CH # 17

Flexible Mechanical Elements

ME-305 Mechanical Engineering Design II

Introduction

- Belts, ropes, chains, and other similar elastic or flexible machine elements are used in conveying systems and in the transmission of power over comparatively long distances
- Can be used as a replacement for gears, shafts, bearings, and other relatively rigid power transmission devices
- Simplifies the design machine and substantially reduce the cost

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Introduction

Advantages

- Flexible, absorb shocks, eliminate vibration, increase life of the machine

Disadvantages

- Life is limited
- Need regular inspection for wear, aging and loss of elasticity
- The elements should be replaced at the first sight of deterioration

17-1 Belts

- May be used for long centre distances
- Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shaft is neither constant nor exactly equal to the ratio of the pulley diameters
- In some cases an idler or tension pulley can be used to avoid adjustments in centre distance that are ordinarily necessitated by age or the installation of new belts

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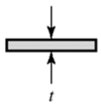
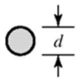
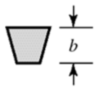
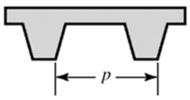
Types of Belts

Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round		Yes	$d = \frac{1}{8}$ to $\frac{3}{4}$ in	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing		None	$p = 2$ mm and up	Limited

Types of Pulleys

- *Crowned pulleys* are used for flat belts
- *Grooved pulleys or sheaves* are used for round and V belts
- *Toothed wheels or sprockets* are used for timing belts

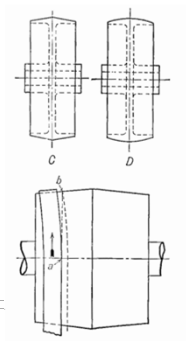
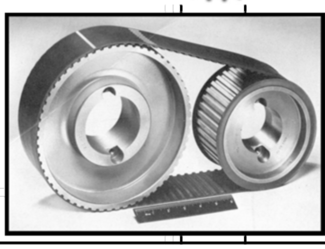
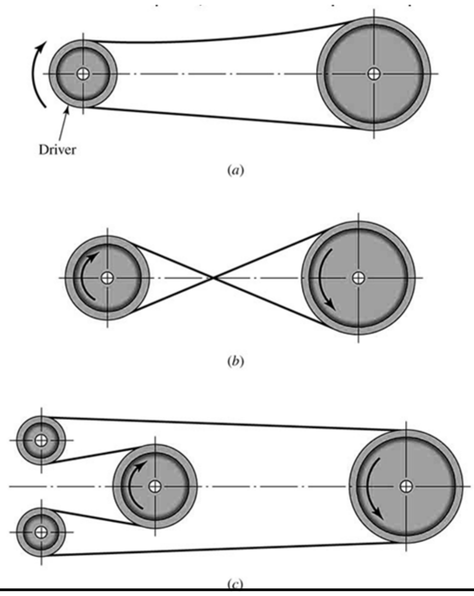


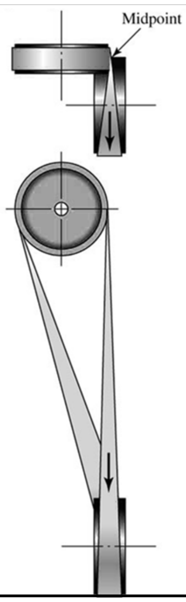
Fig. 52



Non-reversing and reversing belts



Quarter twist belt drive



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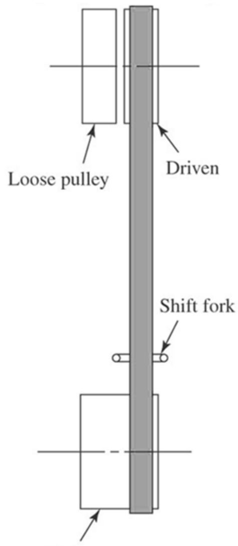
Eliminating the need for a clutch



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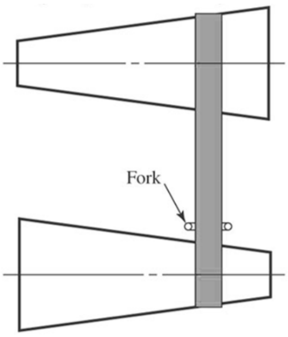
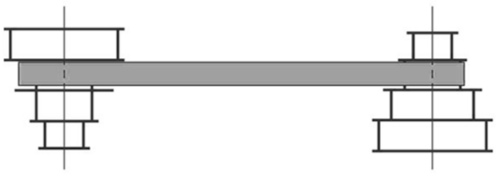
Variable-speed belt drives



