Introduction

• Belts, ropes, chains, and other similar elastic or flexible machine elements are used in conveying systems and in the transmission of power over comparatively long distances
• Can be used as a replacement for gears, shafts, bearings, and other relatively rigid power transmission devices
• Simplifies the design machine and substantially reduce the cost
### Introduction

#### Advantages
- Flexible, absorb shocks, eliminate vibration, increase life of the machine

#### Disadvantages
- Life is limited
- Need regular inspection for wear, aging and loss of elasticity
- The elements should be replaced at the first sight of deterioration

### 17-1 Belts
- May be used for long centre distances
- Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shaft is neither constant nor exactly equal to the ratio of the pulley diameters
- In some cases an idler or tension pulley can be used to avoid adjustments in centre distance that are ordinarily necessitated by age or the installation of new belts
Types of Belts

<table>
<thead>
<tr>
<th>Belt Type</th>
<th>Figure</th>
<th>Joint</th>
<th>Size Range</th>
<th>Center Distance</th>
</tr>
</thead>
</table>
| Flat      | ![Flat Belt](image) | Yes   | \( t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\
                      0.75 \text{ to } 5 \text{ mm} \end{cases} \) | No upper limit |
| Round     | ![Round Belt](image) | Yes   | \( d = \frac{1}{8} \text{ to } \frac{1}{4} \text{ in} \) | No upper limit |
| V         | ![V Belt](image) | None  | \( b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\
                        8 \text{ to } 19 \text{ mm} \end{cases} \) | Limited |
| Timing    | ![Timing Belt](image) | None  | \( p = 2 \text{ mm and up} \) | Limited |

Types of Pulleys

- *Crowned pulleys* are used for flat belts
- *Grooved pulleys or sheaves* are used for round and V belts
- *Toothed wheels or sprockets* are used for timing belts
Non-reversing and reversing belts

Quarter twist belt drive
Eliminating the need for a clutch

Variable-speed belt drives
Timing belts and chains
17-2 Fat- and Round-Belt Drives

• Belt geometry - Open

\[
\begin{align*}
\theta_s &= \pi - 2 \sin^{-1} \frac{D - d}{2C} \\
\theta_L &= \pi + 2 \sin^{-1} \frac{D - d}{2C} \\
L &= \sqrt{4C^2 - (D - d)^2} + \frac{1}{2} (D\theta_L - d\theta_s)
\end{align*}
\]

17-2 Fat- and Round-Belt Drives

• Belt geometry - Crossed

\[
\begin{align*}
\theta &= 2 \sin^{-1} \frac{D + d}{2C} \\
L &= \sqrt{4C^2 - (D + d)^2} + \frac{1}{2} (D + d)
\end{align*}
\]
17-2 Flat and Round belt drives

- Firbank explains that the relative motion between the belt and pulley by *Elastic creep*.
- Due to which *Velocity ratio* is not constant.
- There is *sliding friction* as opposed to *static friction*.
- The belt first contacts with pulley with tight side.

\[ \theta \]

- The velocity of the *tight side* of the belt is the same as the pulley.
- The belt tension as well as the velocity of the belt changes at the *loose side*.
- *Firbank* used this theory to express the mechanics of the *flat-belt drives*.
- The results were verified through experiments.
- The c.o.f. was found 0.7 up to 0.9
- The relationship is similar to band brake, additional is the centrifugal force.
Forces in Flat-Belts

- \( ds = m r^2 \omega^2 d\theta \)
- \( \sum F_r = 0 \) and \( \sum F_H = 0 \)
- From which
  \[
  \frac{dF}{d\theta} - fF = -f m r^2 \omega^2
  \]
- Which is a 1st order differential equation and the general solution is given by;
  \[
  F = A \exp(f \theta) + m r^2 \omega^2
  \]
- A is constant and can be obtained from the B. Conditions as;
- \( F = F_2 \) if \( \theta = 0 \) and \( F = F_1 \) if \( \theta = \emptyset \)
- From which
  \[
  \frac{F_1 - m r^2 \omega^2}{F_2 - m r^2 \omega^2} = \exp(f \emptyset)
  \]

