ME 305: Mechanical Engineering Design (2)

Flexible Mechanical Elements

- Belts, ropes, chains are used to convey power over a long distance

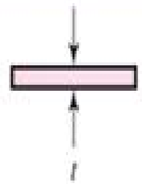
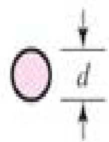
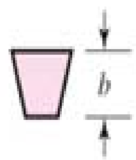
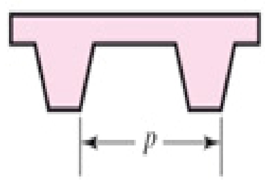
They are often used as a replacement for gears. They are less noisy, and absorb shocks and vibration

In contrast to other systems that friction is no good, here we rely on the friction to transmit power

Belts

Table 17-1

Characteristics of Some Common Belt Types. Figures are Cross Sections except for the Timing Belt, which is a Side View

Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round		Yes	$d = \frac{1}{8}$ to $\frac{3}{4}$ in	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing		None	$p = 2$ mm and up	Limited

- They may be used for long center distances.

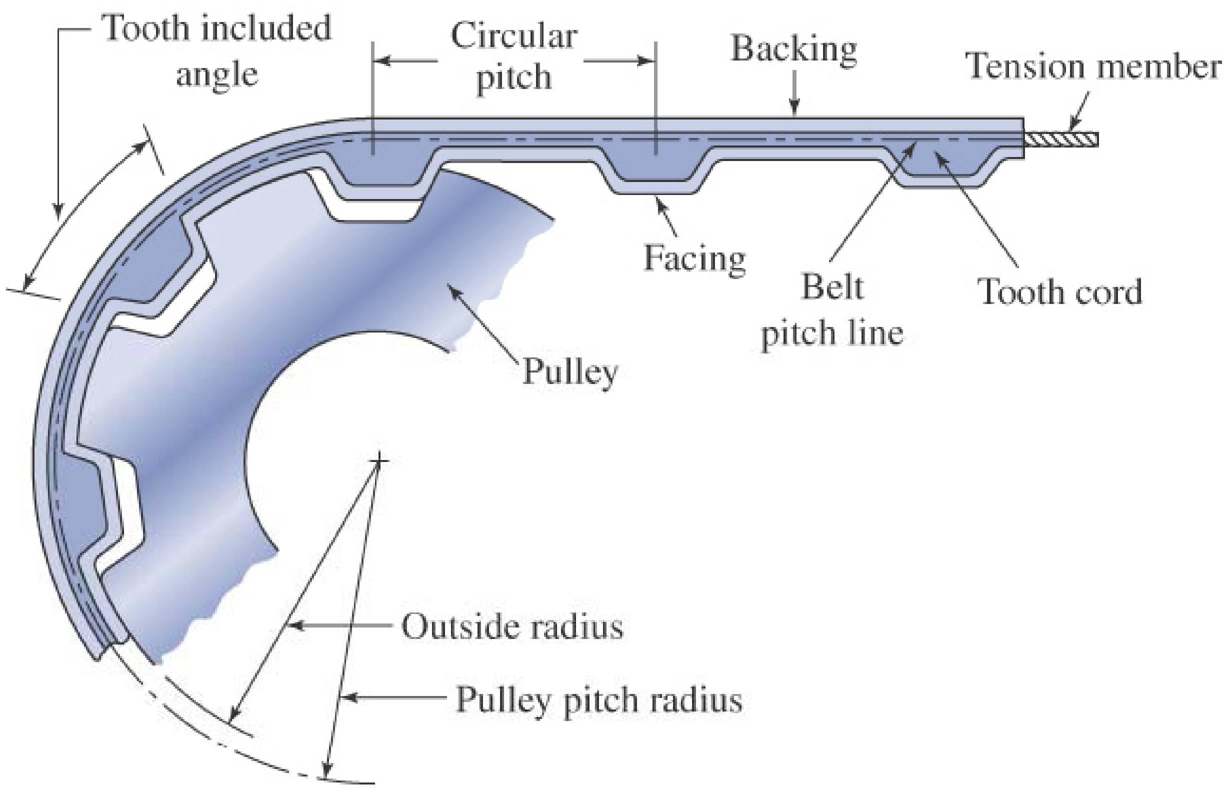
- Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shafts is neither constant nor exactly equal to the ratio of the pulley diameters.

- In some cases an idler or tension pulley can be used to avoid adjustments in center distance that are ordinarily necessitated by age or the installation of new belts.

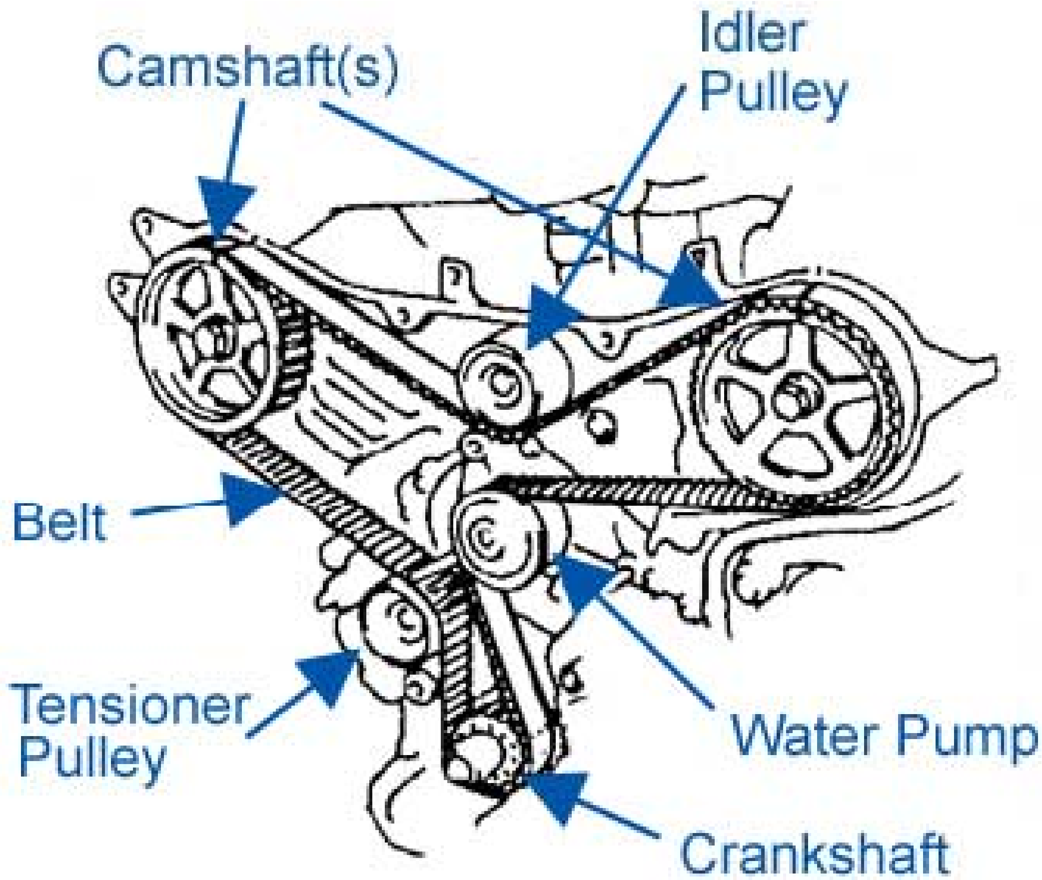
Belts are made from fiber reinforce urethane or rubber-impregnated fabric reinforced with steel or Nylon

- Flat belts has to operate at higher tension than the V belt
- Flat belt drive has an efficiency of about 98% which is about the same as for a gear drive.
- V belt speed should be in the range of 2300 m/min.
- V belts are slightly less efficient than flat belts, but it can transmit more power.

Timing Belt



Timing belts transmit power at the constant angular velocity ratio, application when precise speed ratio is important



Typical V6 Timing Belt

Belts

Belt drive is specially suited for applications where the center distance between rotating shafts are large

Advantages

- Eliminates the need for a more complicated arrangement of gears, bearings, and shafts.
- Runs relatively quiet.
- Reduce transmission of shock and vibration between shafts.
- Simple to install.
- It has high reliability and warning to failure.
- Requires minimum maintenance.
- Belt drives are adaptable to variety of applications

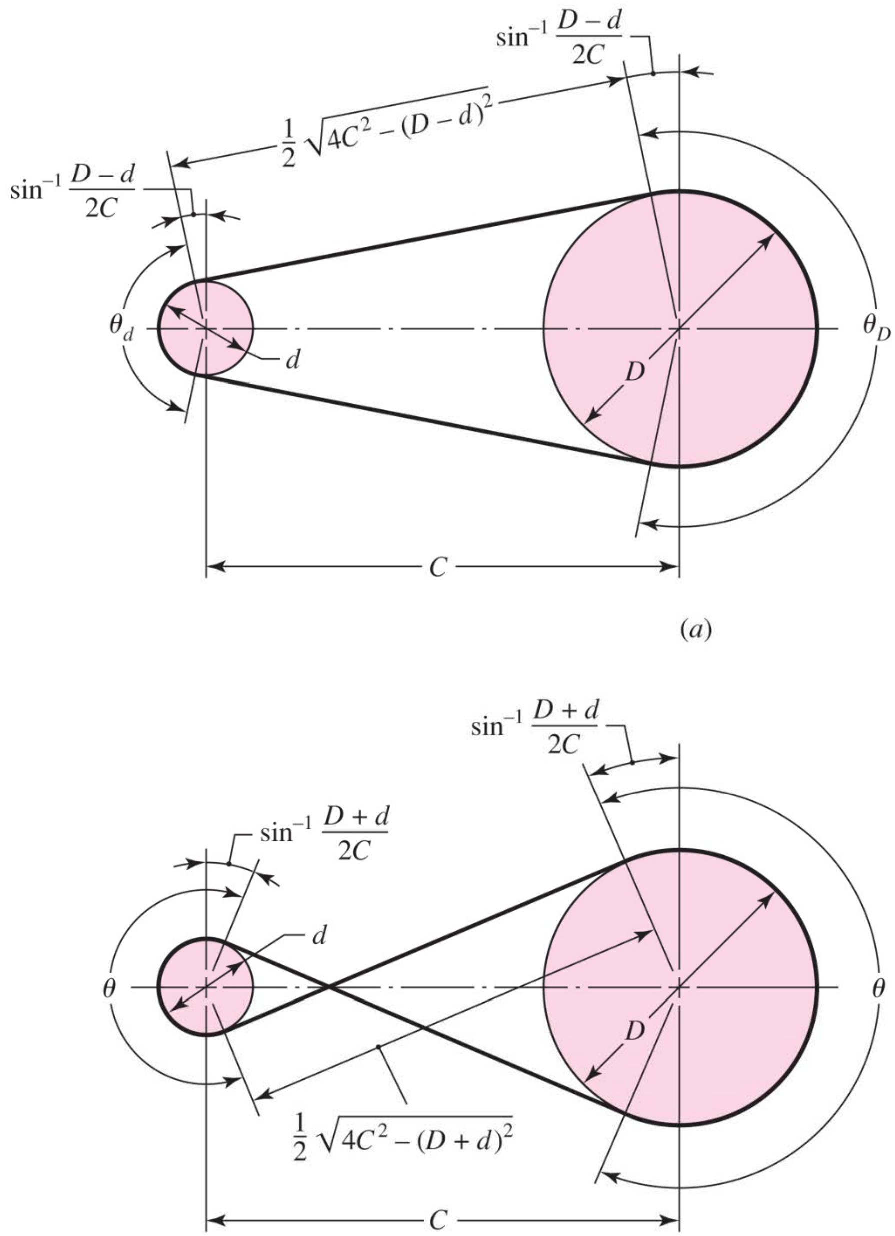
Belts

Disadvantages

- The torque capacity is limited by the coefficient of friction and interfacial pressure between belt and pulley.
- Because of slip and/or creep, the angular velocity ratio will vary between the rotating shafts and may
- Low speed reduction ratio, up to 3:1.
- Belt tension needs to be adjusted periodically.