(a)



(b)

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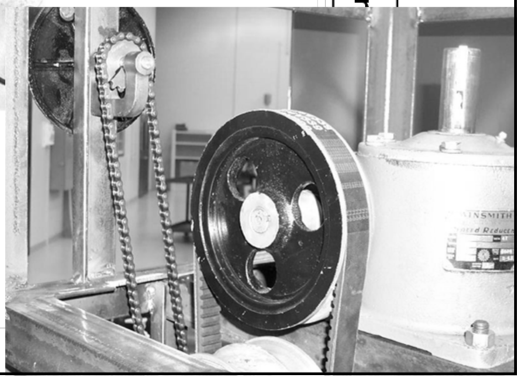
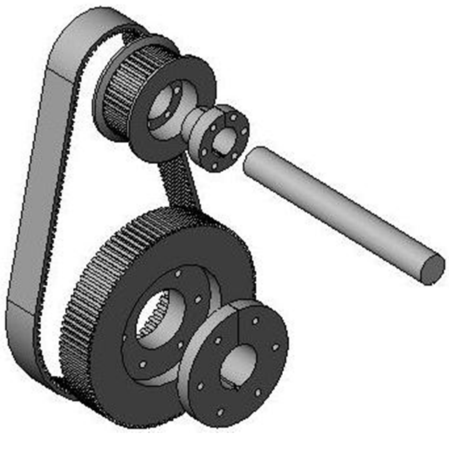
Timing belts and chains

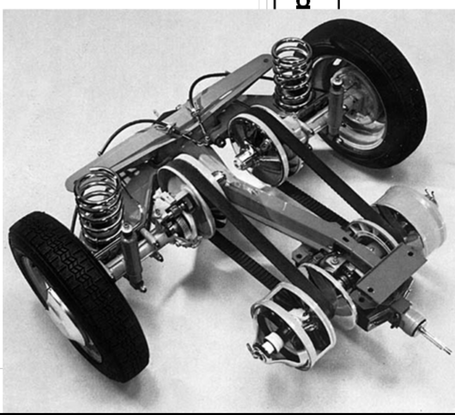
ME-305 Mechanical E



Timing belts and chains

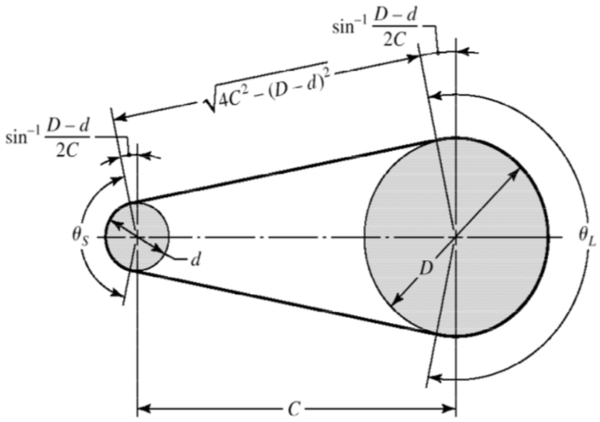
ME-305 Mechanica





17-2 Fat- and Round-Belt Drives

- Belt geometry - Open



$$\theta_S = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_L = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \tfrac{1}{2}(D\theta_L + d\theta_S)$$

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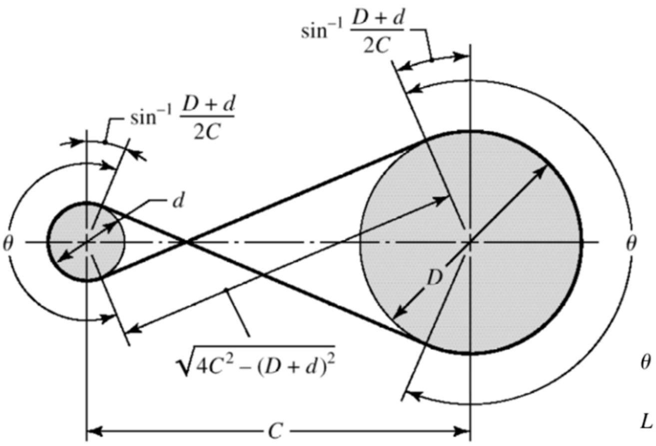
17-2 Fat- and Round-Belt Drives

