King Saud University
Department of Mathematics

244
Final Exam, January 2014

NAME:

Group Number:

ID:

I) Let $\begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$ be the augmented matrix for a linear system. Find for what values of a and b the system has:

a) A unique solution;

b) A one-parameter solution;

c) A two-parameter solution;

d) No solution.

II) If A is a 4×4 matrix, with $\det A = -2$, find $\det(-A)$, $\det(A^{-1})$, $\det(2A^T)$, $\det(A^3)$.

III) a) Show that $v_1 = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $v_2 = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$ are orthonormal vectors and find a third vector v_3, for which $\{v_1, v_2, v_3\}$ is an orthonormal set.

b) If $P(-3, 1, 0, 6)$, $Q(0, 5, 1, -2)$ and $R(-4, 1, 4, 0)$, find the cosine of the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

c) Show that the set of all vectors of the form $(a, -a, 0)$, where $a \in \mathbb{R}$, is a subspace of $\mathbb{R}^3$.

IV) A) Let $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ be the multiplication by

$$\begin{bmatrix} 5 & 3 & 1 \\ 3 & 0 & 2 \\ 4 & -1 & 2. \end{bmatrix},$$

and let e_1, e_2 and e_3 be the standard basis vectors for $\mathbb{R}^3$. Find the following vectors:

a) $T_A(e_1)$, $T_A(e_2)$, $T_A(e_3)$;

b) $T_A(3e_2)$;

c) $T_A(2e_1 - 3e_2 + 5e_3)$.

B) Find the standard matrix for the operator $T : \mathbb{R}^3 \to \mathbb{R}^3$, that reflects a vector about the xy-plane, then reflects that vector about the xz-plane.

V. A) Find a basis for the nullspace of the linear system

$$\begin{cases} 2x - 2y + 6z = 4 \\ 5x - 4y - 4z = 1 \\ 7x - 6y - 2z = -3 \end{cases}$$

B) Are there values for r and s, for which

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & s+3 & 3 \\ 0 & 2r-4 & s-2 \\ 0 & 0 & 1 \end{bmatrix}$$

has rank 1? Has rank 2? If so, find those values.

VI. Given the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix},$$

a) Find the eigenvalues and bases for the eigenspaces of A;

b) Find the eigenvalues and bases for the eigenspaces of A^3;

c) Find the eigenvalues and bases for the eigenspaces of A^{-1}.