Figure 17–1

Flat-belt geometry. (*a*) Open belt. (*b*) Crossed belt.



$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

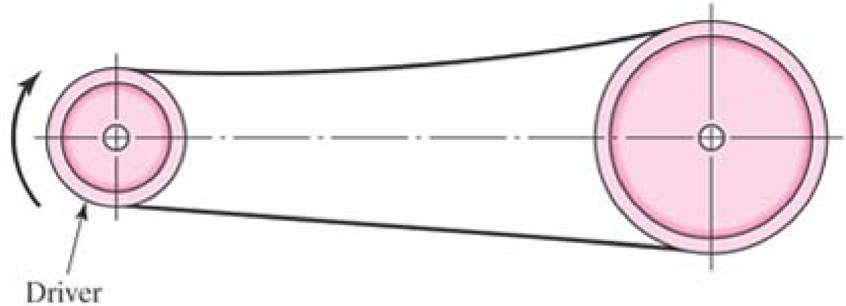
$$L = \sqrt{4C^2-(D-d)^2} + \tfrac{1}{2}(D\theta_D + d\theta_d)$$

(*a*)

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

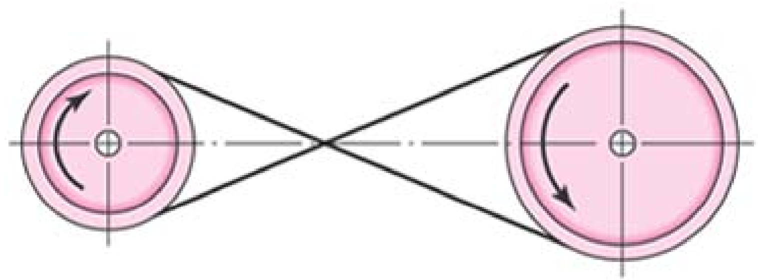
$$L = \sqrt{4C^2-(D+d)^2} + \tfrac{1}{2}(D+d)\theta$$

(*b*)



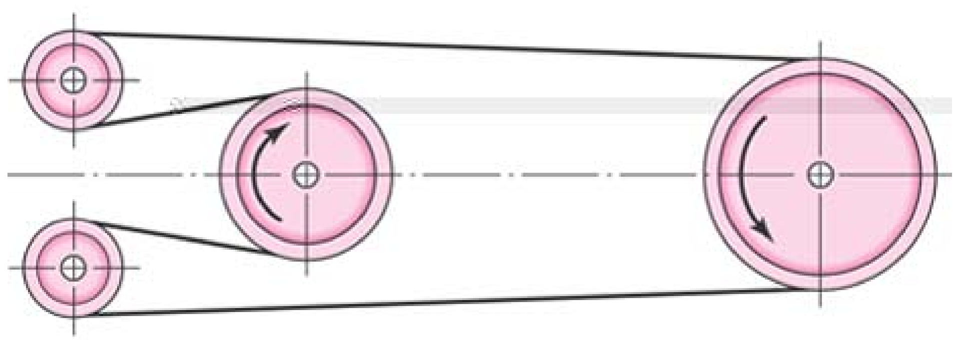
(a)

Non Reversing
Open Belt



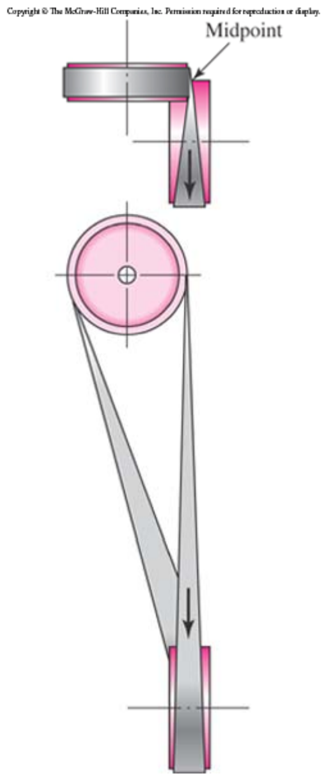
(b)

Reversing Crossed
Belt
(Crossed belts must
be separated to
prevent rubbing)

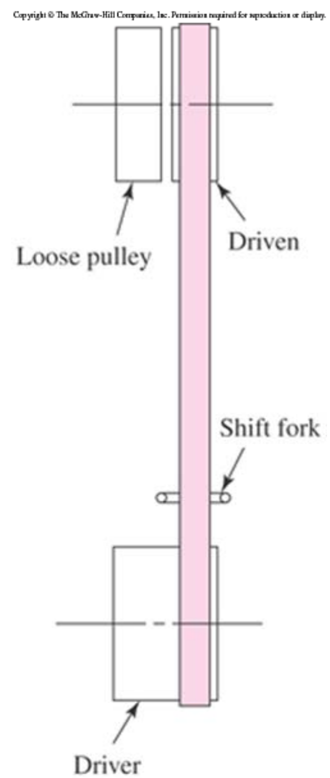


(c)

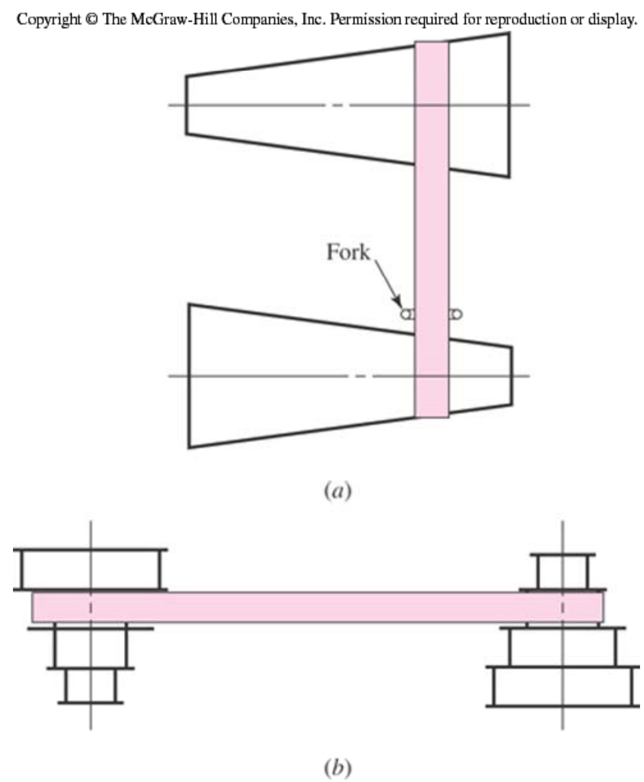
Reversing
Open Belt



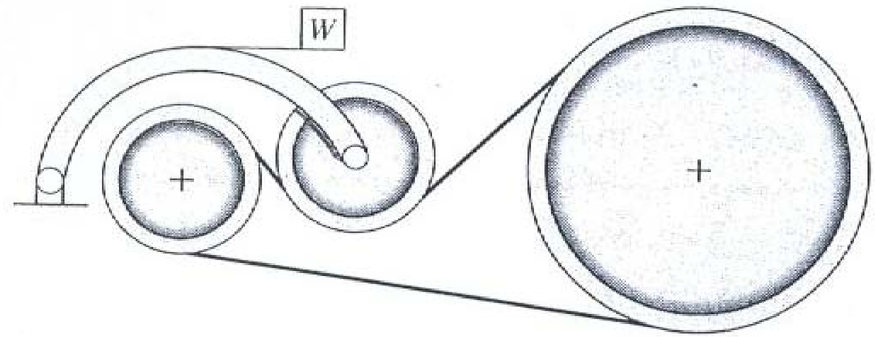
Quarter twist belt drive



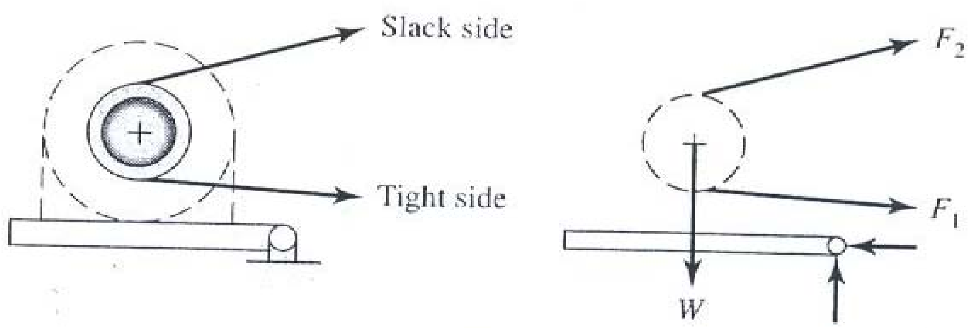
Flat belt can be shifted left or right using a fork



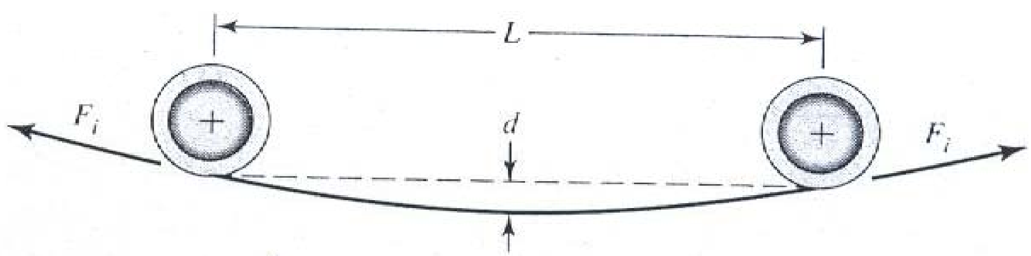
Variable speed belt drives



(a)



(b)



$$d = \frac{L^2 w}{8F_i}$$

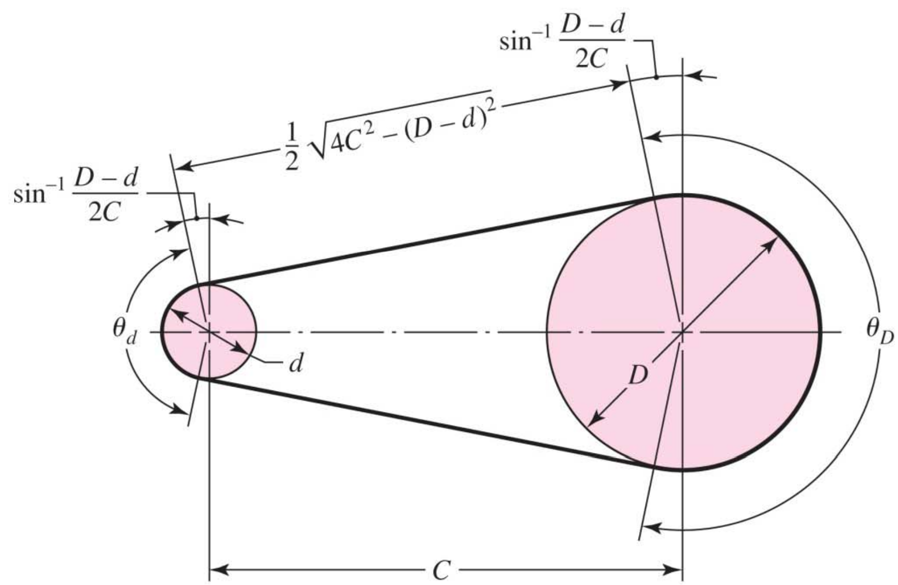
d: dip , m
L: center to center distance, m
w: weight per foot of the belt, N/m
Fi: initial tension, N.