- Belt geometry - Crossed



$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2-(D+d)^2} + \tfrac{\theta}{2}(D+d)$$

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17-2 Flat and Round belt drives

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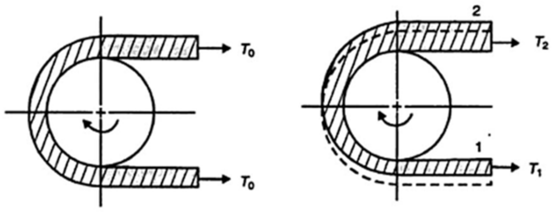
- Firbank explains that the relative motion between the belt and pulley by *Elastic creep.*
- Due to which *Velocity ratio* is not constant.
- There is *sliding friction* as opposed to *static friction.*
- The belt first contacts with pulley with tight side.

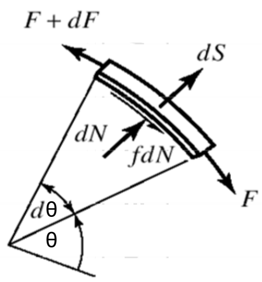


(a) Belt in idle condition (b) Belt in normal running condition

17-2 Flat and Round belt drives

ME-3 ring Design II

- The velocity of the *tight side* of the belt is the same as the pulley
- The belt tension as well as the velocity of the belt changes at the *loose side.*
- *Firbank* used this theory to express the mechanics of the *flat-belt drives.*
- The results were verified through experiments.
- The c.o.f. was found 0.7 up to 0.9
- The relationship is similar to band brake, additional is the centrifugal force.



Forces in Flat-Belts

ME-3

- $ds = mr^2\omega^2 d\theta$
- $\sum F_v = 0$ and $\sum F_H = 0$
- From which

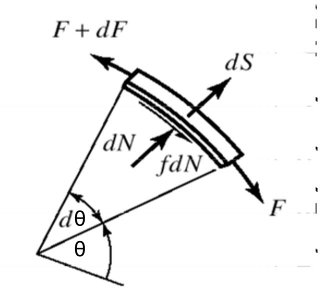
$$\frac{dF}{d\theta} - fF = -fmr^2\omega^2$$

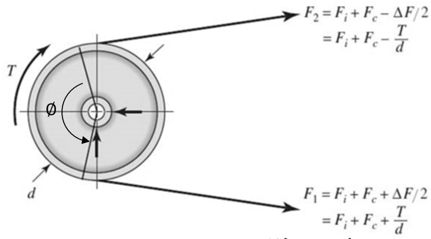
- Which is a 1st order differential equation and the general solution is given by;

$$F = Aexp(f\emptyset) + mr^2\omega^2$$

- A is constant and can be obtained from the B. Conditions as;
- $F = F_2$ if $\theta = 0$ and $F = F_1$ if $\theta = \emptyset$
- From which

$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \exp(f\emptyset)$$

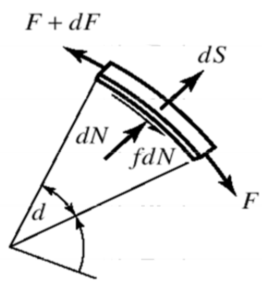




Forces in Flat-Belts

ME-3

- $(F_1 - F_2) = (F_1 - F_c)\frac{\exp(f\emptyset)-1}{\exp(f\emptyset)} \rightarrow (i)$
- $F_c = \mathcal{W}V^2/g$ ($\mathcal{W} = \gamma bt$ N/m)
- $F_1 - F_2 = \frac{2T}{d} \rightarrow (ii)$
- $F_i = \frac{F_1+F_2}{2} - F_c \rightarrow (iii)$
- Divide *(ii)* by *(iii)*, and use *(i)* to get,
- $F_i = \frac{T}{D}\frac{\exp(f\emptyset)+1}{\exp(f\emptyset)-1}$
- $F_1 = F_c + F_i\frac{2\exp(f\emptyset)}{\exp(f\emptyset)+1}$ $F_2 = F_c + F_i\frac{2}{\exp(f\emptyset)+1}$
- $(F_1)_a = bF_aC_pC_v$
- F_a= Manufacturer's allowed tension (N/mm)
- C_p= Pulley correction factor (Table 17-4)
- C_v= Velocity correction factor



