Test of hypothesis about single population mean

Example 1

In the following example, the population distribution is normal with known standard deviation and the hypothesis test is right-tailed.

A hypothesis test is to be performed to determine whether the mean waiting time during peak hours for customers in a supermarket has increased from the previous mean waiting time of 8.2 minutes. Previous experience indicates that the waiting time follows a normal distribution with standard deviation equal 3.8 minutes. To test the hypothesis, a random sample of 25 customers will be selected yields mean $\bar{x} = 9.75$.. Answer the questions 1 to 10.

Question 1

The null and alternative hypotheses are...

(A) $H_0 : \mu \geq 8.2$ & $H_1 : \mu < 8.2$	(B) $H_0 : \mu = 8.2$ & $H_1 : \mu \neq 8.2$
(C) $H_0 : \mu \leq 8.2$ & $H_1 : \mu > 8.2$	(D) $H_0 : \mu \neq 8.2$ & $H_1 : \mu = 8.2$

Question 2:

This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed
(C) Multi-tailed	(D) left-tailed

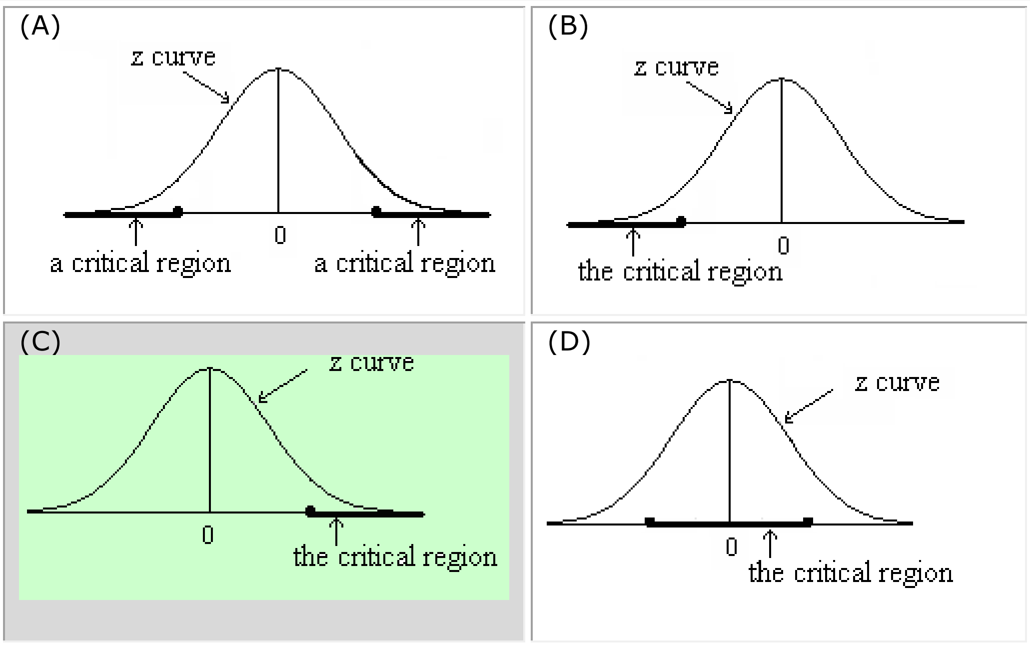
Question 3

The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \approx N(0,1)$	(B) $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$
(C) $T = \dfrac{\bar{X} - \mu_0}{S / \sqrt{n}} \sim t_{24}$	(D) $Z = \dfrac{\bar{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$

Question 4

The critical region is best described by figure....



Question 5

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic is....

(A) Reject H_0 if z_c < -1.645	(B) Reject H_0 if z_c > 1.96
(C) Reject H_0 if Z_c > 1.645 or $Z_c < -1.645$	(D) Reject H_0 if z_c > 1.645

Question 6

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the estimator of the pupation parameter($\bar{x}_c$)is

(A)) 9.45	(B) 3.98	(C) 8.90	(D) 6.95

Solution:

$$\bar{X}_c = \mu_0 + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = 8.2 + 1.645\left(\frac{3.8}{\sqrt{25}}\right) = 9.45$$

Question 7:

The computed value of our test statistic is....

(A) -2.04	(B) 3.98	(C) 2.04	(D) 0.54

Solution:

$$z_c = \frac{9.75-8.2}{3.8/\sqrt{25}} = 2.04$$

Question 8

The decision would be to....

