King Saud University
Department of Mathematics

151
Final Exam, January 2016

NAME:

Group Number/Instructor's Name:

ID:

| Question | Grade |
|---|---|
| I | |
| II | |
| III | |
| IV | |
| V | |
| VI | |
| Total | |

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Answer | | | | | | | | | | |

I) Choose the correct answer (write it on the table above):

1) If a conditional statement is true, then

| (A) Its inverse is true | (B) Its converse is true | (C) Its contrapositive is true | (D) None |
|---|---|---|---|

2) The argument

$\neg p \vee q$
$p$
$- - - - -$
$\therefore q$
is

| (A) valid | (B) invalid |
|---|---|

3) Let $D = \{2, 3, 4, 5\}$. The truth value of $(\exists x \in D)(2^x < x\,!)$ is

| (A) False | (B) True |
|---|---|

4) If $M$ and $N$ are subsets of the universal set $U$, such that $\overline{M} \cup N = \emptyset$, then $M \cap \overline{N}$ is

| (A) $\emptyset$ | (B) $U$ | (C) $M \cup \overline{N}$ | (D) None |
|---|---|---|---|

5) If $A = \{1, 2, 3\}$ and $R = \{(1,1), (1,3), (2,1), (2,3), (3,2), (3,3)\}$, then the reflexive closure of $R$ is

| (A) $\{(1,1), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ | (B) $\{(1,1), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$ | (C) $\{(1,1), (2,2), (3,3)\}$ | (D) None of the previous |
|---|---|---|---|

6) If $f(x, y, z) = x \cdot y \cdot z + x \cdot \overline{y} \cdot z$, then $f(x, y, z)$ is equivalent to

| (A) $x \cdot y$ | (B) $x \cdot \overline{y} \cdot \overline{z}$ | (C) $x \cdot z$ | (D) None |
|---|---|---|---|

7) The dual of $F(x, y) = x \cdot \overline{y} + x \cdot 0 + \overline{x} \cdot 1$ is

| (A) $(x+\overline{y})\cdot(x+0)\cdot(\overline{x}+1)$ | (B) $(x+\overline{y})\cdot(x+1)\cdot(\overline{x}+0)$ | (C) $(x+y)\cdot(x+1)\cdot(x+0)$ | (D) None |
|---|---|---|---|

8) If $f(x, y, z) = x \cdot y + y + \overline{x} \cdot \overline{z}$ is a Boolean function, then $f(1, 0, 0)$ equals

| (A) 0 | (B) 1 | (C) None of the previous |
|---|---|---|

9) A tree with 20 vertices has

| (A) 20 edges | (B) 19 edges | (C) 21 edges | (D) None of the previous |
|---|---|---|---|

10) If a connected, planar, simple graph $G$ has 6 vertices with degrees 2, 2, 2, 3, 3, 4, then the number of regions in the planar representation of $G$ is

| (A) $r = 6$ | (B) $r = 8$ | (C) $r = 4$ | (D) None of the previous |
|---|---|---|---|

II) A) Prove, **without using truth tables**, that

$$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$$

B) Prove the following theorem: *If $m$, $n$ and $p$ are integer numbers, such that $m + n$ is even and $n + p$ is even, then $m + p$ is even.*

C) Use the first principle of mathematical induction to show that 2 is a divisor of $n^2 + n$, for every integer $n \geq 1$.

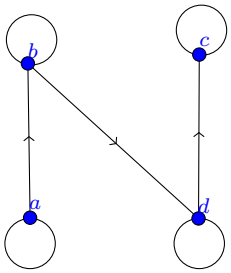
III) A) Let $\mathbb{Z}$ be the set of all integers. Define the relation $R$ on $\mathbb{Z}$ by

$$xRy \iff x - y \text{ is a multiple of 3.}$$

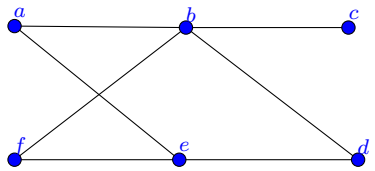
(Note that 0 is a multiple of 3). Prove that $R$ is an equivalence relation on $\mathbb{Z}$.

B) Let $A = \{0, 1, 2, 3\}$ and $R = \{(0, 0), (1, 1, ), (1, 2), (2, 2), (3, 3)\}$. Prove that $R$ is a partial order relation on $A$.

C) List the ordered pairs in the relation $R$ represented by the graph below.



IV) Let $G$ be the graph below:



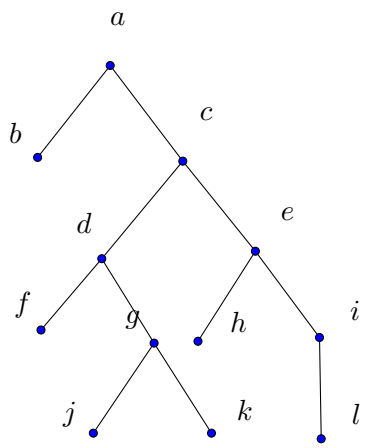

i) Is the graph $G$ connected? Justify your answer.

ii) Is the graph $G$ simple? Justify your answer.

iii) Is the graph $G$ planar? If $G$ is planar, draw a planar representation for $G$.

iv) Is the graph $G$ bipartite? Justify your answer.

V) A) Consider the tree below:




i) Which vertex is the root?

ii) List the internal vertices;

iii) List the leaves;

iv) What is the parent of $e$?

v) What are the siblings of $c$?

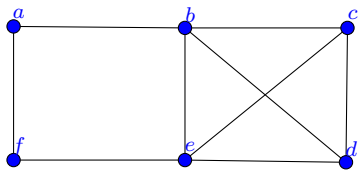
vi) Is this tree a binary tree? Justify your answer;

vii) Find the level of each vertex of the tree;

viii) What is the height of the tree?

ix) Is the tree balanced? Justify your answer.

B) Find a spanning tree for the graph




C) Form a binary search tree for the words: **map**, **pencil**, **eraser**, **ruler**, **paper** (using alphabetical order).

VI) A) Find an expression for the Boolean function $F(x, y, z)$, whose values are given in the table below:

| $x$ | $y$ | $z$ | $F(x, y, z)$ |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

B) Find a sum-of-product expansion for the Boolean function given by

$$F(x, y, z) = x \cdot \overline{y} + x \cdot z.$$

C) a) Use $K$-maps to minimize the Boolean function

$$F(x, y, z) = xyz + \overline{x}yz + x\overline{y}z + x\overline{y}\,\overline{z}.$$

b) Draw the logic gates (circuits) representing the function

$$F(x, y, z) = x \cdot \overline{y} + \overline{x} \cdot \overline{y} \cdot z + y \cdot z.$$