

King Saud University
Department of Mathematics

244
First Midterm, March 2016

NAME:

Group Number/Instructor name:

ID:

- Duration of the exam: 90 minutes
- Simple calculators are allowed

| Question | Grade |
|----------|-------|
| I | |
| II | |
| III | |
| IV | |
| Total | |

| | | | | | |
|----------|---|---|---|---|---|
| Question | 1 | 2 | 3 | 4 | 5 |
| Answer | | | | | |

I) Choose the correct answer (write it on the table above):

1) If $A^3 - 2B^T = \begin{bmatrix} 18 & -2 \\ -6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 3 \\ 1 & 0 \end{bmatrix}$, then the matrix A is

| | | | |
|--|--|--|----------|
| (A) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ | (B) $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ | (C) $A = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ | (D) None |
|--|--|--|----------|

2) If $A^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $p(x) = x^2 - x + 3$, then $p(A)$ equals

| | | | |
|---|---|---|----------|
| (A) $\begin{bmatrix} 5 & 3 \\ 6 & 11 \end{bmatrix}$ | (B) $\begin{bmatrix} 5 & 11 \\ 3 & 6 \end{bmatrix}$ | (C) $\begin{bmatrix} 5 & 6 \\ 3 & 11 \end{bmatrix}$ | (D) None |
|---|---|---|----------|

3) The values of x and y for which the matrix $\begin{bmatrix} x^2 & 0 & x^2 - 4 \\ -1 & 3 & 2y - 6 \\ 1 & 7 & 2x - 5y \end{bmatrix}$ is lower triangular are

| | | | |
|--------------------|------------------------|----------------------------|----------|
| (A) $x = 2, y = 3$ | (B) $x = \pm 2, y = 3$ | (C) $x = \pm 2, y = \pm 3$ | (D) None |
|--------------------|------------------------|----------------------------|----------|

4) For any $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, the solution of the system $\begin{cases} -x + 2y = b_1 \\ 2x + 6y = b_2 \end{cases}$ is

| | | | |
|-------------|------------|-------------|----------|
| (A) a point | (B) a line | (C) a plane | (D) None |
|-------------|------------|-------------|----------|

5) The trace of of the matrix $\begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & 0 \\ 0 & -3 & 1 \end{bmatrix}$ is

| | | | |
|---------------|-------|-------|----------|
| (A) (2, 4, 1) | (B) 7 | (C) 6 | (D) None |
|---------------|-------|-------|----------|

II) Determine whether the following is **True** or **False**.

(1) If A is a symmetric matrix, then A^2 is symmetric. ()

(2) The following system of equations is linear. ()

$$\begin{aligned}\sqrt{3}x + 2y - \cos z &= 0 \\ \sqrt{3} - x + 2y + z &= 1\end{aligned}$$

(3) If A and B are $n \times n$ matrices, then

$$(A + B)^2 - (A - B)^2 = 4AB.$$

()

(4) If A is an upper triangular matrix, then the matrix $A - A^T$ is diagonal. ()

(5) If A and B are square matrices of the same size, such that $A + B$ is symmetric, then both A and B are symmetric. ()

(6) If $D^3 = \begin{bmatrix} 8 & 0 \\ 0 & -1 \end{bmatrix}$, then $D^2 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$. ()

III) Solve the linear system of equations

$$\begin{cases} x - y + 3z + 2w = 1 \\ -2x + y + 5z + w = 2 \\ -3x + 2y + 2z - w = 1 \\ 4x - 4y + 7z + 18w = 9 \end{cases}$$

IV) Let $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

a) Find A^{-1} .

b) Find the matrix B , if $AB = C$, where $C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ -2 & 0 \end{bmatrix}$.

Scrap paper. This page will not be graded.