

Question 1(2+2+3)

a) Find the number  $c$  in the mean value theorem for  $f(x) = -x^2 + 4x$  on  $[0, 3]$

b) Compute the integral  $\int \frac{dx}{\sqrt{5^x - 16}}$

c) Evaluate  $\int \frac{\cot x dx}{\sqrt{9 - (\sin x)^4}}$

Question 2(3+3+3)

a) Compute  $\lim_{x \rightarrow 3} \left( \frac{1}{x-3} - \frac{1}{\ln(x-2)} \right)$

b) Find  $\int x^2 \tan^{-1}(x) dx$

c) Evaluate the integral  $\int (\tan x)^4 (\sec x)^6 dx$

Question 3(3+3+3)

a) Compute the following integral  $\int \frac{x^2 dx}{(x^2 + 9)^{3/2}}$

b) Find the integral  $\int \frac{(3x-2)dx}{(x^2+4)(x+2)}$

c) Evaluate the integral  $\int \frac{dx}{3 - \sin x + \cos x}$

**Question 4(3+2+1)**

- a) Sketch the region bounded by the curves:  $y = 4 - x^2$ ,  $y = x + 2$ ,  $x = -3$ ,  $x = 0$  and find its area.
- b) Find the volume obtained by revolving the region bounded by the curves  $y = -x^2 + 2$ ,  $y = 1$  about the line of equation  $x = 3$
- c) Set up an integral for the volume obtained by revolving the region in part b) about the line of equation  $y = 4$ .

**Question 5(3+3+3)**

- a) Find the length of the curve given by  $r = (\cos(\frac{\theta}{2}))^2$ ,  $0 \leq \theta \leq \pi$ .
- b) Sketch the region R that lies inside the curve  $r = 1 - \sin\theta$  and outside the curve  $r = 1$  and find its area.
- c) Find the area of the surface obtained by revolving the curve  $r = 2\cos\theta$   $0 \leq \theta \leq \pi/4$  about the y-axis.