



**Question I:**

**A) Use the Bisection Method to find solution accurate to within  $10^{-3}$  for**

$$x + 3 \cos(x) - e^x = 0 \text{ on the interval } [0,1]$$

**B) Use the Newton's Method to find solution accurate to within  $10^{-3}$  for**

$$(x - 2)^2 - \ln(x) = 0 \text{ for } e \leq x \leq 4$$

**Question II:**

$$\text{let } f(x) = \sqrt[3]{x-1}$$

and  $x_0=1$ ;  $x_1=1.25$ ;  $x_2=1.6$

(a) Construct interpolation polynomials of degree at most one and at most two to approximate  $f(1.4)$  and find the absolute error.

(b) Find an error bound for this approximation

**Question III:**

(a) Use the composite Trapezoidal rule with  $n = 6$  to approximate:

$$\int_0^2 \frac{2}{4+x^2} dx$$

(b) Approximate  $\int_0^2 \frac{2}{4+x^2} dx$  using Composite Simpson's rule and find a bound for the error.

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**Question IV:** For  $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

- (a) Find P, L and U that satisfies  $PA = LU$  where P is a permutation matrix, L and U are lower and upper triangular matrices, respectively.

(b) Can you Factorize A as  $A = L D L^T$  where D and L are diagonal and lower triangular matrices, respectively? Justify your answer.

**Question V:** *For the system :*

$$Ax = b$$
$$\text{with } A = \begin{bmatrix} -2 & 1 & 1/2 \\ 1 & -2 & -1/2 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix}$$

(a) Find the first two iterations of the Jacobi method using  $X^{(0)} = 0$

(b) Find the first two iterations of the Gauss-Seidel method using  $X^{(0)} = 0$

**Question VI:** *For*

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

Determine if the matrix A is symmetric, singular, strictly diagonally dominant, positive definite. (justify your answer)

Good Luck 😊