ring Design II

Design steps for Flat-Belts

1. Find $\exp(f\emptyset)$ from belt-drive geometry and friction
2. From belt geometry and speed find F_c
3. From $T = H_{nom}K_s n_d/(2\pi n)$ find necessary *torque*
4. From torque T find the necessary $(F_1)_a - F_2 = 2T/d$
5. From Tables 17–2 and 17–4, and Eq. $(F_1)_a = bF_aC_pC_v$ determine $(F_1)_a$.
6. Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$ i.e. $\Delta F = F_1 - F_2$
7. From Eq. $F_i = \frac{(F_1)a+F_2}{2} - F_c$ find the necessary initial tension F_i.
8. Check the friction development, $f' < f$.
$$f' = \frac{1}{\emptyset} ln \frac{(F_1)_a - F_c}{F_2 - F_c}$$
9. Find the factor of safety from $n_{fs} = H_{Tran}/H_{nom}K_s$

Table 17-2

Properties of Some Flat- and Round-Belt Materials. (Diameter = d, thickness = t, width = w)

Material	Specification	Size, mm	Minimum Pulley Diameter, mm	Allowable Tension per Unit Width at 3 m/s, (10^3) N/m	Specific Weight, kN/m³	Coefficient of Friction
Leather	1 ply	$t = 4.5$	75	5	9.5–12.2	0.4
		$t = 5$	90	6	9.5–12.2	0.4
	2 ply	$t = 7$	115	7	9.5–12.2	0.4
		$t = 8$	150	9	9.5–12.2	0.4
		$t = 9$	230	10	9.5–12.2	0.4
Polymide	F–0[c]	$t = 0.8$	15	1.8	9.5	0.5
	F–1[c]	$t = 1.3$	25	6	9.5	0.5
	F–2[c]	$t = 1.8$	60	10	13.8	0.5
	A–2[c]	$t = 2.8$	60	10	10.0	0.8
	A–3[c]	$t = 3.3$	110	18	11.4	0.8
	A–4[c]	$t = 5.0$	240	30	10.6	0.8
	A–5[c]	$t = 6.4$	340	48	10.6	0.8
Urethane	$w = 12.7$	$t = 1.6$	See	1.0[e]	10.3–12.2	0.7
	$w = 19$	$t = 2.0$	Table	1.7[e]	10.3–12.2	0.7
	$w = 32$	$t = 2.3$	17–3	3.3[e]	10.3–12.2	0.7
	Round	$d = 6$	See	1.4[e]	10.3–12.2	0.7
		$d = 10$	Table	3.3[e]	10.3–12.2	0.7
		$d = 12$	17–3	5.8[e]	10.3–12.2	0.7
		$d = 20$		13[e]	10.3–12.2	0.7

… pulley size for belts 8 in wide or more.

Table 17–4

Pulley Correction Factor C_P for Flat Belts*

Material	Small-Pulley Diameter, mm 40 — 100	115 – 200	220 – 310	355 – 405	460 – 800	Over 800
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

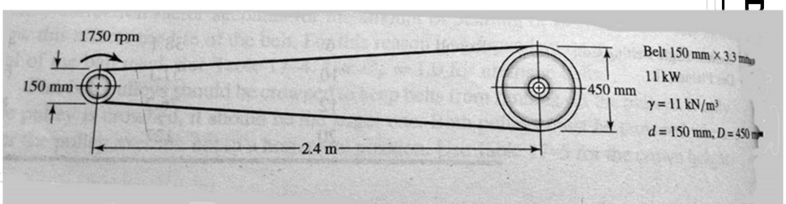
Example 17-1

A polyamide A-3 flat belt 150 mm wide is used to transmit 11 kW under light shock conditions where Ks = 1.25, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 2.4 m apart. The 150 mm driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 450 mm in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(a) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a .

(*c*) Estimate the factor of safety. Is it satisfactory?



