

كلية العلوم
جامعة الملك سعود

215 احص: احتمال (1)
الاختبار النهائي
الفصل الثاني: 1437 – 1438 هـ

اليوم: الاربعاء 1438/8/28 هـ الوقت: 8:00 – 11:00

Version (A)	
اسم الطالبة:	
الرقم الجامعي:	
الرقم التسلسلي:	
رقم الشعبة:	
اسم الأستاذة:	

ملاحظات:

- 1- الاختبار يحتوي على 34 سؤال و 7 صفحات.
- 2- من الضروري كتابة جميع البيانات على الورقة.

Question	1	2	3	4	5	6	7	8	9	10
Answer										
Question	11	12	13	14	15	16	17	18	19	20
Answer										
Question	21	22	23	24	25	26	27	28		
Answer										

Question:

If $E(X) = 0.4$, $E(Y) = 0.7$, $E(XY) = 0.5$, $V(X) = 0.24$, $V(Y) = 0.37$. Find:

1. $E(X^2) =$
(A) 0.86 (B) 0.53 (C) 0.4 (D) 0.08
 2. $E(X^2 - 2X + 3) =$
(A) 2.73 (B) 2.6 (C) 2.28 (D) 3.06
 3. $Cov(X, Y) =$
(A) 0.22 (B) 0.04 (C) 0.41 (D) 0.36
 4. $V(X - Y) =$
(A) 1.05 (B) 0.17 (C) 0.83 (D) 1.43
 5. $\rho_{X,Y} =$
(A) 2.48 (B) 0.74 (C) 2.89 (D) 3.67
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Question:

Let $f(x, y) = cy^2 + x$, $0 \leq x \leq 1, 0 \leq y \leq 1$. Then,

6. $c =$
(A) $\frac{1}{6}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$
 7. $f(x) =$
(A) $x + \frac{1}{2}$ (B) $x^2 + \frac{1}{2}$ (C) $\frac{3}{2}y^2 + \frac{1}{2}$ (D) $\frac{1}{3}y^2 + \frac{1}{2}$
 8. $f(y) =$
(A) $x + \frac{1}{2}$ (B) $x^2 + \frac{1}{2}$ (C) $\frac{3}{2}y^2 + \frac{1}{2}$ (D) $\frac{1}{3}y^2 + \frac{1}{2}$
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Question:

If $E(E(Y|X)) = 2$, $V(E(Y|X)) = 3$, $E(V(Y|X)) = 3$ and $Cov(X, Y) = 2$. Then

9. $E(Y) =$
 (A) 2 (B) 0 (C) 6 (D) 3
10. $V(Y) =$
 (A) 0 (B) 6 (C) 3 (D) 2
11. $Cov(X + Y, Y) =$
 (A) 2 (B) 4 (C) 5 (D) 8
12. $Cov(3, Y) =$
 (A) 2 (B) 3 (C) 0 (D) 4

Question:

Let X and Y be random variables that take on values from the set $\{-1, 0, 1\}$:

Y \ X	-1	0	1
-1	3/15	2/15	1/15
0	1/15	1/15	2/15
1	1/15	1/15	3/15

Find:

13. $P(X = 0, Y = 1) =$
 (A) 0 (B) $\frac{2}{15}$ (C) $\frac{1}{15}$ (D) $\frac{3}{15}$
14. $P(X < 0, Y > -1) =$
 (A) $\frac{2}{15}$ (B) $\frac{5}{15}$ (C) $\frac{4}{15}$ (D) $\frac{3}{15}$
15. $P(X + Y < 0) =$
 (A) $\frac{3}{15}$ (B) $\frac{5}{15}$ (C) $\frac{4}{15}$ (D) $\frac{6}{15}$
16. $f_X(x) =$

(A)

x	-1	0	1
$f(x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{6}{15}$

(B)

x	-1	0	1
$f(x)$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

(C)

x	-1	0	1
$f(x)$	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$

(D)

x	-1	0	1
$f(x)$	$\frac{6}{15}$	$\frac{5}{15}$	$\frac{4}{15}$

17. $E(X) =$
 (A) $\frac{3}{15}$ (B) $\frac{10}{15}$ (C) $\frac{11}{15}$ (D) $\frac{1}{15}$
18. $V(X) =$
 (A) $\frac{10}{15^2}$ (B) $\frac{1}{15}$ (C) $\frac{11}{15}$ (D) $\frac{164}{15^2}$
19. $f(x = 0|y = -1) =$
 (A) $\frac{3}{15}$ (B) $\frac{1}{3}$ (C) $\frac{2}{15}$ (D) $\frac{6}{15}$

