King Saud University | Math 253 | Second term 1432/1433
Mathematics Department | Final Exam | 3 hours

I. Determine if the statement is true or false, and justify your answer

a) The Bisection method needs 13 iterations to compute a root of $f(x)=\sqrt{x}-\cos x$ for $\mathbf{0 \le x \le 1}$ with $\mathbf{10^{-3}}$ accuracy. ( )

b) The sequence $x_{n+1}=x_n+1-0.2x_n^2$ will converge faster than the sequence $x_{n+1}=\frac{1}{3}\left[3x_n+1-0.2x_n^2\right]$ to $\sqrt{5}$. ( )

c) $\mathbf{A}=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, can be factored to LDL^t with L lower triangular with 1's on it's diagonal and D is a diagonal matrix with diagonal positive entries . ( )

d) If $\sqrt{2}$ is approximated by a cubic Lagrange polynomial wiyh **f(x)=$\mathbf{2^x}$** through the points -1, 0,1,2 the approximate value is 1.4209 ( )

e) If the Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives 4, and the Midpoint rule gives 5 then Simpson rule gives 8/3.
. ( )

II. a) Show that the Newton method applied to find the reciprocal of a number **S** on the equation **S=1/x** gives the iterative formula

$$\mathbf{x_{n+1} = x_n (2 - x_n S)}$$

b) Find the approximation of the reciprocal of **S = 3**, correct to four digit using this formula and starting point p_0= 0.28.

c) If $g(x)=\sqrt[4]{3x^2+3}$ is used as an iterative scheme $\boldsymbol{x_{n+1} = g(x_n)}$, How many iterations are needed to reach an accuracy $1\text{x}10^{-3}$ in [1, 2] and with x_0=1

III. Use the Composite Simpson Rule to find the following integral with accuracy of at most $1\text{x}10^{-4}$, How many intervals are needed?

$$\int_0^1 e^{2t} \sin 3t \, dt$$

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IV. Consider the system A x = b,

s.t. $\mathbf{A} = \begin{bmatrix} 3 & -1 & c \\ 3 & 6 & c \\ 3 & 3 & 7 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

a) Find the least positive nonzero integer c that will guarantee convergence of the Gauss -Seidel method on Ax=b.

b) Find the second approximation $\mathbf{x}^{(2)}$ by Gauss-Seidel method, where $\mathbf{x}^{(0)}= [1,0,1]^t$

c) Calculate an error bound for $\mathbf{x}^{(2)}$

V. **a**) Suppose the system

$$\begin{aligned} 1.011x_1 + 2x_2 &= 3.01 \\ 1.0001x_1 + 2x_2 &= 3.00 \end{aligned}$$

has exact solution $x=[1,1]^{t,}$ compute an approximate solution using Gaussian elimination with three digit rounding arithmetic

b). Is A ill or well conditioned?

c)) calculate an upper error bound for the approximate solution using $\|\cdot\|_\infty$

d) Improve the approximated solution by one step of the error refinement technique.