Solution

ME-3

Solution (a) Eq. (17–1):

$$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{450 - 150}{2(2400)}\right] = 3.0165 \text{ rad}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(0.15)1750/60 = 13.7 \text{ m/s}$$

Table 17–2:

$$w = \gamma bt = 11\,000(0.15)0.0033 = 5.4 \text{ N/m}$$

Answer Eq. (e):

$$F_c = \frac{w}{g}V^2 = \frac{5.4}{9.81}(13.7)^2 = 103 \text{ N}$$

$$T = \frac{H_{\text{nom}} K_s n_d}{2\pi n} = \frac{1.25(1.1)11000}{2\pi 1750/60}$$

Answer

$$= 82 \text{ N} \cdot \text{m}$$

(b) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (h), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(82)}{0.15} = 1093 \text{ N}$$

From Table 17–2 $F_a = 18$ kN/m. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

Solution…

ME-

Answer

$$(F_1)_a = bF_aC_pC_v = 0.15(18000)0.70(1) = 1890 \text{ N}$$

then

Answer

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 1890 - 1093 = 797 \text{ N}$$

and from Eq. (i)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{1890 + 797}{2} - 103 = 1240 \text{ N}$$

Answer The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $11(1.25)(1.1) = 15.125$ kW and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165}\ln\frac{1890 - 103}{797 - 103} = 0.314$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.314 < 0.80$, there is no danger of slipping.

Answer

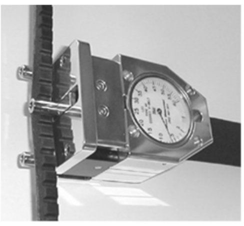
$$n_{fs} = \frac{H}{H_{\text{nom}}K_s} = \frac{15.125}{11(1.25)} = 1.1 \qquad \text{(as expected)}$$

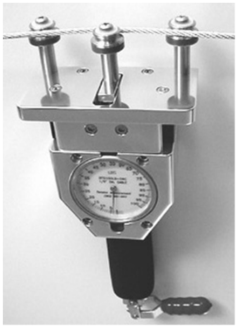
Answer The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 15.125 kW.

Example 17-2

ME-305 Mechanical Engineering Design II

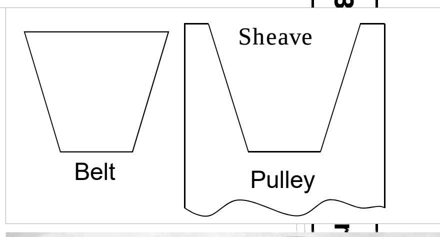
Design a flat-belt drive to connect horizontal shafts on 5 m centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 45 kW under very light shock.





17-3 V Belts

- The dimensions are standardized by the manufacturers (by a letter of the alphabet in inch and numbers in metric) given in Table 17-10.
- B875 is a B section belt having 875mm inside circumference
- Used for relatively large torque over small distances
- Groove angle of Sheave is slightly smaller than the X-Section of the Belt
- For greater torque more than one belts are used on the same pulley
- Calculations are based on pitch length. The Pitch length of B875 belt will be 875+45 (from Table 17-11) = 920mm.



17-3 V Belts

- The pitch length is given by;

$$L_p = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}$$

$$cos\theta = 1 - 2sin^2\frac{\theta}{2}$$

- The center distance is given by;

$$C = 0.25\left\{\left[L_p - \frac{\pi}{2}(D+d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\}$$

- The centrifugal tension is calculated as

$$F_c = K_c\left(\frac{V}{2.4}\right)^2$$

- K_c is a parameter determined from Table 17-16

Table 17–11

Length Conversion Dimensions (Add the listed quantity to the inside circumference to obtain the pitch length in mm).

Belt section	A	B	C	D	E
Quantity to be added	32	45	72	82	112

17-3 V Belts

- The belt life is expressed in hours (i.e. 24000) or in life (i.e. 10^8 or 10^9)
- The power rating of the belt is determined by the manufacturer for belt on equal diameter pulleys, moderate length and steady load.
- Deviation from this needs correction as;

$$H_a = K_1 K_2 H_{tab}$$

- Where
  - H_a= allowable power/belt
  - K_1= angle of wrap correction factor (Table 17-13)
  - K_2= belt length correction factor (Table 17-14)
- The design power is given by