(A) Cannot be determined
(B) Do not reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 9

Suppose that in fact the waiting time is increased to 9 minutes ($\mu_1 = 9.9$), then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision($1-\alpha$)	(D) Correct decision($1-\beta$)

Question 10

The power of our test at μ_1 = 9.9 minutes is....

(A) 0.2224	(B) 0.7224	(C) 0.7776	(D) 0.2776

Solution:

Power equals the probability of rejecting the null hypothesis while the alternative hypothesis is false. We compute it as follows:

At $\mu_1 = 9.9$

$$\beta = P(Z < Z_c) = P\left(Z < \frac{\bar{X}_c - \mu_1}{\sigma/\sqrt{n}}\right) = P\left(Z < \frac{9.45-9.9}{3.8/\sqrt{25}}\right) = P(Z < -0.59)$$

= 0.5-0.2224=0.2776

Power=1- β =1-0.2776=7224

Question 11

A 95% confidence interval is $8.3 < \mu < 11.2$. The null hypothesis is $\begin{array}{l} H_0 : \mu = 8.2 \\ H_1 : \mu \neq 8.2 \end{array}$

What is the decision?

(A) Reject the null hypothesis.
(B) Do not Reject the null hypothesis.
(C) Can not be determined
(D) Reject the alternative hypothesis.

End of example 1

Example 2

In the following example, the population distribution is approximately normal with known standard deviation and the hypothesis test is left-tailed

The students at the university claim that the average student must travel for at least 25 minutes in order to reach the university. The university admissions office obtained a random sample of 36 one-way travel times from students. The sample had a mean of 19.4 minutes. Assume that the population standard deviation is 9.6 minutes. Does the admissions office have sufficient evidence to reject the student's claim?
Answer the following questions

Question 1
The null and alternative hypotheses are...

(A) $H_0: \mu < 25$ & $H_1: \mu \geq 25$	(B) $H_0: \mu = 25$ & $H_1: \mu > 25$
(C) $H_0: \mu \neq 25$ & $H_1: \mu < 25$	(D) $H_0: \mu \geq 25$ & $H_1: \mu < 25$

Question 2:
This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed
(C) Multi-tailed	(D) left-tailed

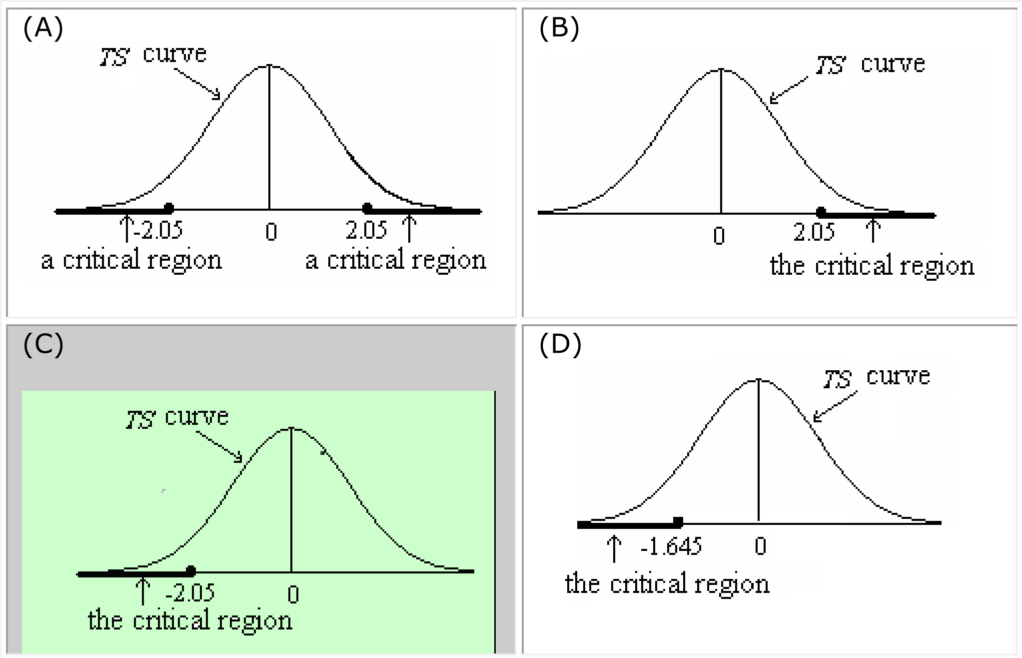
Question 3:
The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{P-\pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$	(B) $T = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$
(C) $Z = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$	(D) $Z = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

Question 4
With significance level equal 0.02, the decision criterion for the hypothesis test in terms of the computed value of the test statistic (Z_c) is....