2 Flat- and Round-Belt Drives

Modern flat-belt drives consist of a strong elastic core surrounded by an elastomer; these drives have distinct advantages over gear drives or V-belt drives. A flat-belt drive has an efficiency of about 98 percent, which is about the same as for a gear drive. On the other hand, the efficiency of a V-belt drive ranges from about 70 to 96 percent.[1] Flat-belt drives produce very little noise and absorb more torsional vibration from the system than either V-belt or gear drives.

Figure 17–1

Flat-belt geometry. (*a*) Open belt. (*b*) Crossed belt.



$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2 - (D-d)^2} + \tfrac{1}{2}(D\theta_D + d\theta_d)$$

When an open-belt drive (Fig. 17–1*a*) is used, the contact angles are found to be

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$
$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} \tag{17-1}$$

where D = diameter of large pulley
d = diameter of small pulley
C = center distance
θ = angle of contact

The length of the belt is found by summing the two arc lengths with twice the distance between the beginning and end of contact. The result is

$$L = [4C^2 - (D-d)^2]^{1/2} + \frac{1}{2}(D\theta_D + d\theta_d) \tag{17-2}$$

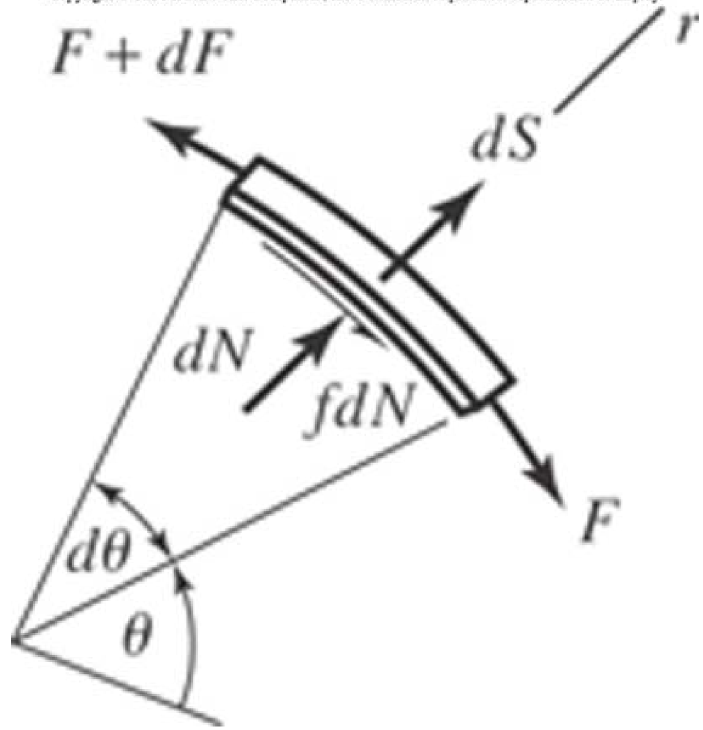
A similar set of equations can be derived for the crossed belt of Fig. 17–2*b*. For this belt, the angle of wrap is the same for both pulleys and is

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C} \tag{17-3}$$

The belt length for crossed belts is found to be

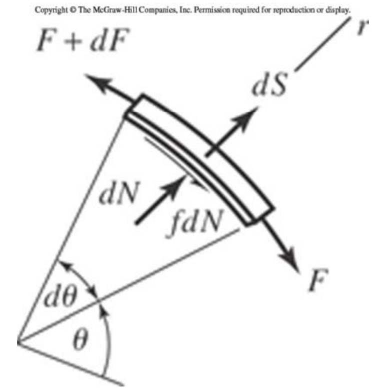
$$L = [4C^2 - (D+d)^2]^{1/2} + \frac{1}{2}(D+d)\theta \tag{17-4}$$

The angle of contact is made up of the effective arc, through which power is transmitted and the idle arc.



free body of a small segment of the belt. The differential force dS is due to centrifugal force, dN is the normal force between the belt and pulley, and $f\,dN$ is the shearing traction due to friction at the point of slip. The belt width is b and the thickness is t. The belt mass per unit length is m. The centrifugal force dS can be expressed as

through which power is transmitted, and the *idle arc*. For the driving pulley the belt first contacts the pulley with a *tight-side tension* F_1 and a velocity V_1, which is the same as the surface velocity of the pulley. The belt then passes through the idle arc with no change in F_1 or V_1. Then creep or sliding contact begins, and the belt tension changes in accordance with the friction forces. At the end of the effective arc the belt leaves the pulley with a *loose-side tension* F_2 and a reduced speed V_2.



The centrifugal force dS can be expressed as

$$dS = (mr\,d\theta)r\omega^2 = mr^2\omega^2\,d\theta = mV^2\,d\theta = F_c\,d\theta \qquad (a)$$

F_c = hoop tension due to centrifugal force

where V is the belt speed. Summing forces radially gives

$$\sum F_r = -(F + dF)\frac{d\theta}{2} - F\frac{d\theta}{2} + dN + dS = 0$$

Ignoring the higher-order term, we have

$$dN = F\,d\theta - dS \qquad (b)$$

Summing forces tangentially gives

$$\sum F_t = -f\,dN - F + (F + dF) = 0$$

from which, incorporating Eqs. (a) and (b), we obtain

$$dF = f\,dN = fF\,d\theta - f\,dS = fF\,d\theta - fmr^2\omega^2\,d\theta$$

or

$$\frac{dF}{d\theta} - fF = -fmr^2\omega^2 \qquad (c)$$

The solution to this nonhomogeneous first-order linear differential equation is

$$F = A\exp(f\theta) + mr^2\omega^2 \qquad (d)$$

where A is an arbitrary constant. Assuming θ starts at the loose side, the boundary condition that F at $\theta = 0$ equals F_2 gives $A = F_2 - mr^2\omega^2$. The solution is

$$F = (F_2 - mr^2\omega^2)\exp(f\theta) + mr^2\omega^2 \qquad (17\text{–}5)$$

At the end of the angle of wrap ϕ, the tight side,

$$F|_{\theta=\phi} = F_1 = (F_2 - mr^2\omega^2)\exp(f\phi) + mr^2\omega^2 \qquad (17\text{–}6)$$

Now we can write

$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f\phi) \qquad (17\text{–}7)$$

where, from Eq. (a), $F_c = mr^2\omega^2$. It is also useful that Eq. (17–7) can be written as

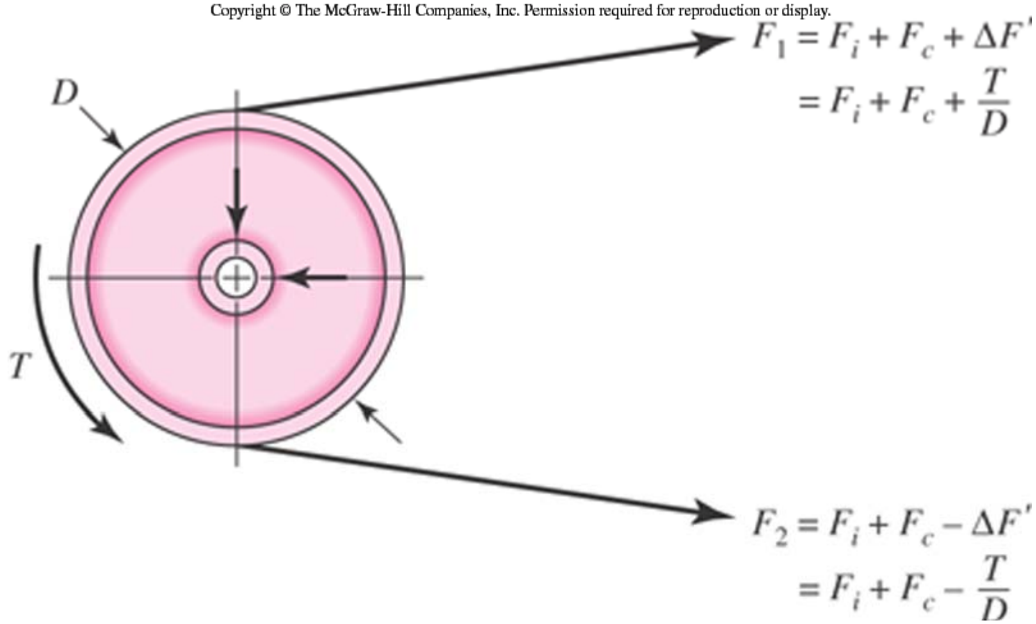
$$F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\phi) - 1}{\exp(f\phi)} \qquad (17\text{–}8)$$

F_c is found as follows: with n being the rotational speed, in rpm, of the pulley of diameter d, the belt speed is

$$V = \pi\ d\ n/60 \quad \text{m/s}$$

The weight w of a meter of belt is given in terms of the weight density γ in N/m^3 as w = γ bt N/m where b and t in meters. F_c is written as

$$F_c = w\ V^2 / g$$



$$F_1 = F_i + F_c + \Delta F' = F_i + F_c + T/D \quad (f)$$

$$F_2 = F_i + F_c - \Delta F' = F_i + F_c - T/D \quad (g)$$

where F_i = initial tension

F_c = hoop tension due to centrifugal force

$\Delta F'$ = tension due to the transmitted torque T

D = diameter of the pulley

The difference between F_1 and F_2 is related to the pulley torque. Subtracting Eq. (*g*) from Eq. (*f*) gives

$$F_1 - F_2 = \frac{2T}{D} = \frac{T}{D/2} \quad (h)$$

Adding Eqs. (*f*) and (*g*) gives

$$F_1 + F_2 = 2F_i + 2F_c$$

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (i)$$

$$F_i = \frac{T}{D}\,\frac{\exp(f\phi) + 1}{\exp(f\phi) - 1} \qquad (17\text{–}9)$$