$$H_d = H_{nom} K_s n_d$$

17-3 V Belts

- Table 17-12 for Power (kW) i.e. H_{tab}

Belt Section	Sheave Pitch Diameter, mm	Belt Speed, m/s 5	10	15	20	25
A	65	0.35	0.46	0.40	0.11	
	75	0.49	0.75	0.84	0.69	0.28
	85	0.60	0.98	1.17	1.64	0.84
	95	0.69	1.16	1.43	1.49	1.28
	105	0.77	1.30	1.64	1.78	1.63
	115	0.83	1.41	1.82	2.01	1.93
	125 and up	0.87	1.51	1.97	2.21	2.16
B	105	0.80	1.18	1.25	0.94	0.16
	115	0.95	1.48	1.71	1.55	0.92
	125	1.07	1.74	2.09	2.06	1.57
	135	1.19	1.95	2.42	2.49	2.10
	145	1.28	2.14	2.69	2.87	2.57
	155	1.36	2.31	2.94	3.19	2.98
	165	1.43	2.45	3.16	3.48	3.34
	175 and up	1.50	2.58	3.35	3.74	3.66
C	150	1.37	1.98	2.03	1.40	
	175	1.85	2.94	3.46	3.31	2.33
	200	2.21	3.66	4.54	4.74	4.12
	225	2.49	4.21	5.38	5.86	5.51
	250	2.72	4.66	6.05	7.16	6.63

17-3 V Belts

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Table 17-13 for K_1

$\frac{D-d}{C}$	θ, deg	K_1 VV	V Flat
0.00	180	1.00	0.75
0.10	174.3	0.99	0.76
0.20	166.5	0.97	0.78
0.30	162.7	0.96	0.79
0.40	156.9	0.94	0.80
0.50	151.0	0.93	0.81
0.60	145.1	0.91	0.83
0.70	139.0	0.89	0.84
0.80	132.8	0.87	0.85
0.90	126.5	0.85	0.85
1.00	120.0	0.82	0.82
1.10	113.3	0.80	0.80
1.20	106.3	0.77	0.77
1.30	98.9	0.73	0.73
1.40	91.1	0.70	0.70
1.50	82.8	0.65	0.65

*A curve fit for the VV column in terms of θ is
$K_1 = 0.143\,543 + 0.007\,46\,8\,\theta - 0.000\,015\,052\,\theta^2$
in the range $90° \leq \theta \leq 180°$.

Table 17-14 for K_2

Length Factor	Nominal Belt Length, m A Belts	B Belts	C Belts	D Belts	E Belts
0.85	Up to 0.88	Up to 1.15	Up to 1.88	Up to 3.2	
0.90	0.95–1.15	1.2–1.5	2.03–2.4	3.6–4.05	Up to 4.88
0.95	1.2–1.38	1.55–1.88	2.63–3.0	4.33–5.25	5.25–6.0
1.00	1.5–1.88	1.95–2.43	3.2–3.95	6.0	6.75–7.5
1.05	1.95–2.25	2.63–3.0	4.05–4.88	6.75–8.25	8.25–9.75
1.10	2.4–2.8	3.2–3.6	5.25–6.0	9.0–10.5	10.5–12.0
1.15	3.0 and up	3.95–4.5	6.75–7.5	12.0	13.5–15.0
1.20		4.88 and up	8.25 and up	13.5 and up	16.5

*Multiply the rated power per belt by this factor to obtain the corrected power.

17-3 V Belts

- The transmitted power per belt is

$$H_d = \frac{\Delta F \pi d n}{N_b}$$

- The largest tension is given by (done for flat belts)

$$F_1 = F_c + \frac{\Delta F exp(f\emptyset)}{\exp(f\emptyset)-1}$$

The Gates Rubber Company declares the effective COF to be 0.5123

- The initial tension is

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

- The factor of safety is

$$n_{fs} = \frac{H_a N_b}{H_{nom} K_s}$$

- K_s is determined from Table 17-15

	Source of Power	
Driven Machinery	Normal Torque Characteristic	High or Nonuniform Torque
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

17-3 V Belts

- The life of the belt is determined by adding the equivalent forces $(F_b)_1$ and $(F_b)_2$ (due to the induced flexural stresses in the belt) into the belt tension as;

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}$$

- K_b is given in Table 17-16
- Where

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$

- N_P is the number of passes. L_p is the pitch length. K and b are the durability parameters given in Table 17-17.
- The life of v-belt (in hours) is determined as;

$$t = \frac{N_P L_p}{3600V}$$

Some parameters

This column in the book (9th Ed.) is not correct for SI units. Use these values for K_b

Table 17-16 (for SI units)

Belt Section	K_b	K_c
A	25	0.561
B	65.0	0.965
C	180	1.716
D	642	3.498
E	1226	5.041
3V	26	0.425
5V	124	1.217
8V	546	3.288

*Data courtesy of Gates Rubber Co., Denver, Colo.