(A) Reject H_0 if $Z_c <$ -2.05
(B) Reject H_0 if $Z_c >$ 1.96
(C) Reject H_0 if $Z_c >$ 2.05 or $Z_c < -2.05$
(D) Reject H_0 if $Z_c <$-1.645

Question 5
With level of significance 2%, the critical region is best described by figure....



Question 6:
With significance level equal 0.02, the decision criterion for the hypothesis test in terms of the computed value of $\overline{X}$, $\overline{X}_c$ is....

(A) 4.67	(B) 3.98	(C) 22.37	(D) 21.72

$$\overline{X}_c = \mu_0 + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = 25 - 2.05\left(\frac{9.6}{\sqrt{36}}\right) = 21.72$$

Question 7:
The computed value of our test statistic is....

(A) -3.5	(B) 1.24	(C) 3.5	(D) -2.36

Question 8:
The decision would be to....

(A) Cannot be determined
(B) Do not Reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 9:
Suppose that in fact the traveling time is 20 minutes ($\mu_1 = 20$), then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision($1-\alpha$)	(D) Correct decision($1-\beta$)

Question 10:

The p-value of this test statistic is....

(A) 0.0002	(B) 2.05	(C) 0.025	(D) 0.05

Solution:

P-value= $P(Z < z_c | H_0)$

$= P(Z < -3.5 | \mu = 25)$

$= P(Z < -3.5) = 0.5 - P(0 < Z < 3.5)$

$= 0.5 - .4998 = 0.0002$

p-value=0.0002< α =0.02

$\therefore$ Reject H_0

Question 11:

The power of our test at μ_1 = 20 minutes is....

(A) 0.2224	(B) 0.8599	(C) 0.7776	(D) 0.7224

Solution:

$$\beta = P(Z > Z_c) = P\left(Z > \frac{\bar{X}_c - \mu_1}{\sigma / \sqrt{n}}\right) = P\left(Z > \frac{21.72 - 20}{9.6/6}\right) = P\left(Z > \frac{1.72}{1.6}\right)$$

$$= P(Z > 1.08) = 0.5 - \phi(1.08) = 0.5 - 0.3599 = 0.1401$$

$$power = 1 - \beta = 1 - 0.1401 = 0.8599$$

End of example 2

Test of Hypothesis about Single Population Proportion (large Samples)

Example 3

In the following example, the sample is large such that $n\pi > 5$& $n(1-\pi) > 5$ and the hypothesis test is Right-tailed.

It assumed from last experience that 75% of sports viewers are male. A famous sport newspaper reports that this proportion is greater than 0.75. A random sample of 400 season ticket holders reveals that 350 are male. We wish to test the above hypothesis.

Answer the following questions

Question 1:

The null and alternative hypotheses are...

(A) $H_0: \pi \geq 0.75$ & $H_1: \pi < 0.75$	(B) $H_0: \pi < 0.75$ & $H_1: \pi \geq 0.75$
(C) $H_0: \pi \leq 0.75$ & $H_1: \pi > 0.75$	(D) $H_0: \pi \neq 0.75$ & $H_1: \pi = 0.75$

Question2:

This hypothesis test is classifies as...

(A) Two-tailed	(B) Right-tailed
(C) Opposite-tailed	(D) left-tailed

Question 3:

The appropriate test statistic and its distribution under the null hypothesis is...