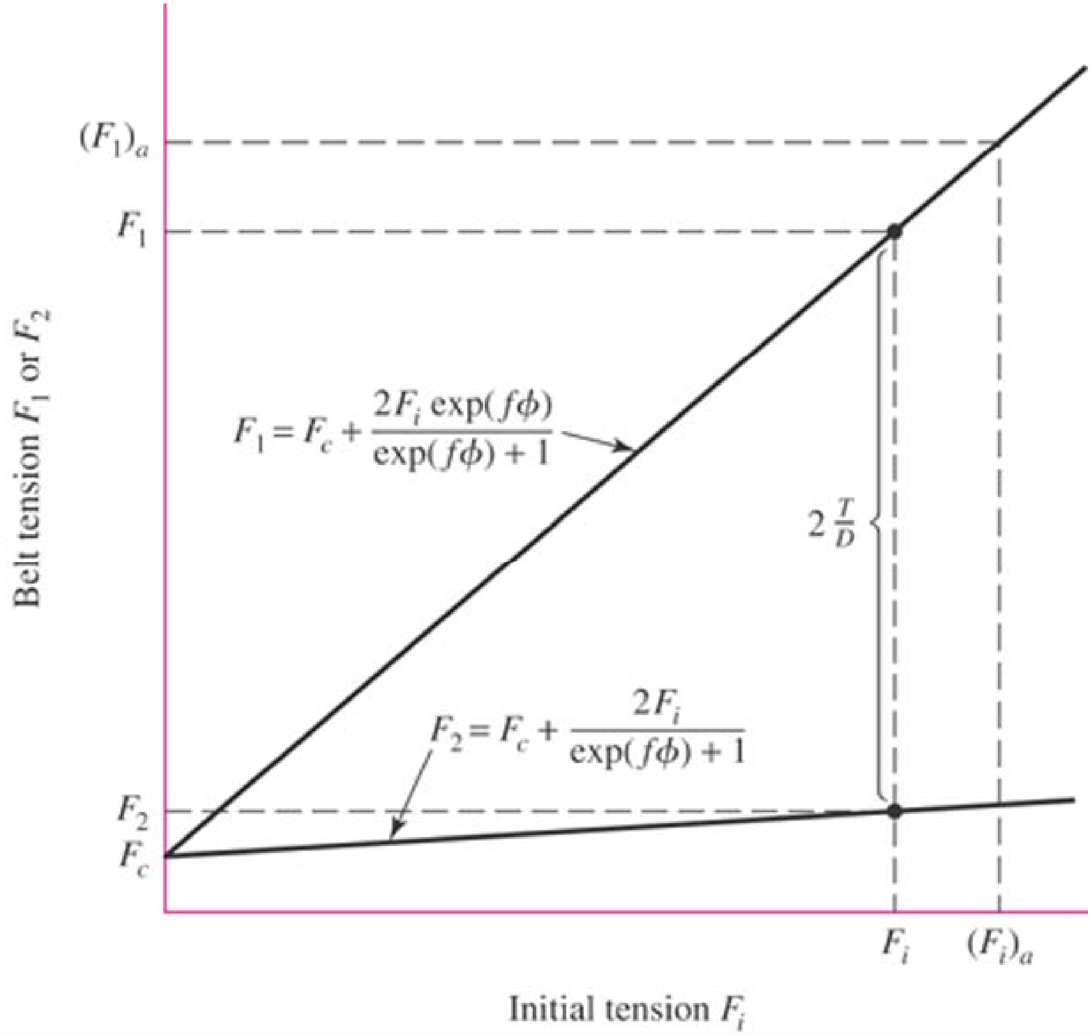
Equation (17–9) give us a fundamental insight into flat belting. If F_i equals zero, then T equals zero: no initial tension, no torque transmitted. The torque is in proportion to the initial tension. This means that if there is to be a satisfactory flat-belt drive, the initial tension must be (1) provided, (2) sustained, (3) in the proper amount, and (4) maintained by routine inspection.

$$F_1 = F_i + F_c + \frac{T}{D} = F_c + F_i + F_i\,\frac{\exp(f\phi) - 1}{\exp(f\phi) + 1}$$

$$= F_c + \frac{F_i[\exp(f\phi) + 1] + F_i[\exp(f\phi) - 1]}{\exp(f\phi) + 1}$$

$$F_1 = F_c + F_i\,\frac{2\exp(f\phi)}{\exp(f\phi) + 1} \qquad (17\text{–}10)$$

$$F_2 = F_c + F_i\,\frac{2}{\exp(f\phi) + 1} \qquad (17\text{–}11)$$



Equation (17–7) is called the *belting equation,* but Eqs. (17–9), (17–10), and (17–11) reveal how belting works. We plot Eqs. (17–10) and (17–11) as shown in Fig. 17–8 against F_i as abscissa. The initial tension needs to be sufficient so that the difference between the F_1 and F_2 curve is $2T/D$. With no torque transmitted, the least possible belt tension is $F_1 = F_2 = F_c$.

Table 17–2

Properties of Some Flat- and Round-Belt Materials. (Diameter = d, thickness = t, width = w)

Material	Specification	Size, mm	Minimum Pulley Diameter, mm	Allowable Tension per Unit Width at 3 m/s, (10^3) N/m	Specific Weight, kN/m^3	Coefficient of Friction
Leather	1 ply	$t = 4.5$	75	5	9.5–12.2	0.4
		$t = 5$	90	6	9.5–12.2	0.4
	2 ply	$t = 7$	115	7	9.5–12.2	0.4
		$t = 8$	150	9	9.5–12.2	0.4
		$t = 9$	230	10	9.5–12.2	0.4
Polyamide[b]	F–0[c]	$t = 0.8$	15	1.8	9.5	0.5
	F–1[c]	$t = 1.3$	25	6	9.5	0.5
	F–2[c]	$t = 1.8$	60	10	13.8	0.5
	A–2[c]	$t = 2.8$	60	10	10.0	0.8
	A–3[c]	$t = 3.3$	110	18	11.4	0.8
	A–4[c]	$t = 5.0$	240	30	10.6	0.8
	A–5[c]	$t = 6.4$	340	48	10.6	0.8
Urethane[d]	$w = 12.7$	$t = 1.6$	See	1.0[e]	10.3–12.2	0.7
	$w = 19$	$t = 2.0$	Table	1.7[e]	10.3–12.2	0.7
	$w = 32$	$t = 2.3$	17–3	3.3[e]	10.3–12.2	0.7
	Round	$d = 6$	See	1.4[e]	10.3–12.2	0.7
		$d = 10$	Table	3.3[e]	10.3–12.2	0.7
		$d = 12$	17–3	5.8[e]	10.3–12.2	0.7
		$d = 20$		13[e]	10.3–12.2	0.7

[a]Add 2 in to pulley size for belts 8 in wide or more.

The values given in Table 17–2 for the allowable belt tension are based on a belt speed of 600 ft/min. For higher speeds, use Fig. 17–9 to obtain C_v values for leather belts. For polyamide and urethane belts, use $C_v = 1.0$.

Table 17–3

Minimum Pulley Sizes for Flat and Round Urethane Belts. (Listed are the Pulley Diameters in mm)

Source: Eagle Belting Co., Des Plaines, Ill.

Belt Style	Belt Size, mm	Ratio of Pulley Speed to Belt Length, rev/(m • s) Up to 14	14 to 27	28 — 55
Flat	12.7 × 1.6	9.7	11.2	12.7
	19 × 2.0	12.7	16	19
	32 × 2.3	12.7	16	19
Round	6	38.1	44.5	50.8
	10	57.1	66.5	76.2
	12	76.2	88.9	101.6
	20	127	152	177.8

Design of Flat Belts

Transmitted power is given by

$$H = (F_1 - F_2)\,V$$

Design power $$H_d = H_{nom}\,K_s\,n_d$$

K_s is the service factor and n_d is the design factor

Table 17–15

Suggested Service Factors K_S for V-Belt Drives

	Source of Power	
Driven Machinery	**Normal Torque Characteristic**	**High or Nonuniform Torque**
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

$$(F_1)_a = b\, F_a\, C_p\, C_v$$

Where $(F_1)_a$ = allowable largest tension, N

b= belt width, mm

F_a = manufacture's allowed tension, N/mm

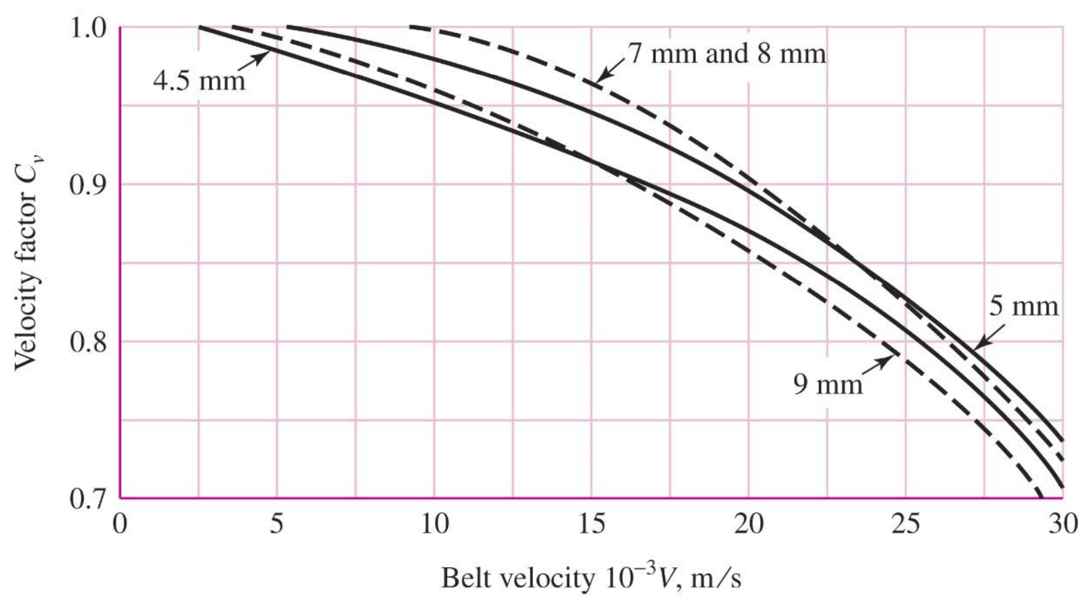
C_p = pulley correction factor (C_p = 1 for urethane belts)

C_v = velocity correction factor

(for polyamide and urethane belts use C_v =1)

Figure 17–9

Velocity correction factor C_v for leather belts for various thicknesses. *(Data source:* Machinery's Handbook, 20th ed., *Industrial Press, New York, 1976, p. 1047.)*



Minimum pulley sizes for the various belts are listed in Tables 17–2 and 17–3. The pulley correction factor accounts for the amount of bending or flexing of the belt and how this affects the life of the belt. For this reason it is dependent on the size and material of the belt used. See Table 17–4. Use $C_p = 1.0$ for urethane belts.

Pulley Correction Factor C_P for Flat Belts*

	Small-Pulley Diameter, mm					
Material	**40 — 100**	**115 – 200**	**220 – 310**	**355 – 405**	**460 – 800**	**Over 800**
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

*Average values of C_P for the given ranges were approximated from curves in the *Habasit Engineering Manual,* Habasit Belting, Inc., Chamblee (Atlanta), Ga.

Flat-belt pulleys should be crowned to keep belts from running off the pulleys. If only one pulley is crowned, it should be the larger one. Both pulleys must be crowned whenever the pulley axes are not in a horizontal position. Use Table 17–5 for the crown height.

Table 17–5

Crown Height and ISO Pulley Diameters for Flat Belts*

ISOCrown Pulley Diameter, mm	ISO Height, mm	Pulley Diameter, mm	*Crown Height,* in $w \le 250$ mm	$w > 250$ mm
40, 50, 62	0.3	315, 355	0.75	0.75
70, 80	0.3	315, 355	1.0	1.0
90, 100, 115	0.3	570, 635, 710	1.3	1.3
125, 142	0.4	800, 900	1.3	1.5
160, 180	0.5	1015	1.3	1.5
200, 230	0.6	1140, 1270, 1420	1.5	2.0
250, 285	0.75	1600, 1800, 2030	1.8	2.5

*Crown should be rounded, not angled; maximum roughness is R_a = AA 1500 μmm.

