**Department of Mathematics**

**College of Sciences**

**King Saud University**

**Math 382**

**Final Exam**

**Second Semester, 1435-1434H**

**Time: 3 hours.**

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| **Name:** |
| **Student No.** |

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| **Question number** | I | II | III | IV | Total |
| **Mark** |  |  |  |  |  |

# Question I

1. Prove that an upper bound u of a nonempty set S in R is the supremum of S if and only if for each > 0 there exists s S such that u -  < s.
2. Prove the Archimedean Property:if xR, then there exists nxN such that

x < nx.

1. Let S := {}. Find inf S and supS and justify your answer.

# Question II

1. Prove that a sequence in R can have at most one limit.
2. Find the limit if it exists. Write all the details.
3. 
4. 

**Question III**

1. Let A, B  R and let f : AR and g: BR be functions such that f(A)  B. If f is continuous at a point c  A and g is continuous at b = f(c)  B, then the composition gf: AR is continuous at c.
2. Let A R and let f : AR. Define uniform continuity of f on A and prove that f(x)=x2 is uniformly continuous on [0,2].
3. Find the limit if it exists. Write all the details.
4. 
5. 
6. 

**Question IV**

1. Prove Rolle's Theorem: Suppose that f is continuous on a closed interval I := [a, b ], that the derivative f' exists at every point of the open interval (a, b), and that f(a) = f(b) = 0. Then there exists at least one point c in

(a, b) such that f' (c) = 0.

1. Suppose that f : [0, 2] R is continuous on [0, 2] and differentiable on (0, 2), and that f(0) = 0, f(l) = 1, f(2) = 1.
2. Show that there exists c1  (0, 1) such that f'(c1) = 1.
3. Show that there exists c2  (1, 2) such that f' (c2) = 0.
4. Show that there exists c  (0, 2) such that f' (c) = 1/3.
5. Find the limit if it exists. Write all the details.
6. 
7. 