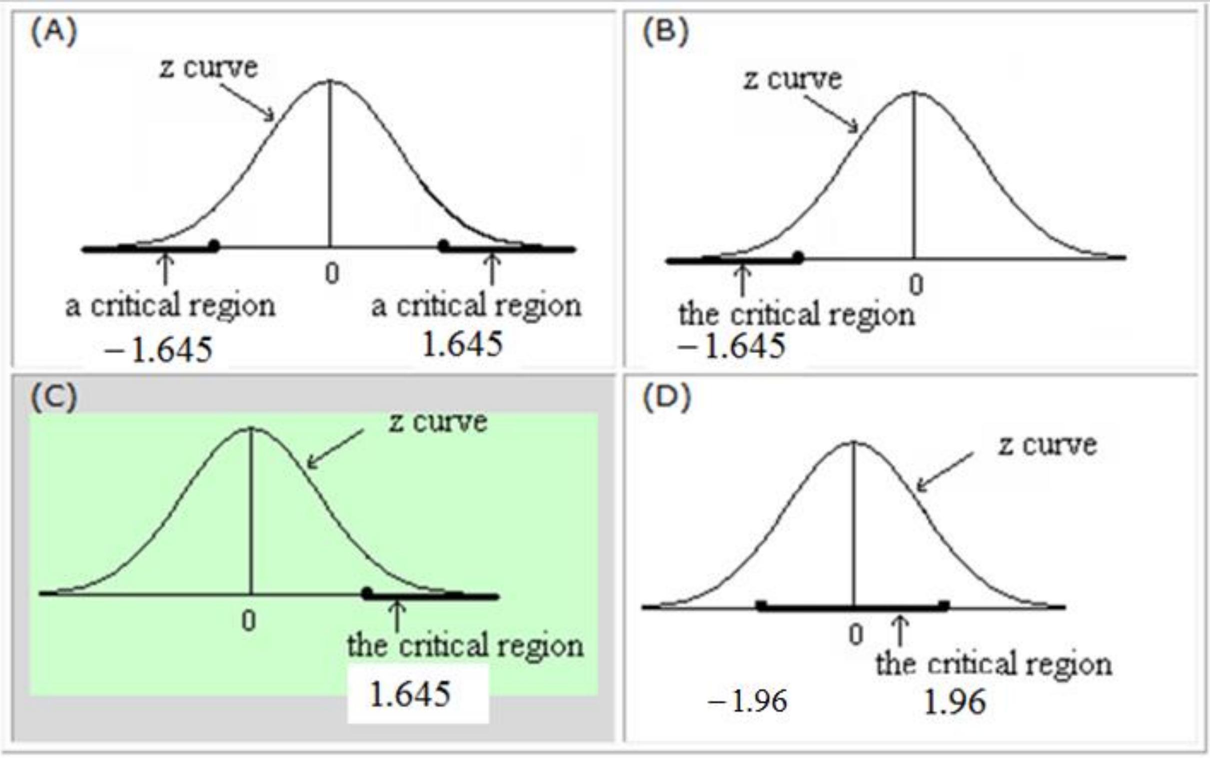
(A) $Z = \dfrac{P-\pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \stackrel{\sim}{\sim} N(0,1)$	(B) $T = \dfrac{\bar{X}-\mu_0}{S/\sqrt{n}} \sim t_8$
(C) $Z = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \stackrel{\sim}{\sim} N(0,1)$	(D) $\chi^2 = \dfrac{P-\pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \sim \chi_1^2$

Question 4:

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic (Z_c) is....

(A) Reject H_0 if Z_c <-1.96	(B) Reject H_0 if Z_c > 1.96
(C) Reject H_0 if Z_c > 1.96 or $Z_c < -1.96$	(D) Reject H_0 if Z_c >1.645

Question 5:
With level of significance 5%, the critical region is best described by figure....



Question 6:
With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of P, p_c is....

(A) 0.708	(B) 0.978	(C) 0.786	(D) 0.6734

Solution:
We reject the null hypothesis if either $z_c > 1.645$

$z_c > 1.645$

$$\Rightarrow P_c = \pi_0 + Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}} \Rightarrow P_c = 0.75 + 1.645\sqrt{\frac{0.75(0.25)}{400}} \Rightarrow P_c = 0.786$$

Question 7:
The computed value of our test statistic is....

(A) 0.01	(B) 5.99	(C) 0.23	(D) -0.01

Solution:

$$z_c = \frac{350/400 - 0.75}{\sqrt{(.75)(.25)/400}} = \frac{0.875 - 0.75}{0.0217} = \frac{0.88 - 0.75}{0.0217} = 5.99$$

Question 8:
The decision would be to....

(A)	Do not Reject the null hypothesis
(B)	Cannot be determined.
(C)	Reject the null hypothesis.
(D	Reject the alternative hypothesis.

Question 9:
Suppose that in fact the true proportion is 0.85, then the decision has been made is... α

(A) Rejecting the true hypothesis(α) type1 error	(B)Do not Rejecting the false hypothesis (β) type11 error.
(C) Do not rejecting the true hypothesis($1-\alpha$)Correct decision	(D) Rejecting the false hypothesis($1-\beta$)Correct decision

Question10:
Suppose that in fact the true proportion is 0.74, then the decision has been made is...

(A) Rejecting the true hypothesis(α) type1 error	(B)Do not Rejecting the false hypothesis (β) type11 error.
(C) Do not rejecting the true hypothesis($1-\alpha$)Correct decision	(D) Rejecting the false hypothesis($1-\beta$)Correct decision

Question11:
The power of our test at π_1 = 0.85 is....