- \( (F_1 - F_2) = (F_1 - F_2) \frac{\exp(f \emptyset) - 1}{\exp(f \emptyset)} \rightarrow (i) \)
- \( F_c = \frac{W V^2}{g} \) \( (W = \gamma b t \text{ N/m}) \)
- \( F_1 - F_2 = \frac{2T}{d} \rightarrow (ii) \)
- \( F_i = \frac{F_1 + F_2}{2} - F_c \rightarrow (iii) \)
- Divide \( (iii) \) by \( (iii) \), and use \( (i) \) to get,
  \[
  F_i = \frac{T}{D} \frac{\exp(f \emptyset) + 1}{\exp(f \emptyset) - 1}
  \]
  \[
  F_1 = F_c + F_i \frac{2 \exp(f \emptyset)}{\exp(f \emptyset) + 1}, F_2 = F_c + F_i \frac{2 \exp(f \emptyset)}{\exp(f \emptyset) + 1}
  \]
  \[ (F_i)_a = b F_a C_p C_v \]
- \( F_{a} \) = Manufacturer’s allowed tension (N/mm)
- \( C_p \) = Pulley correction factor (Table 17-4)
- \( C_v \) = Velocity correction factor
Design steps for Flat-Belts

1. Find $\exp(f \emptyset)$ from belt-drive geometry and friction
2. From belt geometry and speed find $F_c$
3. From $T = H_{nom}K_5n_d/(2\pi n)$ find necessary torque
4. From torque $T$ find the necessary $(F_1)_a - F_2 = 2T/d$
5. From Tables 17–2 and 17–4, and Eq. $(F_1)_a = bF_aC_\thetaC_v$ determine $(F_1)_a$
6. Find $F_2$ from $(F_1)_a - [(F_1)_a - F_2]$ i.e. $\Delta F = F_1 - F_2$
7. From Eq. $F_i = \frac{F_1x_{1i}+F_2}{2} - F_c$ find the necessary initial tension $F_i$
8. Check the friction development, $f' < f$
   \[ f' = \frac{1}{\emptyset} \ln \left( \frac{(F_i)_a - F_c}{F_2 - F_c} \right) \]
9. Find the factor of safety from $n_{fs} = H_{Tran}/H_{nom}K_5$

### Table 17–2

<table>
<thead>
<tr>
<th>Material</th>
<th>Specification</th>
<th>Size, mm</th>
<th>Minimum Pulley Diameter, mm</th>
<th>Allowable Tension per Unit Width of 3 m/s, $10^5$ N/m</th>
<th>Specific Weight, kN/m²</th>
<th>Coefficient of Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leather</td>
<td>1 ply</td>
<td>$t = 4.5$</td>
<td>75</td>
<td>5</td>
<td>9.5–12.2</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>$t = 5$</td>
<td>90</td>
<td>6</td>
<td>9.5–12.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 ply</td>
<td>$t = 7$</td>
<td>115</td>
<td>7</td>
<td>9.5–12.2</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>$t = 8$</td>
<td>150</td>
<td>9</td>
<td>9.5–12.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = 9$</td>
<td>230</td>
<td>10</td>
<td>9.5–12.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polymide</td>
<td>P-0°</td>
<td>$t = 0.8$</td>
<td>15</td>
<td>1.8</td>
<td>9.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>P-1°</td>
<td>$t = 1.3$</td>
<td>25</td>
<td>6</td>
<td>9.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>P-2°</td>
<td>$t = 1.8$</td>
<td>60</td>
<td>10</td>
<td>13.8</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>A-2°</td>
<td>$t = 2.8$</td>
<td>60</td>
<td>10</td>
<td>10.0</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>A-3°</td>
<td>$t = 3.3$</td>
<td>110</td>
<td>18</td>
<td>11.4</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>A-4°</td>
<td>$t = 5.0$</td>
<td>240</td>
<td>30</td>
<td>10.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$t = 6.4$</td>
<td>340</td>
<td>48</td>
<td>10.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urethane</td>
<td>$w = 12.7$</td>
<td>$t = 1.6$</td>
<td>See</td>
<td>1.0°</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$w = 19$</td>
<td>$t = 2.0$</td>
<td>Table</td>
<td>1.7°</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$w = 32$</td>
<td>$t = 2.3$</td>
<td>17–3</td>
<td>3.3°</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td>Round</td>
<td>$d = 6$</td>
<td>See</td>
<td>1.4°</td>
<td>-</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$d = 10$</td>
<td>Table</td>
<td>3.3°</td>
<td>-</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$d = 12$</td>
<td>17–3</td>
<td>5.8°</td>
<td>-</td>
<td>10.3–12.2</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$d = 20$</td>
<td>19°</td>
<td>10.3–12.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: $w$ = width for belts 8 in wide or more.*
A polyamide A-3 flat belt 150 mm wide is used to transmit 11 kW under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 2.4 m apart. The 150 mm driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 450 mm in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(a) Estimate the centrifugal tension $F_c$ and the torque $T$.
(b) Estimate the allowable $F_1$, $F_2$, $F_i$ and allowable power $H_a$.
(c) Estimate the factor of safety. Is it satisfactory?
Solution