Analysis of Flat-Belt Drives

- The transmitted horsepower is given by $H(F_1 - F_2)V$
- Corrections on allowable tension give $$(F_1)_a = bF_aC_pC_v$$

  where $(F_1)_a$ = allowable largest tension, N

  b = belt width, mm

  F_a = manufacturer's allowed tension, N/mm

  C_p = pulley correction factor (Table 17–4)

  C_v = velocity correction factor
- The steps in analyzing a flat-belt drive can include
  - Find exp(f φ) from belt-drive geometry and friction
  - From belt geometry and speed find F_c
  - From $T = H_{nom}K_s n_d/(2\pi n)$ find necessary torque
  - From torque T find the necessary $(F_1)_a - F_2 = 2T/D$
  - *Determine* $(F_1)_a = bF_aC_pC_v$
  - Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$
  - From Eq. (i) find the necessary initial tension Fi
  - Check the friction development, $f' < f$. Use Eq. (17–7) solved for f': $$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c}$$
  - Find the factor of safety from

    $n_{fs} = H_a/(H_{nom}K_s)$

The steps in analyzing a flat-belt drive can include

1. Find $\exp(f\phi)$ from belt-drive geometry and friction
2. From belt geometry and speed find F_c
3. From $T = 63\,025 H_{\text{nom}} K_s n_d / n$ find necessary torque
4. From torque T find the necessary $(F_1)_a - F_2 = 2T/D$
5. Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$
6. From Eq. (i) find the necessary initial tension F_i
7. Check the friction development, $f' < f$. Use Eq. (17–7) solved for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c}$$

8. Find the factor of safety from $n_{fs} = H_a/(H_{\text{nom}} K_s)$

From $T = H_{nom}\ K_s\ n_d\ /\ (2\ \pi\ n/60)$

Example 1

A 150 mm-wide polyamide F-1 flat belt is used to connect a 50 mm- diameter pulley to drive a larger pulley with an angular velocity ratio of 0.5. The center-to-center distance is 2.7 m. The angular speed of the small pulley is 1750 rpm as it delivers 1500 W. The service is such that a service factor K of 1.25 is appropriate.

a) Find F_c , F_i , F_{1a} , and F_2

b) Find n_{fs} and belt length

c) Find the dip

EXAMPLE 17–2 Design a flat-belt drive to connect horizontal shafts on 4.8 m centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 44 760 W under very light shock.

Solution

- Function: $H_{\text{nom}} = 44\,760$ W, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 4.8$ m
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

The last four could be design variables. Let's make a few more a priori decisions.

EXAMPLE 17–1 A polyamide A-3 flat belt 150 mm wide is used to transmit 11 kW under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 2.4 m apart. The 150-mm driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 450 mm in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

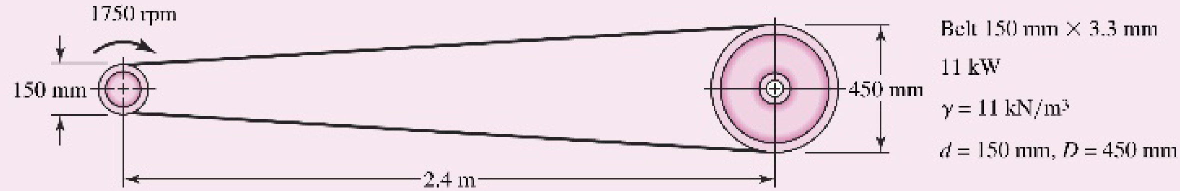
(*a*) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

(*c*) Estimate the factor of safety. Is it satisfactory?

Figure 17–10

The flat-belt drive of Ex. 17–1.



Solution (*a*) Eq. (17–1):

$$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{450 - 150}{2(2400)}\right] = 3.0165 \text{ rad}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(0.15)1750/60 = 13.7 \text{ m/s}$$

Table 17–2:

$$w = \gamma bt = 11\,000(0.15)0.0033 = 5.4 \text{ N/m}$$

Answer Eq. (*e*):

$$F_c = \frac{w}{g}V^2 = \frac{5.4}{9.81}(13.7)^2 = 103 \text{ N}$$

$$T = \frac{H_{\text{nom}} K_s n_d}{2\pi n} = \frac{1.25(1.1)11000}{2\pi 1750/60}$$

Answer

$$= 82 \text{ N} \cdot \text{m}$$

(*b*) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (*h*), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(82)}{0.15} = 1093 \text{ N}$$

From Table 17–2 $F_a = 18$ kN/m. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

Answer

$$(F_1)_a = bF_aC_pC_v = 0.15(18000)0.70(1) = 1890 \text{ N}$$

then

Answer

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 1890 - 1093 = 797 \text{ N}$$

and from Eq. (*i*)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{1890 + 797}{2} - 103 = 1240 \text{ N}$$

Answer The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $11(1.25)(1.1) = 15.125$ kW and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{1890 - 103}{797 - 103} = 0.314$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.314 < 0.80$, there is no danger of slipping.

(*c*)

Answer

$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{15.125}{11(1.25)} = 1.1 \qquad \text{(as expected)}$$

Answer The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 15.125 kW.

EXAMPLE 17–2 Design a flat-belt drive to connect horizontal shafts on 4.8 m centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 44 760 W under very light shock.

Solution

- Function: $H_{nom} = 44\,760$ W, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 4.8$ m
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

The last four could be design variables. Let's make a few more a priori decisions.

Decision $d = 400$ mm, $D = 2.25d = 900$ mm.

Decision Use polyamide A-3 belt; therefore $t = 3.3$ mm and $C_v = 1$.
Now there is one design decision remaining to be made, the belt width b.

Table 17–2: $\gamma = 11.4$ kN/m^3 $\quad f = 0.8 \quad F_a = 18$ kN/m at 600 rev/min

Table 17–4: $C_p = 0.94$

Eq. (17–12): $F_{1a} = b(18\,000)0.94(1) = 16\,920b$ N (1)

$$H_d = H_{nom}K_s n_d = 44\,760(1.15)1.05 = 54\,047 \text{ W}$$

$$T = \frac{H_d}{2\pi n} = \frac{54\,047}{2\pi 860/60} = 600 \text{ N}\cdot\text{m}$$

Estimate $\exp(f\phi)$ for full friction development:

Eq. (17–1):
$$\phi = \theta_d = \pi - 2\sin^{-1}\frac{900-400}{2(4800)} = 3.037 \text{ rad}$$

$$\exp(f\phi) = \exp[0.80(3.037)] = 11.35$$

Estimate centrifugal tension F_c in terms of belt width b:

$$w = \gamma bt = (11\,400)b(0.0033) = 37.6b \text{ N/m}$$

$$V = \pi dn = \pi(0.4)860/60 = 18 \text{ m/s}$$

Eq. (e):
$$F_c = \frac{w}{g}V^2 = \frac{(37.6)b(18)^2}{9.81} = 1241.8b \text{ N} \qquad (2)$$

For design conditions, that is, at H_d power level, using Eq. (h) gives

$$(F_1)_a - F_2 = 2T/d = 2(600)/0.4 = 3000 \text{ N} \qquad (3)$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 16\,920b - 3000 \text{ N} \qquad (4)$$

Using Eq. (i) gives

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{16\,920b + 16\,920b - 3000}{2} - 1241.8b = 15\,678.2b - 1500 \text{ N} \qquad (5)$$

Place friction development at its highest level, using Eq. (17–7):

$$f\phi = \ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \ln\frac{16\,920b - 1241.8b}{16\,920b - 3000 - 1241.8b} = \ln\frac{15\,678.2b}{15\,678.2b - 3000}$$

Solving the preceding equation for belt width b at which friction is fully developed gives

$$b = \frac{3000}{15\,678.2}\frac{\exp(f\phi)}{\exp(f\phi)-1} = \frac{3000}{15\,678.2}\frac{11.38}{11.38-1} = 0.210 \text{ m} = 210 \text{ mm}$$

A belt width greater than 210 mm will develop friction less than $f = 0.80$. The manufacturer's data indicate that the next available larger width is 250 mm.

Decision Use 250 mm-wide belt.

It follows that for a 250-mm-wide belt

Eq. (2): $F_c = 1241.8(0.25) = 310$ N

Eq. (1): $(F_1)_a = 16\,920(0.25) = 4230$ N

Eq. (4): $F_2 = 4230 - 3000 = 1230$ N

Eq. (5): $F_i = 15\,678.2(0.25) - 1500 = 2420$ N

The transmitted power, from Eq. (3), is

$$H_t = [(F_1)_a - F_2]V = 3000(18) = 54\,000 \text{ W}$$

and the level of friction development f', from Eq. (17–7) is

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.037} \ln \frac{4230 - 310}{1230 - 310} = 0.477$$

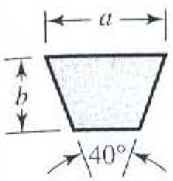
which is less than $f = 0.8$, and thus is satisfactory. Had a 225-mm belt width been available, the analysis would show $(F_1)_a = 3807$ N, $F_2 = 811$ N, $F_i = 2260$ N, and $f' = 0.63$. With a figure of merit available reflecting cost, thicker belts (A-4 or A-5) could be examined to ascertain which of the satisfactory alternatives is best. From Eq. (17–13) the catenary dip is

$$dip = \frac{L^2 w}{8F_i} = \frac{4.8^2(37.6)0.25}{8(2420)} = 0.011 \text{ m} = 11 \text{ mm}$$

V Belts

Table 17-9

Standard V-Belt Sections



Belt Section	Width *a*, mm	Thickness *b*, mm	Minimum Sheave Diameter, mm	kW Range, One or More Belts
A	12	8.5	75	0.2–7.5
B	16	11	135	0.7–18.5
C	22	13	230	11–75
D	30	19	325	37–186
E	38	25	540	75 and up

Table 17–10

Inside Circumferences of Standard V Belts

Section	Circumference, mm
A	650, 775, 825, 875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1650, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200
B	875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1650, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3275, 3400, 3450, 3950, 4325, 4500, 4875, 5250, 6000, 6750, 7500
C	1275, 1500, 1700, 1875, 2025, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3400, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500
D	3000, 3200, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500, 12 000, 13 500, 15 000, 16 500
E	4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500, 12 000, 13 500, 15 000, 16 500

Table 17–11

Length Conversion Dimensions (Add the Listed Quantity to the Inside Circumference to Obtain the Pitch Length in mm)

Belt section	A	B	C	D	E
Quantity to be added	32	45	72	82	112

To specify a V belt, give the belt-section letter, followed by the inside circumference in mm (standard circumferences are listed in Table 17–10). For example, B875 is a B-section belt having an inside circumference of 875 mm.

Calculations involving the belt length are usually based on the pitch length. For any given belt section, the pitch length is obtained by adding a quantity to the inside circumference (Tables 17–10 and 17–11). For example, a B875 belt has a pitch length of (875+45) 920 mm.

- The cross-sectional dimensions of V belts have been standardized by manufacturers, with each section designated by a letter of the alphabet for sizes in inch dimensions.
- To specify a V belt, give the belt-section letter, followed by the inside circumference in inches.
- The pitch length is obtained by adding a quantity to the inside circumference.
- For best results, a V belt should be run quite fast: 20 m/s is a good speed. Trouble may be encountered if the belt runs much faster than 25 m/s or much slower than 5 m/s .

The *pitch length* L_p and the center-to-center distance C are

$$L_p = 2C + \pi(D+d)/2 + (D-d)^2/(4C) \tag{17–16a}$$

$$C = 0.25\left\{\left[L_p - \frac{\pi}{2}(D+d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\} \tag{17–16b}$$

where D = pitch diameter of the large sheave and d = pitch diameter of the small sheave.

The groove angle of a sheave is made somewhat smaller than the belt-section angle. This causes the belt to wedge itself into the groove, thus increasing friction. The exact value of this angle depends on the belt section, the sheave diameter, and the angle of contact. If it is made too much smaller than the belt, the force required to pull the belt out of the groove as the belt leaves the pulley will be excessive. Optimum values are given in the commercial literature.

