Math 246 **Name:** ____________________

Fall 2015

Final Term Exam

24/6/2015

Time Limit: 3 hours Student Number ____________________

Grade Table (for teacher use only)

Questions	Points	Student degree
I	10	
II	5	
III	8	
IV	7	
V	7	
VI	3	
Total	40	

Course Work Grades	
Final exam Grades	
Total	

Question I [10 points]

Choose the correct answer. Write your answer in the following table.

1	2	3	4	5	6	7	8	9	10

1. If $W = \text{span}\{(2,4,-2),\ (-2,-2,2),\ (1,3,-1)\}$ then $\dim W$ is

   A. 3 B. 2 C. 1 D. None of the previous

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2. If $T : \mathbb{R}^2 \to \mathbb{R}^4$ is given by $T(x_1, x_2) = (x_2,\ -x_1,\ x_1+3x_2,\ x_1-x_2)$ where $(x_1, x_2) \in \mathbb{R}^2$, the standard matrix for the transformation T is given by

   A. $\begin{bmatrix} 1 & 4 & 2 & 3 \\ 5 & 1 & 2 & 6 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$ C. $\begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & 3 & -1 \end{bmatrix}$ D. None of the previous

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3. If $\lambda^2(\lambda+3)^2(\lambda-4) = 0$ is the characteristic equation of a matrix A then size A is

   A. 3 B. 4 C. 5 D. 0

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4. The image of $(6, -\sqrt{3})$ when it is rotated through an angle $\theta = \frac{\pi}{3}$ is

   A. $(-\frac{9}{2}, \frac{5\sqrt{3}}{2})$ B. $(\frac{9}{2}, \frac{5\sqrt{3}}{2})$ C. $(\frac{5\sqrt{3}}{2}, \frac{9}{2})$ D. None of the previous

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If M_{22} has the inner product

$$< A, B >= \text{tr}(A^T B)$$

and

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} -2 & 3 \\ -2 & 5 \end{bmatrix}$$

then

5. $d(A, B) =$

   A. 5 B. 0 C. 6 D. 2

and

6. $\|A\|$ is

A. 14 B. 1 C. $2\sqrt{7}$ D. $\sqrt{14}$

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7. If $A = \begin{bmatrix} 5 & 1 \\ -2 & 2 \end{bmatrix}$ then the eigenvalues of A are

A. $\{3, 4\}$ B. $\{-3, -4\}$ C. $\{3, -4\}$ D. None

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8. Let $\mathbb{R}^3$ have the Euclidean inner product. If $u = (k, -2, 4)$, $v = (k, k, -2)$ are orthogonal then

A. $k \in \{-4, 2\}$ B. $k = 2$ C. $k \in \{-2, 4\}$ D. None

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9. Let $\mathbf{P}_2$ have the standard norm then the cosine of the angle between

$$P = 2x + x^2, \quad \text{and} \quad q = 1 - x + 2x^2$$

is

A. $\frac{\pi}{2}$ B. 0 C. 1 D. $-\frac{\pi}{2}$

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10. If $T : M_{22} \to \mathbb{R}$ is given by

$$T(M) = \mathrm{tr}(M)$$

then $M \in KerT$ if and only if

A. $M = 0$ B. $M = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}$, b, $c \in \mathbb{R}$ C. $M = M^T$ D. None of the previous

Question II[5 points]

Determine whether the following is True or False.

1. The vectors $\{(1,0,0),\ (2,0,0),\ (3,3,3)\}$ is a basis of $\mathbb{R}^3$ ( )

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2. The set $\{(x,y) : x,y \in \mathbb{R},\ x \geq 0\}$ with the standard operations on $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$ ( )

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3. If 5 is an eigenvalue of a matrix A then $\frac{1}{25}$ is an eigenvalue of A^{-2}. ( )

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4. If A is invertible then Nullity $A = 0$. ( )

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5. $T(A) = \det A$ is a linear transformation from M_{nn} into $\mathbb{R}$. ( )

Question III[8 points]

(a) Let S be a finite set of vectors in a finite dimensional vector space V. Prove that if S spans V but is not a basis for V, then S can be reduced to a basis for V by removing appropriate vectors from S.

Let $A = \begin{bmatrix} 1 & -2 & 3 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

(d) Find Rank A and Nullity A.

Question IV [7 points]

Let $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

(a) Find the eigenvalues of A.

(b) Find the eigenspace of each eigenvalue.

(c) Is A diagonalize? (Justify your answer). If yes, find a matrix P that diagonalize A, and find $P^{-1}AP$.

Question V [7 points]

(a) Prove Cauchy Schwartz inequality: If u and v are vectors in a real inner product space V, then

$$| < u, v > | \leq \|u\| \; \|v\| \, .$$

(b) Apply the Gram-Schmidt process to transform the basis

$$\{u_1 = (1, 0),\ u_2 = (3, 5)\}$$

into an orthonormal basis.

Question VI [3 points]

Consider the basis $\{v_1, v_2\}$ for $\mathbb{R}^2$, where $v_1 = (1,1)$ and $v_2 = (1,0)$ and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator for which

$$T(v_1) = (2,3) \qquad \text{and} \qquad T(v_2) = (-2,0).$$

(a) Find a formula for $T(x_1, x_2)$.

(b) Use (a) to find $T(5,-3)$.