(a) Eq. (17.1): 
\[ \phi = \theta_d = \pi - 2 \sin^{-1} \left( \frac{450 - 150}{2(2400)} \right) = 3.0165 \text{ rad} \]

\[ \exp(f \phi) = \exp(0.8(3.0165)) = 11.17 \]

\[ V = \pi (0.15)(1750)/60 = 13.7 \text{ m/s} \]

Table 17-2: 
\[ w = \gamma h u = 11000(0.15)(0.0033) = 5.4 \text{ N/m} \]

Answer Eq. (e):
\[ F_s = \frac{w}{g} V^2 = \frac{5.4}{9.81} (13.7)^2 = 103 \text{ N} \]

\[ T = \frac{H_{\text{min}} K_r n_d}{2 \pi n} = \frac{1.25(1.1)(1000)}{2 \pi (1750)/60} = 82 \text{ N} \cdot \text{m} \]

(b) The necessary \((F_1)_h - F_2\) to transmit the torque \(T\), from Eq. (b), is

\[ (F_1)_h - F_2 = \frac{2T}{d} = \frac{2(82)}{0.15} = 1093 \text{ N} \]

From Table 17-2 \(F_2 = 18 \text{ kN/m}\). For polyamide belts \(C_s = 1\), and from Table 17-4 \(C_p = 0.70\). From Eq. (17-12) the allowable largest belt tension \((F_1)_h\) is

Solution...

Answer

then

\[ (F_1)_h = bF_2 C_p C_s = 0.15(18000)(0.70)(1) = 1890 \text{ N} \]

\[ F_3 = (F_1)_h - [(F_1)_h - F_2] = 1890 - 1093 = 797 \text{ N} \]

and from Eq. (d)

\[ F_1 = \frac{(F_1)_h + F_2}{2} = \frac{1890 + 797}{2} = 1440 \text{ N} \]

Answer The combination \((F_1)_h\), \(F_2\), and \(F_1\) will transmit the design power of 11(1.25)(1.1) = 15.125 kW and protect the belt. We check the friction development by solving Eq. (17-7) for \(f'\):

\[ f' = \frac{1}{\phi} \ln \left( \frac{(F_1)_h - F_2}{F_1 - F_2} \right) = \frac{1}{3.0165} \ln \left( \frac{1890 - 103}{797 - 103} \right) = 0.314 \]

From Table 17-2, \(f = 0.8\). Since \(f' < f\), that is, 0.314 < 0.80, there is no danger of slipping:

Answer

\[ n_{fr} = \frac{H}{H_{\text{min}} K_s} = \frac{15.125}{11(1.25)} = 1.1 \] (as expected)

Answer The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 15.125 kW.
Example 17-2

Design a flat-belt drive to connect horizontal shafts on 5 m centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 45 kW under very light shock.