Table 17–12

Horsepower Ratings of Standard V Belts

Belt Section	Sheave Pitch Diameter, in	Belt Speed, ft/min 1000	2000	3000	4000	5000
A	2.6	0.47	0.62	0.53	0.15	
	3.0	0.66	1.01	1.12	0.93	0.38
	3.4	0.81	1.31	1.57	1.53	1.12
	3.8	0.93	1.55	1.92	2.00	1.71
	4.2	1.03	1.74	2.20	2.38	2.19
	4.6	1.11	1.89	2.44	2.69	2.58
	5.0 and up	1.17	2.03	2.64	2.96	2.89
B	4.2	1.07	1.58	1.68	1.26	0.22
	4.6	1.27	1.99	2.29	2.08	1.24
	5.0	1.44	2.33	2.80	2.76	2.10
	5.4	1.59	2.62	3.24	3.34	2.82
	5.8	1.72	2.87	3.61	3.85	3.45
	6.2	1.82	3.09	3.94	4.28	4.00
	6.6	1.92	3.29	4.23	4.67	4.48
	7.0 and up	2.01	3.46	4.49	5.01	4.90
C	6.0	1.84	2.66	2.72	1.87	
	7.0	2.48	3.94	4.64	4.44	3.12
	8.0	2.96	4.90	6.09	6.36	5.52
	9.0	3.34	5.65	7.21	7.86	7.39
	10.0	3.64	6.25	8.11	9.06	8.89
	11.0	3.88	6.74	8.84	10.0	10.1
	12.0 and up	4.09	7.15	9.46	10.9	11.1
D	10.0	4.14	6.13	6.55	5.09	1.35
	11.0	5.00	7.83	9.11	8.50	5.62
	12.0	5.71	9.26	11.2	11.4	9.18
	13.0	6.31	10.5	13.0	13.8	12.2
	14.0	6.82	11.5	14.6	15.8	14.8
	15.0	7.27	12.4	15.9	17.6	17.0
	16.0	7.66	13.2	17.1	19.2	19.0
	17.0 and up	8.01	13.9	18.1	20.6	20.7
E	16.0	8.68	14.0	17.5	18.1	15.3
	18.0	9.92	16.7	21.2	23.0	21.5
	20.0	10.9	18.7	24.2	26.9	26.4
	22.0	11.7	20.3	26.6	30.2	30.5
	24.0	12.4	21.6	28.6	32.9	33.8
	26.0	13.0	22.8	30.3	35.1	36.7
	28.0 and up	13.4	23.7	31.8	37.1	39.1

Table 17–12

Power (kW) Ratings of Standard V Belts

Belt Section	Sheave Pitch Diameter, mm	Belt Speed, m/s 5	10	15	20	25
A	65	0.35	0.46	0.40	0.11	
	75	0.49	0.75	0.84	0.69	0.28
	85	0.60	0.98	1.17	1.64	0.84
	95	0.69	1.16	1.43	1.49	1.28
	105	0.77	1.30	1.64	1.78	1.63
	115	0.83	1.41	1.82	2.01	1.93
	125 and up	0.87	1.51	1.97	2.21	2.16
B	105	0.80	1.18	1.25	0.94	0.16
	115	0.95	1.48	1.71	1.55	0.92
	125	1.07	1.74	2.09	2.06	1.57
	135	1.19	1.95	2.42	2.49	2.10
	145	1.28	2.14	2.69	2.87	2.57
	155	1.36	2.31	2.94	3.19	2.98
	165	1.43	2.45	3.16	3.48	3.34
	175 and up	1.50	2.58	3.35	3.74	3.66
C	150	1.37	1.98	2.03	1.40	
	175	1.85	2.94	3.46	3.31	2.33
	200	2.21	3.66	4.54	4.74	4.12
	225	2.49	4.21	5.38	5.86	5.51
	250	2.72	4.66	6.05	7.16	6.63
	275	2.89	5.03	6.59	7.46	7.53
	300 and up	3.05	5.33	7.06	8.13	8.28
D	250	3.09	4.57	4.89	3.80	1.01
	275	3.73	5.84	6.80	6.34	4.19
	300	4.26	6.91	8.36	8.50	6.85
	325	4.71	7.83	9.70	10.30	9.10
	350	5.09	8.58	10.89	11.79	11.04
	375	5.42	9.25	11.86	13.13	12.68
	400	5.71	9.85	12.76	14.32	14.17
	425 and up	5.98	10.37	13.50	15.37	15.44
E	400	6.48	10.44	13.06	13.50	11.41
	450	7.40	12.46	15.82	17.16	16.04
	500	8.13	13.95	18.05	20.07	19.69
	550	8.73	15.14	19.84	22.53	22.75
	600	9.25	16.11	21.34	24.54	25.22
	650	9.70	17.01	22.60	26.19	27.38
	700 and up	10.00	17.68	23.72	27.68	29.17

The rating, whether in terms of hours or belt passes, is for a belt running on equal-diameter sheaves (180° of wrap), of moderate length, and transmitting a steady load.

$$H_a = K_1 K_2 H_{\text{tab}}$$

where H_a = allowable power, per belt, Table 17–12

K_1 = angle-of-wrap correction factor, Table 17–13

K_2 = belt length correction factor, Table 17–14

Table 17–13

Angle of Contact Correction Factor K_1 for VV* and V-Flat Drives

$\frac{D-d}{C}$	θ, deg	K_1 VV	K_1 V Flat
0.00	180	1.00	0.75
0.10	174.3	0.99	0.76
0.20	166.5	0.97	0.78
0.30	162.7	0.96	0.79
0.40	156.9	0.94	0.80
0.50	151.0	0.93	0.81
0.60	145.1	0.91	0.83
0.70	139.0	0.89	0.84
0.80	132.8	0.87	0.85
0.90	126.5	0.85	0.85
1.00	120.0	0.82	0.82
1.10	113.3	0.80	0.80
1.20	106.3	0.77	0.77
1.30	98.9	0.73	0.73
1.40	91.1	0.70	0.70
1.50	82.8	0.65	0.65

*A curvefit for the VV column in terms of θ is
$K_1 = 0.143\,543 + 0.007\,46\,8\,\theta - 0.000\,015\,052\,\theta^2$
in the range $90° \leq \theta \leq 180°$.

Table 17–14

Belt-Length Correction Factor K_2*

Length Factor	A Belts	B Belts	C Belts	D Belts	E Belts
	Nominal Belt Length, in				
0.85	Up to 35	Up to 46	Up to 75	Up to 128	
0.90	38–46	48–60	81–96	144–162	Up to 195
0.95	48–55	62–75	105–120	173–210	210–240
1.00	60–75	78–97	128–158	240	270–300
1.05	78–90	105–120	162–195	270–330	330–390
1.10	96–112	128–144	210–240	360–420	420–480
1.15	120 and up	158–180	270–300	480	540–600
1.20		195 and up	330 and up	540 and up	660

*Multiply the rated horsepower per belt by this factor to obtain the corrected horsepower.

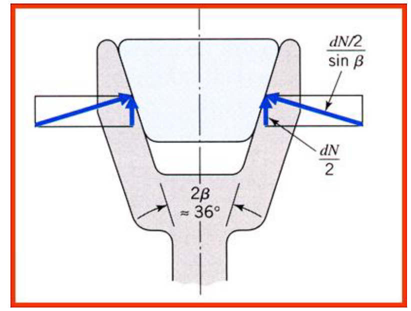
Table 17–14

Belt-Length Correction Factor K_2^*

Length Factor	A Belts	B Belts	C Belts	D Belts	E Belts
	Nominal Belt Length, m				
0.85	Up to 0.88	Up to 1.15	Up to 1.88	Up to 3.2	
0.90	0.95–1.15	1.2–1.5	2.03–2.4	3.6–4.05	Up to 4.88
0.95	1.2–1.38	1.55–1.88	2.63–3.0	4.33–5.25	5.25–6.0
1.00	1.5–1.88	1.95–2.43	3.2–3.95	6.0	6.75–7.5
1.05	1.95–2.25	2.63–3.0	4.05–4.88	6.75–8.25	8.25–9.75
1.10	2.4–2.8	3.2–3.6	5.25–6.0	9.0–10.5	10.5–12.0
1.15	3.0 and up	3.95–4.5	6.75–7.5	12.0	13.5–15.0
1.20		4.88 and up	8.25 and up	13.5 and up	16.5

*Multiply the rated power per belt by this factor to obtain the corrected power.

In a V belt the effective coefficient of friction f' is $f/\sin(\phi/2)$, which amounts to an augmentation by a factor of about 3 due to the grooves. The effective coefficient of friction f' is sometimes tabulated against *sheave* groove angles of 30°, 34°, and 38°, the tabulated values being 0.50, 0.45, and 0.40, respectively, revealing a belt material-on-metal coefficient of friction of 0.13 for each case. The Gates Rubber Company declares its effective coefficient of friction to be 0.5123 for grooves. Thus



$$\frac{F_1 - F_c}{F_2 - F_c} = \exp(0.5123\phi) \tag{17–18}$$

The design power is given by

$$H_d = H_{\text{nom}} K_s n_d \tag{17–19}$$

where H_{nom} is the nominal power, K_s is the service factor given in Table 17–15, and n_d is the design factor. The number of belts, N_b, is usually the next higher integer to H_d/H_a.

Table 17–15

Suggested Service Factors K_S for V-Belt Drives

	Source of Power	
Driven Machinery	**Normal Torque Characteristic**	**High or Nonuniform Torque**
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

$N_b > H_d / H_a \qquad N_b = 1, 2, 3, ..$

The centrifugal tension, F_c is given by

$$F_c = K_c\,(V/2.4)^2$$

The power transmitted per belt is based on $\Delta F = F_1 - F_2$

$$\Delta F = (H_d/N_b)/\ \pi\, nd$$

Table 17–16

Some V-Belt Parameters*

Belt Section	K_b	K_c
A	220	0.561
B	576	0.965
C	1 600	1.716
D	5 680	3.498
E	10 850	5.041
3V	230	0.425
5V	1098	1.217
8V	4830	3.288

*Data courtesy of Gates Rubber Co., Denver, Colo.

