x (0.85)^{7-x}$

18- $P(X=2) = \binom{7}{2} (0.15)^2 (0.85)^5 = 0.20965$

$\boxed{0.2097}$

19- $P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0)$

$= 1 - \binom{7}{0} (0.85)^7 = 1 - 0.3206 = 0.679$

20- $E(X) = np = 7 * 0.15 = 1.05$

21- $Var(X) = npq = 7 * 0.15 * 0.85 = 0.8925$

Q7

$X \sim Poisson(\mu)$

$f(x) = e^{-\mu} \frac{\mu^x}{x!} \quad , \quad x = 0, 1, 2, \ldots$

$\mu = \lambda t = 1 \qquad t = 1$ (10 seconds)

→ 22- $t = 1$ $\qquad P(X = 2) = e^{-1} \cdot \frac{1^2}{2!} = 0.184$

→ 23- $t = 1$ $\qquad P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - e^{-1} \left[\frac{1^0}{0!} + \frac{1}{1!} \right]$

$= 0.264$

24- (30 seconds) $t = 3 \Rightarrow \mu = \lambda t = 3$

$P(X > 1) = 1 - [P(X=0) + P(X=1)]$

$= 1 - e^{-3} \left[\frac{3^0}{0!} + \frac{3^1}{1!} \right] = 0.801$

25- (60 seconds) $t = 6 \Rightarrow \mu = \lambda t = (1)(6) = 6$

Q8

26- $X \sim N(\mu, \sigma) \qquad \mu = 2.5 \ , \ \sigma = 0.02$

$Z \sim N(0, 1) \qquad Z = \frac{X - \mu}{\sigma} = \frac{X - 2.5}{0.02}$

$P(X > 2.54) = P\left(Z > \frac{2.54 - 2.5}{0.02}\right) = P(Z > 2)$

$= 1 - \Phi(2)$

$= 1 - 0.9772$

$= \underline{0.0228}$

27- $P(X < 2.52) = P\left(Z < \frac{2.52 - 2.5}{0.02}\right)$

$= P(Z < 1) = \Phi(1) = 0.8413$

28 - $P(2.52 < X < 2.54) = P(1 < Z < 2)$

$= \Phi(2) - \Phi(1)$

$= 0.9772 - 0.8413$

$= 0.1359$

Q9 $n = 6$, $\bar{X} = 0.95$, $S = 0.251$

29 $s.e(\bar{X}) = \frac{\sigma}{\sqrt{n}} \simeq \frac{S}{\sqrt{n}} = \frac{0.251}{\sqrt{6}} = 0.102$

30 point estimate for $\mu = \bar{X} = 0.95$

31 90% C.I. for μ $\left[\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}, \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right]$

$L = 0.95 - t_{0.05} * 0.102 = 0.744$

$1 - \alpha = 0.9$
$\alpha = 0.1$
$\frac{\alpha}{2} = 0.05$

Q10 $n_1 = 5$ $n_2 = 10$

$\bar{X}_1 = 10.3$, $S_1 = 1.6$ $\bar{X}_2 = 10.7$, $S_2 = 2.3$

32 $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{4(1.6)^2 + 9(2.3)^2}{13}$

$S_p^2 = 4.45 \longrightarrow S_p = 2.11$

33 $df = n_1 + n_2 - 2 = 13$

34 99% C.I. for $\mu_1 - \mu_2$ $(\bar{X}_1 - \bar{X}_2) - t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
$(\bar{X}_1 - \bar{X}_2) + t_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

35 $(-0.4) \mp 3.012 (2.11) \sqrt{\frac{1}{5} + \frac{1}{10}}$
$[-3.881, 3.081]$

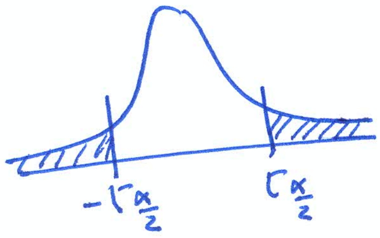
$\alpha = 0.01$
$\frac{\alpha}{2} = 0.005$

(36) $\alpha = 0.01$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{-0.3}{1.156} = -0.26$$



± 3.012

accept H_0

no significant difference between outlets.

Q11

$n_1 = 100$ $\qquad n_2 = 400$

$P_1 = 0.8$ $\qquad P_2 = 0.5$

$$\hat{P}_1 - \hat{P}_2 \sim N\left(P_1 - P_2, \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right)$$

(37) $E(\hat{P}_1 - \hat{P}_2) = P_1 - P_2 = 0.3$

(38) $s.e.(\hat{P}_1 - \hat{P}_2) = \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}} = 0.047$

(39) $P(\hat{P}_1 - \hat{P}_2 < 0.2)$

$$P\left(Z < \frac{0.2 - 0.3}{0.047}\right) = P(Z < -2.13)$$

$= 0.0166$

Q12

$n_1 = 81$ $n_2 = 90$

$\bar{X}_1 = 27$ $\bar{X}_2 = 24$

known $\sigma_1 = 6.9$ $\sigma_2 = 6.2$

(40) point estimate for $\mu_1 - \mu_2 = \bar{X}_1 - \bar{X}_2 = 3$

(41) $\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

$$s.e.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{6.9^2}{81} + \frac{6.2^2}{90}} = 1.007$$

(42) 95% C.I. for $\mu_1 - \mu_2$ $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$

$$\left[\bar{X}_1 - \bar{X}_2 - Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \bar{X}_1 - \bar{X}_2 + Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

$$\max e = Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= 1.96(1.007) = \underline{1.975}.$$

(43) width of C.I. $= 2(1.975) = \underline{3.949}$

Q13

$n = 25$, $\bar{X} = 77$, $S = 10$

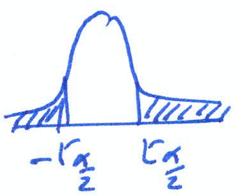
(44) mean score is different from 80

$H_0: \mu = 80$ & $H_1: \mu \neq 80$

(45) $$T = \frac{\bar{X} - 80}{S/\sqrt{n}} = \frac{77 - 80}{10/\sqrt{25}} = -1.5$$

(46) $\alpha = 0.05$, rejection region

$t_{\frac{\alpha}{2}, n-1} = 2.064$ $(-\infty, -2.064) \cup (2.064, \infty)$



(47) accept H_0

<u>Q14</u>

$n_1 = 200$ $\qquad$ $n_2 = 300$

$x_1 = 100$ $\qquad$ $x_2 = 120$

$\hat{P}_1 = \frac{100}{200} = 0.5$, $\hat{P}_2 = \frac{120}{300} = 0.4$

(48) percetage of smokers differs in two populations:

$H_0 : P_1 = P_2$ و $H_1 : P_1 \neq P_2$

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - 0}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} = \frac{(\hat{P}_1 - \hat{P}_2) - 0}{\sqrt{\hat{p}\hat{q}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\hat{P} = \frac{n_1\hat{P}_1 + n_2\hat{P}_2}{n_1 + n_2}$$

$$\hat{P} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{100 + 120}{500} = 0.44$$

$Z = 2.21$ $\qquad$ accept H_0

$Z_{\frac{\alpha}{2}} = 2.58$, $-Z_{\frac{\alpha}{2}} = -2.58$