17-3 V Belts

- The dimensions are standardized by the manufacturers (by a letter of the alphabet in inch and numbers in metric) given in Table 17-10.
- B875 is a B section belt having 875mm inside circumference
- Used for relatively large torque over small distances
- Groove angle of Sheave is slightly smaller than the X-Section of the Belt
- For greater torque more than one belts are used on the same pulley
- Calculations are based on pitch length. The Pitch length of B875 belt will be 875+45 (from Table 17-11) = 920mm.
17-3 V Belts

• The pitch length is given by:
  \[ L_p = 2C + \frac{\pi(D + d)}{2} + \frac{(D - d)^2}{4C} \]

• The center distance is given by:
  \[ C = 0.25 \left\{ L_p - \frac{\pi}{2} (D + d) \right\} + \sqrt{\left[ L_p - \frac{\pi}{2} (D + d) \right]^2 - 2(D - d)^2} \]

• The centrifugal tension is calculated as
  \[ F_c = K_c \left( \frac{V}{2A} \right)^2 \]

• \( K_c \) is a parameter determined from Table 17-16

<table>
<thead>
<tr>
<th>Belt section</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity to be added</td>
<td>32</td>
<td>45</td>
<td>72</td>
<td>82</td>
<td>112</td>
</tr>
</tbody>
</table>

Table 17-11
Length Conversion Dimensions (Add the listed quantity to the inside circumference to obtain the pitch length in mm).

17-3 V Belts

• The belt life is expressed in hours (i.e. 24000) or in life (i.e. \( 10^8 \) or \( 10^9 \))

• The power rating of the belt is determined by the manufacturer for belt on equal diameter pulleys, moderate length and steady load.

• Deviation from this needs correction as;
  \[ H_a = K_1 K_2 H_{tab} \]

• Where
  - \( H_a \) = allowable power/belt
  - \( K_1 \) = angle of wrap correction factor (Table 17-13)
  - \( K_2 \) = belt length correction factor (Table 17-14)

• The design power is given by
  \[ H_d = H_{nom} K_s n_d \]
### Table 17-13 for \( K_f \) and Table 17-14 for \( K_2 \)

| \( D-d \) | \( \theta \), deg | \( VV \) | \( K_f \) | \( V \) Flat |
|----------|-----------------|---------|-----|---------|------|
| 0.00     | 180             | 1.00    | 0.75|        |      |
| 0.10     | 174.3           | 0.99    | 0.76|        |      |
| 0.20     | 166.5           | 0.97    | 0.78|        |      |
| 0.30     | 162.7           | 0.96    | 0.79|        |      |
| 0.40     | 156.9           | 0.94    | 0.80|        |      |
| 0.50     | 151.0           | 0.93    | 0.81|        |      |
| 0.60     | 145.1           | 0.91    | 0.83|        |      |
| 0.70     | 139.0           | 0.89    | 0.84|        |      |
| 0.80     | 132.8           | 0.87    | 0.85|        |      |
| 0.90     | 126.5           | 0.85    | 0.85|        |      |
| 1.00     | 120.0           | 0.82    | 0.82|        |      |
| 1.10     | 113.3           | 0.80    | 0.80|        |      |
| 1.20     | 106.3           | 0.77    | 0.77|        |      |
| 1.30     | 98.9            | 0.73    | 0.73|        |      |
| 1.40     | 91.1            | 0.70    | 0.70|        |      |
| 1.50     | 82.8            | 0.65    | 0.65|        |      |

### Note

*Multiply the net power per belt by this factor to obtain the nominal power*.

\[ K_f = 0.143 \times 345 + 0.007 \times 1 = 0.900 \times 0.052 \]  
\[ \text{in the range } 90^\circ \leq \theta \leq 180^\circ \]
17-3 V Belts

