King Saud University

Department of Mathematics

2nd Semester 1432-1433 H

MATH 111 (Integral Calculus)

Final Exam

Duration: 3 Hours

Student's Name	Student's ID	Group Number	Lecturer's Name

Question Number	I	II	III	IV	Total
Mark					

Question I: A. Choose the correct answer. **(8 Marks)**

(1) $\lim_{n\to\infty}\left(\frac{1}{n^3}\sum_{i=1}^{n}\left(3i^2-1\right)\right)$ **is equal to**

(a) $\mathbf{0}$ **(b)** $\frac{1}{2}$

(b) $\mathbf{1}$ **(d) None of the previous**

(2) If $F(x)=\int_{2x}^{3x}\frac{t-1}{t+1}dt$ **then** $F'(0)$ **is equal to**

(a) 1 **(b)** -1

(b) 5 **(d) None of the previous**

(3) $2\ln|x|-\ln|x+1|-\ln|x+1|^{-1}$ **equals**

(a) $2\ln|x|$ **(b)** $\ln\left|\frac{x^2}{x+1}\right|$

(c) $\ln\left|x^2(x+1)\right|$ **(d) None of the previous**

(4) The value of $\int_0^{ln2} \frac{1}{(coshx+sinhx)^2} dx$ is

(a) $\frac{-3}{4}$ **(b) $\frac{3}{8}$**

(c) $ln2$ **(d) None of the previous**

(5) The partial fractions of $\frac{3}{x^4+9x^2}$ are

(a) $\frac{A}{x} + \frac{Bx+C}{x^2+9}$ **(b) $\frac{A}{x^2} + \frac{Bx+C}{(x+3)^2}$**

(c) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}$ **(d) None of the previous**

(6) If $(x, y) = (2, 1)$ is the rectangular coordinates representation of a point, then a corresponding polar coordinates representation (r, θ) is

(a) $\left(\sqrt{5}, tan^{-1}\left(\frac{1}{2}\right)\right)$ **(b) $\left(-\sqrt{5}, tan^{-1}\left(\frac{1}{2}\right)\right)$**

(c) $\left(\sqrt{5}, tan^{-1}\left(\frac{1}{2}\right) + \pi\right)$ **(d) None of the previous**

(7) If $(r, \theta) = (5, \pi)$ is a polar coordinates representation of a point, then the corresponding rectangular representation (x, y) is

(a) (5,0) **(b) (-5,0)**

(c) (0,5) **(d) None of the previous**

(8) The plane curve $x = 2 + 4cost$, $y = 3 + 4sint$, $0 \leq t \leq 2\pi$, is a circle with center

(a) $(-2, -3)$ **(b) $(4, 4)$**

(c) $(2, 3)$ **(d) None of the previous**

B. Prove that for any real numbers r *and* s **(2 Marks)**

$$e^r e^s = e^{r+s}$$

C. Find the value of c that satisfies the conclusion of the Integral Mean Value Theorem on [0,3] for

$$f(x) = x^2 + 1$$

(2.5 Marks)

D. Find the value of x that satisfies the equation $\boldsymbol{ln\left(\frac{e^{-4x}}{e^{2x}}\right)=3}$**.** **(1.5 Marks)**

<u>Question II</u>: A. Compute the following integrals

(i) $\int \sqrt{2x-x^2}\,dx$ **(4 Marks)**

(ii) $\int \frac{lnx}{x^3}\,dx$ **(3 Marks)**

(iii) $\int \frac{e^x}{\sqrt{16+e^{2x}}}\,dx$ **(4 Marks)**

(iv) $\int tan^{\frac{3}{2}}(x)sec^{4}(x)dx$ **(3 Marks)**

B. Determine whether the following improper integrals converge or diverge

(i) $\int_{1}^{5} \frac{1}{1-x} dx$ **(2.5 Marks)**

(ii) $\int_0^{\infty} x^2 e^{-x^3}\, dx$ **(2.5 Marks)**

QUESTION III

A. ***Sketch*** **and** ***Find the area*** **of the region bounded by the graphs of** **(5 Marks)**

$y = x^2$ **and** $y = -x$.

B. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$, $y = 0$ and $y = 2 - x$
Sketch R **and** ***Find the volume*** **of the solid resulting by revolving R about**

(i) the x −axis.
(ii) the y −axis. (6 Marks)

C. Find the arc length of the portion of the curve of $y = \left(1 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$ from $x = \frac{1}{8}$ to $x = 1$. (4 Marks)

QUESTION IV

Sketch **and** ***find the area*** **of the region bounded by the graph of $r = 3 - 3sin\theta$ for $0 \le \theta \le 2\pi$. (5 Marks)**

Good Luck☺