King Saud University
College of Engineering
Department of Civil Engineering

FINAL EXAM

CE302 Mechanics of Materials – 2nd Semester 1431-32H

Sunday, 10th Rajab 1432 H – 12th June 2011
Time allowed: 3 hours

Student Name

Student Number

Section (put X please)  □ 30629 (from 9:00 to 10:00 A.M.)
□ 30170 (from 10:00 to 11:00 A.M.)

<table>
<thead>
<tr>
<th>Questions</th>
<th>Maximum Marks</th>
<th>Marks obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q ≠ 1</td>
<td>7</td>
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<tr>
<td>Q ≠ 2</td>
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<td>Q ≠ 3</td>
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<td>Q ≠ 4</td>
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<td>Q ≠ 5</td>
<td>10</td>
<td></td>
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<tr>
<td>Q ≠ 6</td>
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</table>

Total marks

50

Total marks obtained (in words):

Instructor’s Signature
Question ≠ 1 (7 points):

The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine:

(a) the stress in the steel,
(b) the maximum stress in the concrete.

\[ \eta = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0 \]

\[ A_s = 4 \cdot \frac{\pi}{4} d^2 = \left(4 \cdot \frac{\pi}{4}\right)(25)^2 = 1.9685 \times 10^3 \text{ mm}^2 \]

\[ nA_s = 15.708 \times 10^3 \text{ mm}^2 \]

Locate the neutral axis:

\[ 300 \times \frac{x}{2} - (15.708 \times 10^3)(480-x) = 0 \]

\[ 150 x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0 \]

Solve for \( x \).

\[ x = \frac{-15.708 \times 10^3 \pm \sqrt{(15.708 \times 10^3)^2 + 4 \times 150 \times 7.5398 \times 10^6}}{2 \times 150} \]

\[ x = 177.87 \text{ mm}, \quad 480 - x = 302.13 \text{ mm} \]

\[ I = \frac{1}{12} 300 \times x^3 + (15.708 \times 10^3)(480-x)^2 \]

\[ = \frac{1}{12} (300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2 \]

\[ = 1.9966 \times 10^{-3} \text{ m}^4 \]

\[ \sigma = \frac{My}{I} \]

(a) Steel:

\[ y = -302.45 \text{ mm} = -0.30245 \text{ m} \]

\[ \sigma = \frac{(8.0 \times 175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6 \text{ Pa} = 212 \text{ MPa} \]

(b) Concrete:

\[ y = 177.87 \text{ mm} = 0.17787 \text{ m} \]

\[ \sigma = \frac{(15.708 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6 \text{ Pa} = -15.59 \text{ MPa} \]
A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 30$ mm, $d = 20$ mm and $\sigma_{\text{all}} = 60$ MPa, determine the magnitude $P$ of the largest load that can be safely applied at the centers of the ends of the bar.

$$A = ad, \quad I = \frac{1}{12} ad^3, \quad c = \frac{1}{2} d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

$$G = \frac{P}{A} + \frac{Me}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$G = \frac{P}{ad} + \frac{3P(a-d)}{nd^2} = KP \quad \text{where} \quad K = \frac{1}{ad} + \frac{3(a-d)}{ad^2}$$

Data:
- $a = 30$ mm $= 0.030$ m
- $d = 20$ mm $= 0.020$ m

$$K = \frac{1}{(0.030)(0.020)} + \frac{(3)(0.010)}{(0.030)(0.020)^2} = 4.1667 \times 10^3 \text{ m}^2$$

$$P = \frac{G}{K} = \frac{60 \times 10^6}{4.1667 \times 10^3} = 14.40 \times 10^3 \text{ N} \quad P = 14.40 \text{ kN}$$
The rigid bar DEF is welded at point D to the steel beam AB. For the loading shown, determine:
(a) the equations defining the shear and bending at portion AD of the steel beam AB,
(b) the location and magnitude of the largest bending moment.
(Hint: Replace the 700 N load applied at F by an equivalent force-couple system at D)

**SOLUTION**

**Reactions.** We consider the beam and bar as a free body and observe that the total load is 4300 N. Because of symmetry, each reaction is equal to 2150 N.

**Modified Loading Diagram.** We replace the 700-N load applied at F by an equivalent force-couple system at D. We thus obtain a loading diagram consisting of a concentrated couple, three concentrated loads (including the two reactions), and a uniformly distributed load.

a) Cut the beam somewhere in between portion AD

\[ M = -375x + 2150x \]

\[ 0 < x < 3.3 \text{ m} \]

b) Moment will be maximum when \( V = 0 \)

\[ V = 2150 - 750x = 0 \]

\[ x = \frac{2150}{750} = 2.86 \text{ m} \]

\[ \frac{M_{\text{max}}}{4} = -375(2.86) + 2150(2.86) \]

\[ M_{\text{max}} = +3081 \text{ N.m} \]
Question 4 (8 points):

For the beam & loading shown and considering the given cross-section through section n-n, determine:

(a) the shearing stresses at points a and b,
(b) the largest shearing stress.

Determine section properties.