• The transmitted power per belt is
\[ H_d = \frac{DF\pi dn}{N_b} \]

• The largest tension is given by (done for flat belts)
\[ F_1 = F_c + \frac{DF\exp(f\theta)}{\exp(f\theta)-1} \]

• The initial tension is
\[ F_i = \frac{F_1 + F_2}{2} - F_c \]

• The factor of safety is
\[ n_{fs} = \frac{H_d N_b}{H_{nom} K_s} \]

• \( K_s \) is determined from Table 17-15

The Gates Rubber Company declares the effective COF to be 0.5123

<table>
<thead>
<tr>
<th>Source of Power</th>
<th>Normal Torque Characteristic</th>
<th>High or Nonuniform Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven Machinery</td>
<td>Uniform</td>
<td>1.0 to 1.2</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.1 to 1.3</td>
<td>1.2 to 1.4</td>
</tr>
<tr>
<td>Medium shock</td>
<td>1.2 to 1.4</td>
<td>1.4 to 1.6</td>
</tr>
<tr>
<td>Heavy shock</td>
<td>1.3 to 1.5</td>
<td>1.5 to 1.8</td>
</tr>
</tbody>
</table>

17-3 V Belts

• The life of the belt is determined by adding the equivalent forces \( (F_{b1}) \) and \( (F_{b2}) \) (due to the induced flexural stresses in the belt) into the belt tension as:
\[ T_1 = F_1 + (F_{b1})_1 = F_1 + \frac{K_b}{D} \]
\[ T_2 = F_1 + (F_{b2})_2 = F_1 + \frac{D}{K_b} \]

• \( K_s \) is given in Table 17-16

• Where
\[ N_p = \left[ \left( \frac{K}{T_1} \right)^{-b} + \left( \frac{K}{T_2} \right)^{-b} \right]^{-1} \]

• \( N_p \) is the number of passes. \( L_p \) is the pitch length. \( K \) and \( b \) are the durability parameters given in Table 17-17.

• The life of v-belt (in hours) is determined as:
\[ t = \frac{N_p L_p}{3600V} \]
The analysis of a V-belt consists the following steps:

- Find $V$, $L_p$, $C$, $\theta_s$ and $\exp(0.5123\theta_s)$
- Find $H_a$, $H_d$, and $N_b$ from $H_d/H_a$ and round up
- Find $F_c$, $\Delta F$, $F_n$, $F_z$, and $F_1$ and FOS
- Find the belt life in number of passes or hours if possible
Exp 17-4

A 7.46kW split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 188mm small sheave, a 280mm large sheave, and three B2800 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

Timing belts

- Made of rubberized fabric coated with a nylon and steel wires
- Has teeth and can be used for any speed
- Doesn’t need initial tension.
- Transmit power at constant velocity ratio
- Efficiency upto 99%
- No lubrication needed
- Analysis procedure is similar to V-belts
Roller chains

- No slippage or creep
- Constant velocity ratio
- Long life
- Can drive a number of shafts from a single source of power
- Standardized by ANSI
- Manufactured in single, double, triple and quadruple stands
- From figure
  \[ D = \frac{p}{\sin\left(\frac{180^\circ}{N}\right)} \]

Roller chains...

- Angle \( \frac{\gamma}{2} \) is called angle of articulation which depends on the number of teeth.
- Rotation of the link “strand” through this angle causes impact between rollers and the teeth.
- \( \frac{\gamma}{2} \) should be as minimum as possible to minimize the impact.
- Fails due to wear of the teeth or pin.
- Minimum number of teeth are 13, 17 or 21
- Center distance
  \[ C = \frac{p}{2} \left[ -A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2} \right] \]
- Where \( A = \frac{N_1 + N_2}{2} - \frac{L}{p} \)
<table>
<thead>
<tr>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Problems:</td>
</tr>
<tr>
<td>– 17-1 to 17-8, 17-10</td>
</tr>
<tr>
<td>– 17-17 to 17-22</td>
</tr>
</tbody>
</table>

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