Question:

Let $f(x) = 3x^2$, $0 < x < 1$ and $f(y|x) = \frac{3y^2}{x^3}$, $0 < y < x$. Determine:

20. $f(x, y) =$
 (A) $\frac{9y^2}{x}$, $0 < x < 1; 0 < y < 1$ (B) $\frac{9y^2}{x}$, $0 < y < x < 1$
 (C) $\frac{3y^2}{x^3}$, $0 < y < x < 1$ (D) $3x^2$, $0 < y < 1, 0 < x < 1$
21. $f(y) =$
 (A) $-9y^2 \ln y$, $0 < y < x$ (B) $-9y^2 \ln y$, $0 < y < 1$
 (C) $9y^2 \ln y$, $0 < y < x$ (D) $9y^2 \ln y$, $0 < y < 1$
22. $f(x|y) =$
 (A) $\ln(x)$, $0 < y < x < 1$ (B) $\frac{x^2}{12}$, $0 < y < x < 1$
 (C) $\frac{-1}{x \ln y}$, $0 < y < x < 1$ (D) $\frac{y^2}{2}$, $0 < y < x < 1$
23. $E(Y|X) =$
 (A) $\frac{3}{4}x$ (B) $\frac{x^2}{12}$ (C) $\frac{x}{2}$ (D) $\frac{y^2}{2}$
24. $F(y|x) =$
 (A) $2x$, $0 < y < x < 1$ (B) 1 , $0 < y < x < 1$
 (C) $\frac{y}{x}$, $0 < y < x < 1$ (D) $\left(\frac{y}{x}\right)^3$, $0 < y < x < 1$
25. X and Y are
 (A) independent (B) dependent

Question:

26. $M_{X,Y}(0, t_2) =$
(A) $M_X(t_1)$ (B) 0 (C) $M_Y(t_2)$ (D) 1
27. If $E(X|Y) = \frac{2}{7}$ and $Z = X + \frac{4}{7}$, then: $E(Z|Y) =$
(A) $\frac{4}{7}$ (B) $\frac{8}{7}$ (C) $\frac{2}{7}$ (D) $\frac{6}{7}$
28. The value of $\rho_{2x, y+8}$ when $\rho_{X,Y} = 0.27$ is equal to
(A) -0.27 (B) 0.27 (C) 0.54 (D) 0.3
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Question:

29. Prove that $\rho_{X,X} = 1$. (1 mark).

30. Prove that $Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z)$. (2 marks).

31. If $f(x) = \theta x^{\theta-1}$ $0 < x < 1$, find the distribution of $Y = -\ln X$ (Use the one-to-one transformation). (2 marks).

32. If $f(x) = 1$, $0 < x < 1$. Find the pdf of $Y = \sqrt{X}$. (Use the CDF method). (2 marks).

33. Let X_1 and X_2 are two random variables have the joint probability distribution as follows:

		X_1	
		0	1
X_2	0	0.05	0.58
	1	0.3	0.07

(a) Prove that X_1 and X_2 are dependent random variables. (1 mark).

(b) Find the pmf of the random variable $Y = 2X_1 + X_2$. (2 marks).

34. If $X_1 \sim \chi_n^2$ and $X_2 \sim \chi_m^2$ are independent random variables. Find the distribution of $Y = X_1 + X_2$ (Use the moment generating function method).

Table of some MGF:

Distribution Name	pdf	MGF
Uniform Distribution	$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$	$M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$
Exponential Distribution	$f(x) = \theta e^{-\theta x}, \quad x \geq 0$	$M_X(t) = \frac{\theta}{\theta - t}$
Gamma Distribution	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x \geq 0$	$M_X(t) = \left(\frac{\beta}{\beta - t} \right)^\alpha$
Chi-Squared Distribution	$f(x) = \frac{1}{2^{(\frac{v}{2})} \Gamma(\frac{v}{2})} x^{\frac{v}{2}-1} e^{-\frac{x}{2}}, \quad x \geq 0$	$M_X(t) = (1 - 2t)^{-\frac{v}{2}}$
Normal Distribution	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, \quad -\infty < x < \infty$	$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

End of Questions