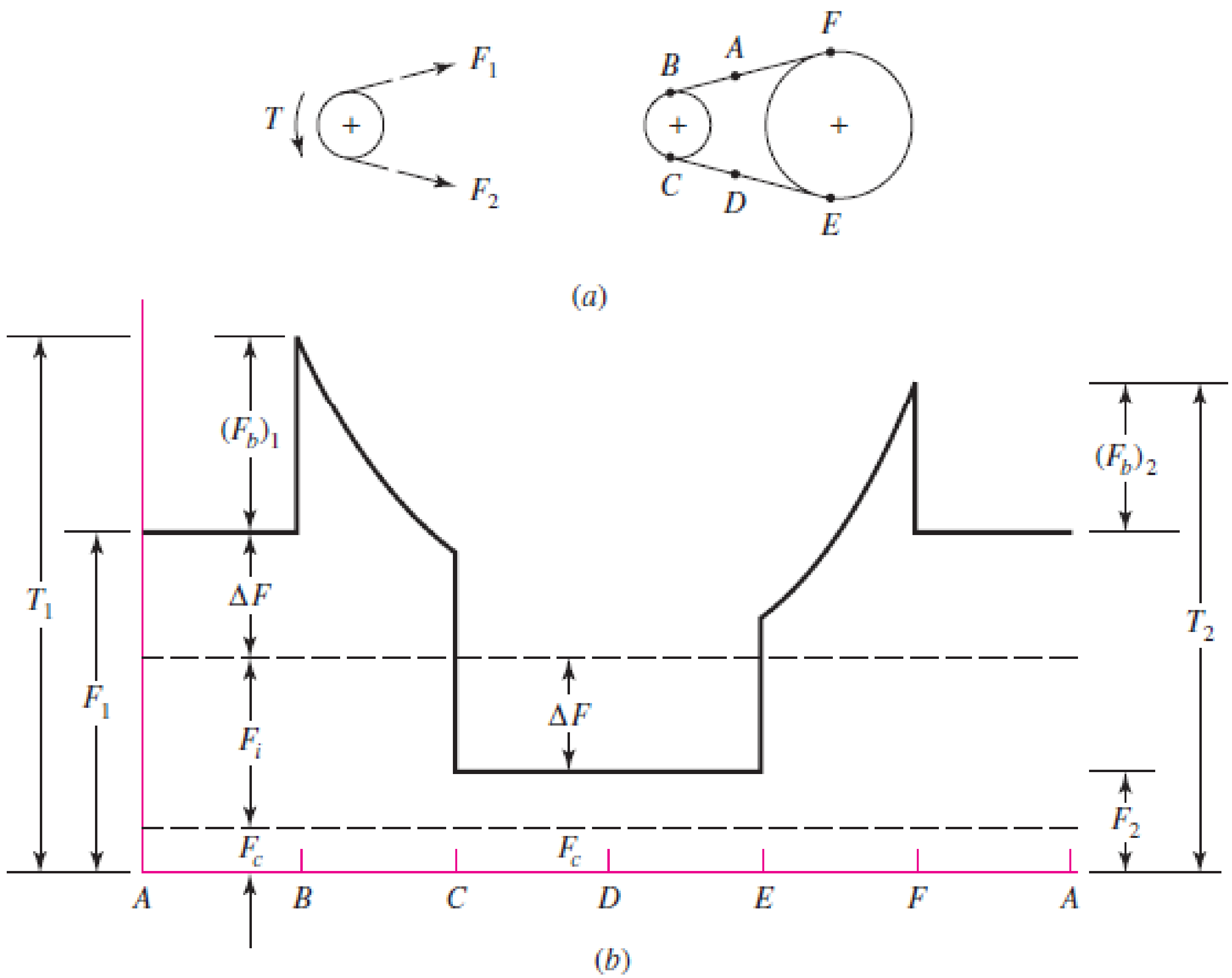
$$F_1 = F_c + \frac{\Delta F \exp(f\phi)}{\exp(f\phi) - 1}$$

From the definition of ΔF, the least tension F_2 is

$$F_2 = F_1 - \Delta F$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$n_{fs} = \frac{H_a N_b}{H_{\text{nom}} K_s}$$



T
F1
F2
F
A
B
C
D
E
(a)
(Fb)1
ΔF
Fi
Fc
T1
F1
ΔF
Fc
(Fb)2
T2
F2
A
B
C
D
E
F
A
(b)

Durability (life) correlations are complicated by the fact that the bending induces flexural stresses in the belt; the corresponding belt tension that induces the same maximum tensile stress is F_{b1} at the driving sheave and F_{b2} at the driven pulley. These equivalent tensions are added to F_1 as

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}$$

where K_b is given in Table 17–16. The equation for the tension versus pass trade-off used by the Gates Rubber Company is of the form

$$T^b N_P = K^b$$

where N_P is the number of passes and b is approximately 11. See Table 17–17. The Miner rule is used to sum damage incurred by the two tension peaks:

$$\frac{1}{N_P} = \left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}$$

or

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} \tag{17–27}$$

The lifetime t in hours is given by $t = N_p L_p / (3600V)$

If $N_p > 10^9$ take $N_p = 10^9$

Table 17–17

Durability Parameters for Some V-Belt Sections

Source: M. E. Spotts, *Design of Machine Elements,* 6th ed. Prentice Hall, Englewood Cliffs, N.J., 1985.

Belt Section	10^8 to 10^9 Force Peaks		10^9 to 10^{10} Force Peaks		Minimum Sheave Diameter, in
	K	*b*	*K*	*b*	
A	674	11.089			3.0
B	1193	10.926			5.0
C	2038	11.173			8.5
D	4208	11.105			13.0
E	6061	11.100			21.6
3V	728	12.464	1062	10.153	2.65
5V	1654	12.593	2394	10.283	7.1
8V	3638	12.629	5253	10.319	12.5

Table 17–17

Durability Parameters for Some V-Belt Sections

Source: M. E. Spotts, *Design of Machine Elements,* 6th ed. Prentice Hall, Englewood Cliffs, N.J., 1985.

Belt Section	10^8 to 10^9 Force Peaks		10^9 to 10^{10} Force Peaks		Minimum Sheave Diameter, mm
	K	*b*	*K*	*b*	
A	2999	11.089			75
B	5309	10.926			125
C	9069	11.173			215
D	18 726	11.105			325
E	26 791	11.100			540
3V	3240	12.464	4726	10.153	66
5V	7360	12.593	10 653	10.283	177
8V	16 189	12.629	23 376	10.319	312

The analysis of a V-belt drive can consist of the following steps:

- Find V, L_p, C, ϕ, and $\exp(0.5123\phi)$
- Find H_d, H_a, and N_b from H_d/H_a and round up
- Find F_c, ΔF, F_1, F_2, and F_i, and n_{fs}
- Find belt life in number of passes, or hours, if possible

Example 1

Two B2125 V belts are used in a drive composed of a 135 mm driving sheave, rotating at 1200 rpm, and a 400 mm driven sheave. Find the power capacity of the drive based on a service factor of 1.25, and the center-to-center distance.

V-belt Drive Design Process

- Need rated power of the driving motor/prime mover. BASE sizing on this.
- Service factor based on type of driver and driven load.
- Center distance (adjustment for center distance must be provided or use idler pulley) nominal range $D < C < 3(D + d)$
- Power rating for one belt as a function of size and speed of the smaller sheave
- Belt length (then choose standard size)
- Sizing of sheaves (use standard size). Most commercially available sheaves should be limited to 30 m/s belt speed.
- Belt length correction factor
- Angle of wrap correction factor. Angle of wrap on smaller sheave should be greater than 120 deg.
- Number of belts
- Initial tension in belts

EXAMPLE 17–4 A 7.46-kW split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 188-mm small sheave, a 280-mm large sheave, and three B2800 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

Solution The peripheral speed V of the belt is

$$V = \pi\, dn = \pi(0.188)1750/60 = 17 \text{ m/s}$$

Table 17–11: $L_p = L + L_c = 2800 + 45 = 2845$ mm

$$\text{Eq. (17–16}b\text{):}\quad C = 0.25\left\{\left[2845 - \frac{\pi}{2}(280 + 188)\right] + \sqrt{\left[2845 - \frac{\pi}{2}(280 + 188)\right]^2 - 2(280 - 188)^2}\right\}$$

$$= 1054 \text{ mm}$$

Eq. (17–1): $\phi = \theta_d = \pi - 2\sin^{-1}(280 - 188)/[2(1054)] = 3.054$ rad

$$\exp[0.5123(3.054)] = 4.781$$

Interpolating in Table 17–12 for $V = 17$ m/s gives $H_{\text{tab}} = 3.5$ kW. The wrap angle ir degrees is $3.054(180)/\pi = 175°$. From Table 17–13, $K_1 = 0.99$. From Table 17–14 $K_2 = 1.05$. Thus, from Eq. (17–17),

$$H_a = K_1 K_2 H_{\text{tab}} = 0.99(1.05)3.5 = 3.64 \text{ kW}$$

Eq. (17–19): $H_d = H_{\text{nom}} K_s n_d = 7.46(1.2 + 0.1)(1) = 9.7$ kW

Eq. (17–20): $N_b \geq H_d/H_a = 9.7/3.64 = 2.67 \rightarrow 3$

From Table 17–16, $K_c = 0.965$. Thus, from Eq. (17–21),

$$F_c = 0.965(17/2.4)^2 = 48.4 \text{ N}$$

Eq.(17–22): $$\Delta F = \frac{9700/3}{\pi(1750/60)0.188} = 188 \text{ N}$$

Eq. (17–23): $$F_1 = 48.4 + \frac{188(4.781)}{4.781 - 1} = 286 \text{ N}$$

Eq. (17–24): $$F_2 = F_1 - \Delta F = 286 - 188 = 98 \text{ N}$$

Eq. (17–25): $$F_i = \frac{286 + 98}{2} - 48.4 = 143 \text{ N}$$

Eq. (17–26): $$n_{fs} = \frac{H_a N_b}{H_{\text{nom}} K_s} = \frac{3.64(3)}{7.46(1.3)} = 1.13$$

Life: From Table 17–16, $K_b = 576$.

$$F_{b1} = \frac{K_b}{d} = \frac{65}{0.188} = 346 \text{ N}$$

$$F_{b2} = 65/0.28 = 232 \text{ N}$$

$$T_1 = F_1 + F_{b1} = 286 + 346 = 632 \text{ N}$$

$$T_2 = F_1 + F_{b2} = 286 + 232 = 518 \text{ N}$$

From Table 17–17, $K = 5309$ and $b = 10.926$.

Eq. (17–27): $$N_P = \left[\left(\frac{5309}{632}\right)^{-10.926} + \left(\frac{5309}{518}\right)^{-10.926}\right]^{-1} = 11(10^9) \text{ passes}$$

Answer Since N_P is out of the validity range of Eq. (17–27), life is reported as greater than 10^9 passes. Then

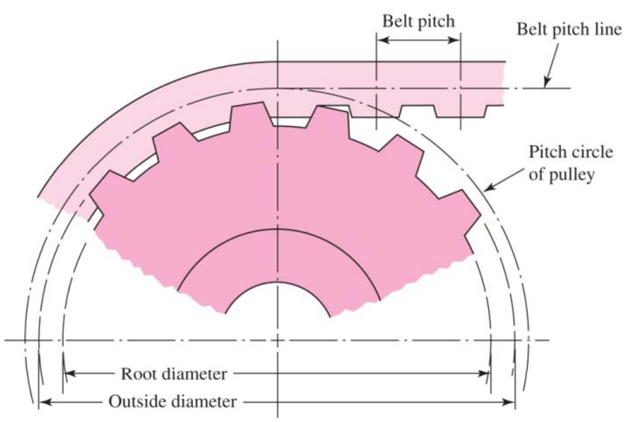
Answer Eq. (17–28): $$t > \frac{10^9(2.845)}{3600(17)} = 46\,500 \text{ h}$$

Example

A 1.5 kW electric motor running at 1760 rev/min is to drive a blower of 220 rev/min. Select a V-belt drive for this application and specify standard V belts, sheave sizes, and the resulting center-to-center distance. The motor size limits the center distance to at least 0.8 m.

Timing Belts

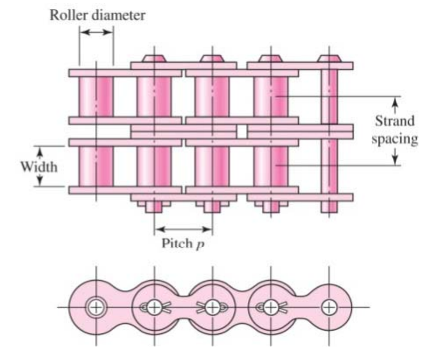
- A timing belt does not stretch appreciably or slip and consequently transmits power at a constant angular-velocity ratio.
- Timing belts can operate over a very wide range of speeds, have efficiencies in the range of 97 to 99 percent, require no lubrication, and are quieter than chain drives.
- The five standard inch-series pitches available are listed in Table 17–18 with their letter designations.
- The design and selection process for timing belts is similar to that for V belts.



Service	Designation	Pitch p, mm
Extra light	XL	5
Light	L	10
Heavy	H	12
Extra heavy	XH	22
Double extra heavy	XXH	30

Roller Chain

- Basic features of chain drives include a constant ratio, since no slippage or creep is involved; long life; and the ability to drive a number of shafts from a single source of power.