<table>
<thead>
<tr>
<th>Part</th>
<th>( A (m^2) )</th>
<th>( I (m^4) )</th>
<th>( A \bar{y} (m^3) )</th>
<th>( A \bar{d} (m^3) )</th>
<th>( A \bar{d}^2 (m^6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.500</td>
<td>100</td>
<td>3120000</td>
<td>60</td>
<td>6280000</td>
</tr>
<tr>
<td>2</td>
<td>5.200</td>
<td>25</td>
<td>125000</td>
<td>-25</td>
<td>3125000</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>12.700</td>
<td>125</td>
<td>4375000</td>
<td>38</td>
<td>940500.0</td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{2A_A}{2A} = \frac{5.75000}{12500} = 0.05 m \]

\[ I = \Sigma A d^2 + \Sigma I = 12.5 \times 10^6 m^4 \]

(a) \[ A = 6.25 m^2 \quad \bar{y} = 87.5 mm \quad Q_a = A \bar{y} = 546.75 m^3 \]

\[ t = 25 mm \]

\[ \tau_a = \frac{VQ_a}{It} = \frac{(50000)(5467.5)}{(12.5 \times 10^6)(25)} = 8.75 MPa \]

(b) \[ A_a = 12.5 m^2 \quad \bar{y} = 75 mm \quad Q_b = A \bar{y} = 92750 m^3 \]

\[ t = 25 mm \]

\[ \tau_b = \frac{VQ_b}{It} = \frac{(50000)(92750)}{(12.5 \times 10^6)(25)} = 15 MPa \]

\[ Q = A \bar{y} = (2500)(50) = 125000 \]

\[ t = 25 mm \]

\[ \tau_{min} = \frac{VQ}{It} = \frac{(50000)(125000)}{(12.5 \times 10^6)(25)} = 20 MPa \]
Question 5 (10 points):

A single horizontal force $P$ of magnitude 500 N is applied to end D of lever ABD. Knowing that portion AB of the lever has a diameter of 30 mm, determine;

(a) the state of plane stress (i.e. the normal and shearing stresses) on an element located at point H, 100 mm above point A and having sides parallel to the x and y axes,

(b) the principal planes (i.e. $\Theta_1$ & $\Theta_2$) and the principal stresses (i.e. $\sigma_{\text{max}}$ & $\sigma_{\text{min}}$) at the same point H.

\[ P = 500 \text{ N} \]

\[ T = 225 \text{ N}\cdot\text{m} \]

\[ M_x = 125 \text{ N}\cdot\text{m} \]

\textbf{SOLUTION}

\textit{Force-Couple System.} We replace the force $P$ by an equivalent force-couple system at the center $C$ of the transverse section containing point H:

\[ P = 500 \text{ N} \quad T = (500 \text{ N})(0.45 \text{ m}) = 225 \text{ N}\cdot\text{m} \]

\[ M_x = (500 \text{ N})(0.25 \text{ m}) = 125 \text{ N}\cdot\text{m} \]

\textit{a. Stresses $\sigma_x$, $\sigma_y$, $\tau_{xy}$ at Point H.} Using the sign convention shown in Fig. 7.2, we determine the sense and the sign of each stress component by carefully examining the sketch of the force-couple system at point C:

\[ \sigma_x = 0 \quad \sigma_y = \frac{M_x}{I} = \frac{(125 \text{ N}\cdot\text{m})(0.015 \text{ m})}{\frac{1}{4} \pi (0.015)^4} \quad \sigma_y = 47.16 \text{ MPa} \]

\[ \tau_{xy} = \frac{T}{J} = \frac{(225 \text{ N}\cdot\text{m})(0.015)}{\frac{1}{4} \pi (0.015)^4} \quad \tau_{xy} = 42.44 \text{ MPa} \]

We note that the shearing force $P$ does not cause any shearing stress at point H.

\textit{b. Principal Planes and Principal Stresses.} Substituting the values of the stress components into Eq. (7.12), we determine the orientation of the principal planes:

\[ \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(42.44)}{0 - 47.16} = -1.80 \]

\[ 2\theta_p = -61.0^\circ \quad \text{and} \quad 180^\circ - 61.0^\circ = +119^\circ \]

\[ \theta_p = -30.5^\circ \quad \text{and} \quad +59.5^\circ \]

Substituting into Eq. (7.14), we determine the magnitudes of the principal stresses:

\[ \sigma_{\text{max, min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]

\[ = \frac{0 + 47.16}{2} \pm \sqrt{\left(\frac{0 - 47.16}{2}\right)^2 + (42.44)^2} = 23.58 \pm 48.55 \]

\[ \sigma_{\text{max}} = 72.13 \text{ MPa} \]

\[ \sigma_{\text{min}} = -24.97 \text{ MPa} \]
Question 6 (10 points):

A wide-flange shape column $AB$ carries a centric load $P$ of magnitude 60 kN. Cables $BC$ and $BD$ are taut and prevent motion of Point $B$ in the $xz$ plane. Using Euler’s formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length $L$. Take $E = 200$ GPa and $I_x = 48.9 \times 10^6$ mm$^4$ & $I_y = 4.73 \times 10^6$ mm$^4$ for W250 x 32.7. (Hint: Consider buckling in $xz$-plane and $yz$-plane separately)

\[
W250 \times 32.7: \quad I_x = 48.9 \times 10^6 \text{ mm}^4, \quad I_y = 4.73 \times 10^6 \text{ mm}^4
\]

\[
P = 60 \text{ kN},
\]

\[
P_{cr} = (F.S.)P = (2.2)(60) = 132 \text{ kN}
\]

Buckling in $xz$-plane. $L_e = 0.7L$

\[
P_{cr} = \frac{\pi^2EI_y}{(0.7L)^2}, \quad L = \frac{\pi}{0.7} \sqrt{\frac{EI_y}{P_{cr}}}
\]

\[
L = \frac{\pi}{0.7} \sqrt{\frac{(200 \times 10^9)(4.73 \times 10^6)}{132000}} = 12.01 \text{ m}
\]

Buckling in $yz$-plane. $L_e = 2L$

\[
P_{cr} = \frac{\pi^2EI_x}{(2L)^2}, \quad L = \frac{\pi}{2} \sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(48.9 \times 10^6)}{132000}} = 13.52 \text{ m}
\]

Smaller value for $L$ governs. $L = 12.01 \text{ m}$