(A) 0.0087	(B) 0.037	(C) 0.352	(D) 0.9998

Solution:

$P_c = 0.786$

$$\beta = P\left(< \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$$

$$\beta = P\left(< \frac{0.786 - 0.85}{\sqrt{\frac{0.85(0.15)}{400}}} \right) = P(Z < -3.58)$$

$$\beta = 0.5 - \phi(3.58) = 0.5 - 0.4998 = 0.0002$$

Power= $1-\beta = 1 - 0.0002 = 0.9998$

Question12: A 95% confidence interval is $0.15 \le \pi \le 0.87$. The null hypothesis is $H_0 : \pi = 0.96$

What is the decision?

(A) Reject the null hypothesis.
(B) Do not Reject the null hypothesis.
(C) Cannot be determined
(D) Reject the alternative hypothesis.

Question13: A 95% confidence interval is $0.15 \le \pi \le 0.87$. The null hypothesis is $H_0 : \pi = 0.77$

What is the decision?

(A) Reject the null hypothesis.
(B) Do not Reject the null hypothesis.
(C) Cannot be determined
(D) Do not Reject Reject the alternative hypothesis.

End of example 3

Test of hypothesis about Tow population means (Independent Samples) Known variances

Example 4
Exercise 30 page 393

A coffee manufacturer is interested in whether the **mean** daily consumption of regular-coffee drinkers is less than that of decaffeinated-coffee drinkers. Assume **the population standard deviation** for those drinking regular coffee is 1.20 cups per day and 1.36 cups per day for those drinking decaffeinated coffee. A random sample of 50 regular-coffee drinkers showed a mean of 4.35 cups per day. Another random sample (independent of the first) of 40 decaffeinated-coffee drinkers showed a mean of 5.84 cups per day.
Answer questions 1 to 15.
Remark: It is preferable to summarize the information as follows:

	Regular-coffee drinkers (1)	decaffeinated-coffee drinkers (2)
σ_i	$\sigma_1 = 1.20$	$\sigma_2 = 1.36$
n_i	$n_1 = 50$	$n_2 = 40$
$\bar{x}_i$	$\bar{x}_1 = 4.35$	$\bar{x}_2 = 5.84$

Question 1
The parameter(s) under testing is (are)...

(A) One population mean	(B) Two population means
(C) One population variance	(D) Two population proportions.

Question 2
The null hypothesis is...

(A) $\mu_1 \le \mu_2$	(B) $\mu_1 = \mu_2$
(C) $\mu_1 \ge \mu_2$	(D) $\pi_1 \ge \pi_2$

Question 3:
This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed	(C) Multi-tailed	(D) left-tailed

Question 4:
The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{P_1 - P_2}{\sqrt{\dfrac{P_1(1-P_1)}{n_1} + \dfrac{P_2(1-P_2)}{n_2}}} \sim N(0,1)$	(B) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$
(C) $T = \dfrac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim N(0,1)$	(D) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$

Question 5:
With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic is....

(A) Reject H_0 if $Z_c <$ -1.645	(B) Reject H_0 if $Z_c >$ 1.96
(C) Reject H_0 if $Z_c >$ 1.645 or $Z_c < -1.645$	(D) Reject H_0 if $Z_c >$ 1.645

Question 6:
A point estimation of the difference $\mu_1 - \mu_2$ is...

(A) -0.16	(B) -10	(C) -1.49	(D) 1.49

Question 7:
The standard deviation of the statistic $\overline{X}_1 - \overline{X}_2$ is given by the formula....

(A) $\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$	(B) $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$
(C) $S_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	(D) $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Question 8:
The value of the standard deviation of the statistic $\overline{X}_1 - \overline{X}_2$ equals...

(A) 0.274	(B) 0.0628	(C) 0.058	(D) 0.241

Question 9:
The computed value of our test statistic is....

(A) -23.726	(B) 23.726	(C) -5.438	(D) 5.936

Solution:

$$z_c = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{-1.49}{0.274} = -5.438$$

Question 10:
The decision would be to....

(A) Can not be determined
(B) Do not reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 11:
Suppose that in fact the mean daily consumption of regular-coffee drinkers is less than that of decaffeinated-coffee drinkers by 2.5 cups, then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision$(1-\alpha)$	(D) Correct decision$(1-\beta)$

Question 12:

Suppose that in fact the mean daily consumption of regular-coffee drinkers is greater than that of decaffeinated-coffee drinkers by 2.5 cups, then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C)) Correct decision$(1-\alpha)$	(D) Correct decision$(1-\beta)$

End of example 4