Table 17–17

Durability Parameters for Some V-Belt Sections

Source: M. E. Spotts, *Design of Machine Elements*, 6th ed. Prentice Hall, Englewood Cliffs, N.J., 1985.

Belt Section	10^8 to 10^9 Force Peaks K	10^8 to 10^9 Force Peaks b	10^9 to 10^{10} Force Peaks K	10^9 to 10^{10} Force Peaks b	Minimum Sheave Diameter, mm
A	2999	11.089			75
B	5309	10.926			125
C	9069	11.173			215
D	18726	11.105			325
E	26791	11.100			540
3V	3240	12.464	4726	10.153	66
5V	7360	12.593	10653	10.283	177
8V	16189	12.629	23376	10.319	312

Analysis of V-Belt

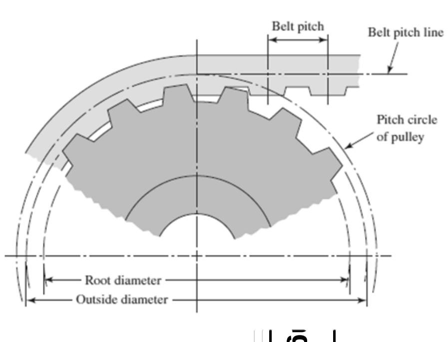
- The analysis of a V-belt consists the following steps;
  - Find V, L_p, C, θ_s and $\exp(0.5123\theta_s)$
  - Find H_d, H_a and N_b from H_d/H_a and round up
  - Find F_c, ΔF, F_1, F_2 and F_i and FOS
  - Find the belt life in number of passes or hours if possible

Exp 17-4

A 7.46kW split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 188mm small sheave, a 280mm large sheave, and three B2800 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

Timing belts

- Made of rubberized fabric coated with a nylon and steel wires
- Has teeth and can be used for any speed
- Doesn't need initial tension.
- Transmit power at constant velocity ratio
- Efficiency upto 99%
- No lubrication needed
- Analysis procedure is similar to V-belts

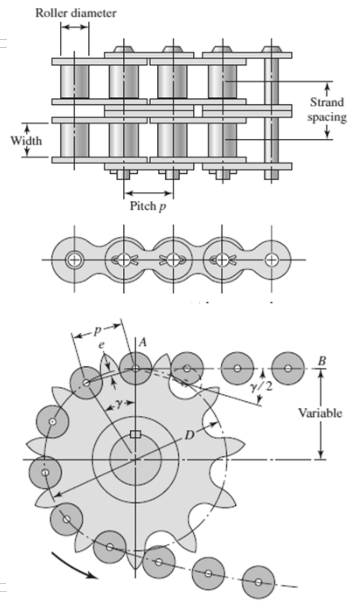


ME-3 ... g Design II

Roller chains

- No slippage or creep
- Constant velocity ratio
- Long life
- Can drive a number of shafts from a single source of power
- Standardized by ANSI
- Manufactured in single, double, triple and quadruple stands
- From figure

$$D = \frac{p}{\sin\left(\frac{180^0}{N}\right)}$$



Roller chains…

- Angle $\frac{\gamma}{2}$ is called angle of articulation which depends o the number of teeth.
- Rotation of the link "strand" through this angle causes impact between rollers and the teeth.
- $\frac{\gamma}{2}$ should be as minimum as possible to minimize the impact.
- Fails due to wear of the teeth or pin.
- Minimum number of teeth are 13, 17 or 21
- Center distance

$$C = \frac{p}{2}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

- Where $A = \frac{N_1+N_2}{2} - \frac{L}{p}$

Problems

- Problems:
  - 17-1 to 17-8, 17-10
  - 17-17 to 17-22

ME-305 Mechanical Engineering Design II