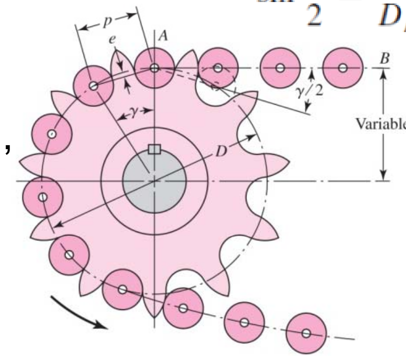
- The pitch diameter of the sprocket by D can be written

$$D = \frac{p}{\sin(180°/N)}$$

- The chain velocity V is defined as the number of feet coming off the sprocket per unit time.

$$V = N\,pn$$

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2}$$



where N = number of sprocket teeth, p = chain pitch, in ,n = sprocket speed, rev/min

- The maximum exit velocity of the chain is

$$v_{\max} = \pi Dn = \frac{\pi np}{\sin(\gamma/2)} \qquad d = D\cos\frac{\gamma}{2}$$

and the minimum exit velocity is

$$v_{\min} = \pi\,dn = \pi\,np\,\frac{\cos(\gamma/2)}{\sin(\gamma/2)}$$

Table 17–19

Dimensions of American Standard Roller Chains—Single Strand

Source: Compiled from ANSI B29.1-1975.

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	— —
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)
160	2.000 (50.80)	1.250 (31.75)	50 000 (222 000)	6.61 (96.5)	1.125 (28.57)	2.305 (58.55)
180	2.250 (57.15)	1.406 (35.71)	63 000 (280 000)	9.06 (132.2)	1.406 (35.71)	2.592 (65.84)
200	2.500 (63.50)	1.500 (38.10)	78 000 (347 000)	10.96 (159.9)	1.562 (39.67)	2.817 (71.55)
240	3.00 (76.70)	1.875 (47.63)	112 000 (498 000)	16.4 (239)	1.875 (47.62)	3.458 (87.83)

Roller chains seldom fail because they lack tensile strength; they more often fail because they have been subjected to a great many hours of service. Actual failure may be due either to wear of the rollers on the pins or to fatigue of the surfaces of the rollers. Roller-chain manufacturers have compiled tables that give the horsepower capacity corresponding to a life expectancy of 15 kh for various sprocket speeds. These capacities are tabulated in Table 17–20 for 17–tooth sprockets. Table 17–21 displays available tooth counts on sprockets of one supplier. Table 17–22 lists the tooth correction factors for other than 17 teeth. Table 17–23 shows the multiple-strand factors K_2.

The capacities of chains are based on the following:

- 15 000 h at full load
- Single strand
- ANSI proportions
- Service factor of unity
- 100 pitches in length
- Recommended lubrication
- Elongation maximum of 3 percent
- Horizontal shafts
- Two 17-tooth sprockets

Table 17–20

Rated Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket

Source: Compiled from ANSI B29.1-1975 Information only section, and from B29.9-1958.

Sprocket Speed, rev/min	ANSI Chain Number 25	35	40	41	50	60
50	0.037	0.12	0.28	0.15	0.54	0.93
100	0.067	0.21	0.51	0.28	0.99	1.72
150	0.097*	0.30*	0.74*	0.42*	1.43*	2.48
200	0.12*	0.40*	0.96	0.53	1.87	3.20
300	0.17	0.58	1.38	0.75	2.69	4.63
400	0.22*	0.75*	1.80	0.98	3.50	6.00
500	0.28	0.93	2.20	1.20	4.25	7.32
600	0.33*	1.10*	2.60*	1.42*	5.01*	8.65
700	0.37	1.25	2.96	1.63	5.77	9.92
800	0.42*	1.40*	3.34*	1.84*	6.5*	11.20
900	0.46	1.56	3.72	2.04	7.23	12.50
1000	0.51*	1.72*	4.1	2.25	7.98	13.65
1200	0.60	2.04	4.81	2.45	9.40	16.11
1400	0.69*	2.33*	5.53	1.95	10.74	13.50
1600	0.78*	2.63*	6.24	1.60	9.55	11.00
1800	0.86	2.93	6.68	1.33	7.98	9.25
2000	0.95*	3.22*	5.76*	1.13*	6.89*	7.90
2500	1.16	3.94	4.11*	0.82*	4.90*	5.64
3000	1.37	4.2	3.11	0.62	3.72	4.30
Type A		Type B			Type C	

*Estimated from ANSI tables by linear interpolation.

Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication.

Table 17–20

Rated Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket (*Continued*)

Sprocket Speed, rev/min		ANSI Chain Number 80	100	120	140	160	180	200	240
50	Type A	2.15	4.11	7	10.7	15.6	21.6	28.6	46.1
100		4.01	7.7	13	20	29.2	40.3	53.4	85.8
150		5.78	11	18.7	29	42	58	76.8	123.8
200		7.46	14.3	24.2	37.5	54.4	75.3	100	160.4
300		10.82	20.7	35	54.4	78.3	108	144	231.3
400		14	26.8	45.2	70	101.5	140.2	185.8	268.0
500	Type B	17	32.7	55.3	85.8	123.8	152.1	165.6	0
600		20.1	38.6	65.1	94.7	105.2	115.6	126	
700		23.1	44.3	66.4	75.3	83.6	91.8	0	
800		26.1	47	54.3	61.5	68.4	75.3		
900		29.8	39.4	45.5	51.5	57.3	63		
1000		28.1	33.6	38.8	44	49	53.8		
1200		21.4	25.6	29.5	33.5	37.2	0		
1400		16.9	20.3	23.5	26.5	0			
1600		13.9	18.6	19.3	0				
1800		11.6	14	16.1					
2000		9.92	11.9	0					
2500		7.13	0.3						
3000		5.4	0						
Type C				**Type C′**					

Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication; type C′—type C, but this is a galling region; submit design to manufacturer for evaluation.

Table 17–22

Tooth Correction Factors, K_1

Number of Teeth on Driving Sprocket	K_1 Pre-extreme Power	K_1 Post-extreme Power
11	0.62	0.52
12	0.69	0.59
13	0.75	0.67
14	0.81	0.75
15	0.87	0.83
16	0.94	0.91
17	1.00	1.00
18	1.06	1.09
19	1.13	1.18
20	1.19	1.28
N	$(N_1/17)^{1.08}$	$(N_1/17)^{1.5}$

Table 17–23

Multiple-Strand Factors, K_2

Number of Strands	K_2
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6
8	6.0

Analysis of Roller Chains

- The chordal speed variation is

$$\frac{\Delta V}{V} = \frac{v_{\max} - v_{\min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180^\circ/N)} - \frac{1}{\tan(180^\circ/N)}\right]$$

- For smooth operation at moderate and high speeds it is considered good practice to use a driving sprocket with at least 17 teeth and no less than 12 teeth.
- The maximum speed (rev/min) for a chain drive is limited by galling between the pin and the bushing.

$$n_1 \le 1000\left[\frac{82.5}{7.95^p(1.0278)^{N_1}(1.323)^{F/1000}}\right]^{1/(1.59\log p + 1.873)} \qquad \text{rev/min}$$

where F is the chain tension in pounds.

- Lubrication of roller chains is essential in order to obtain a long and trouble-free life.

The fatigue strength of link plates governs capacity at lower speeds. The American Chain Association (ACA) publication *Chains for Power Transmission and Materials Handling* (1982) gives, for single-strand chain, the nominal power H_1, link-plate limited, as

$$H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)} \qquad \text{hp} \tag{17–32}$$

and the nominal power H_2, roller-limited, as

$$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}} \qquad \text{hp} \tag{17–33}$$

where N_1 = number of teeth in the smaller sprocket

n_1 = sprocket speed, rev/min

p = pitch of the chain, in

K_r = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

The constant 0.004 becomes 0.0022 for no. 41 lightweight chain. The nominal horsepower in Table 17–20 is $H_{\text{nom}} = \min(H_1, H_2)$. For example, for $N_1 = 17$, $n_1 = 1000$ rev/min, no. 40 chain with $p = 0.5$ in, from Eq. (17–32),

$$H_1 = 0.004(17)^{1.08} 1000^{0.9} 0.5^{[3-0.07(0.5)]} = 5.48 \text{ hp}$$

From Eq. (17–33),

$$H_2 = \frac{1000(17)17^{1.5}(0.5^{0.8})}{1000^{1.5}} = 21.64 \text{ hp}$$

The tabulated value in Table 17–20 is $H_{\text{tab}} = \min(5.48, 21.64) = 5.48$ hp.

Table 17–21

Single-Strand Sprocket Tooth Counts Available from One Supplier*

No.	Available Sprocket Tooth Counts
25	8-30, 32, 34, 35, 36, 40, 42, 45, 48, 54, 60, 64, 65, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
35	4-45, 48, 52, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
41	6-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
40	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
50	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
60	8-60, 62, 63, 64, 65, 66, 67, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
80	8-60, 64, 65, 68, 70, 72, 76, 78, 80, 84, 90, 95, 96, 102, 112, 120
100	8-60, 64, 65, 67, 68, 70, 72, 74, 76, 80, 84, 90, 95, 96, 102, 112, 120
120	9-45, 46, 48, 50, 52, 54, 55, 57, 60, 64, 65, 67, 68, 70, 72, 76, 80, 84, 90, 96, 102, 112, 120
140	9-28, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 48, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 96
160	8-30, 32–36, 38, 40, 45, 46, 50, 52, 53, 54, 56, 57, 60, 62, 63, 64, 65, 66, 68, 70, 72, 73, 80, 84, 96
180	13-25, 28, 35, 39, 40, 45, 54, 60
200	9-30, 32, 33, 35, 36, 39, 40, 42, 44, 45, 48, 50, 51, 54, 56, 58, 59, 60, 63, 64, 65, 68, 70, 72
240	9-30, 32, 35, 36, 40, 44, 45, 48, 52, 54, 60

*Morse Chain Company, Ithaca, NY, Type B hub sprockets.

It is preferable to have an odd number of teeth on the driving sprocket (17, 19, . . .) and an even number of pitches in the chain to avoid a special link. The approximate length of the chain L in pitches is

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p} \tag{17–34}$$

The center-to-center distance C is given by

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right] \tag{17–35}$$

where

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} \tag{17–36}$$

The allowable power H_a is given by

$$H_a = K_1 K_2 H_{\mathrm{tab}} \tag{17–37}$$

where K_1 = correction factor for tooth number other than 17 (Table 17–22)

K_2 = strand correction (Table 17–23)

The horsepower that must be transmitted H_d is given by

$$H_d = H_{\mathrm{nom}} K_s n_d \tag{17–38}$$

Equation (17–32) is the basis of the pre-extreme power entries (vertical entries) of Table 17–20, and the chain power is limited by link-plate fatigue. Equation (17–33) is the basis for the post-extreme power entries of these tables, and the chain power performance is limited by impact fatigue. The entries are for chains of 100 pitch length and 17-tooth sprocket. For a deviation from this

$$H_2 = 1000\left[K_r\left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8}\left(\frac{L_p}{100}\right)^{0.4}\left(\frac{15\,000}{h}\right)^{0.4}\right] \tag{17–39}$$

where L_p is the chain length in pitches and h is the chain life in hours. Viewed from a deviation viewpoint, Eq. (17–39) can be written as a trade-off equation in the following form:

$$\frac{H_2^{2.5}h}{N_1^{3.75}L_p} = \text{constant} \tag{17–40}$$

If tooth-correction factor K_1 is used, then omit the term $N_1^{3.75}$. Note that $(N_1^{1.5})^{2.5} = N_1^{3.75}$.