"From page 14 to 15"

discrete uniform Distribution

(1) X ~ discrete uniform (k=3)

$\therefore f(x) = \frac{1}{k} = \frac{1}{3}$, $x = 0,1,2$

[a] $p(X=1) = f(1) = \frac{1}{3}$

[b] $E(X) = \sum_{i=1}^{3} x_i \cdot f(x_i) = (0)(\frac{1}{3}) + (1)(\frac{1}{3}) + (2)(\frac{1}{3}) = \frac{1}{3} + \frac{2}{3} = 1$

[c] $E(X^2) = \sum_{i=1}^{3} (x_i)^2 f(x_i) = (0)(\frac{1}{3}) + (1)(\frac{1}{3}) + (2)^2(\frac{1}{3}) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$

$\therefore Var(X) = E(X^2) - [E(X)]^2 = \frac{5}{3} - (1) = \frac{5-3}{3} = \frac{2}{3}$

binomial distribution

(1) $p = \frac{4}{12} = \frac{1}{3}$, $q = 1-p = 1-\frac{1}{3} = \frac{2}{3}$, $n = 3$

[a]

$f(x) = \binom{3}{x} (\frac{1}{3})^x (\frac{2}{3})^{3-x}$, $x = 0,1,2,3$

[b]

[i] $f(0) = \binom{3}{0} (\frac{1}{3})^0 (\frac{2}{3})^3 = .2463$

[ii] $f(1) = \binom{3}{1} (\frac{1}{3})^1 (\frac{2}{3})^2 = \frac{4}{9} = .444$

[iii] $p(X \geq 1) = p(X=1) + p(X=2) + p(X=3)$

or

$= 1 - p(X<1)$

$= 1 - p(X=0)$

$= 1 - f(0)$

$= 1 - .2463 = .7037$

[c] $E(X) = np = 3(\frac{1}{3}) = 1$

[d] $Var(X) = npq = \frac{2}{3}$

(2) $p = .9 \Rightarrow q = 1-.9 = .1$, $n = 4$

$f(x) = \binom{4}{x} (.9)^x (.1)^{4-x}$, $x = 0,1,2,3,4$

[a] $p(X \geq 2) = p(X=2) + p(X=3) + p(X=4)$

or

$= 1 - p(X<2) = 1 - p(X \leq 1) = 1 - [p(X=1) + p(X=0)] = 1 - [f(1) + f(0)]$

$= 1 - [\binom{4}{1} (.9)^1 (.1)^3 + \binom{4}{0} (.9)^0 (.1)^4] = .9963$

[b] $E(X) = np = 4(.9) = 3.6$

(3) $n=10$, $p=.4$

$f(x)=\binom{10}{x}(.4)^x(.6)^{10-x}$, $x=0,1,2,3,4,5,6,7,8,9,10$

[1]

$E(X)=np=(.4)(10)=4$

[2] $\sigma_X^2=Var(X)=npq=n(.4)(.6)=2.4$

[3] $P(X=4)=f(4)=\binom{10}{4}(.4)^4(.6)^6=.2142$

[4] $P(X\geq 3)=P(X=3)+P(X=4)+P(X=5)+\cdots+P(X=10)$

or

$=1-P(X<3)$

$=1-P(X\leq 2)=1-[P(X=2)+P(X=1)+P(X=0)]$

$=1-[f(2)+f(1)+f(0)]$

[5] $P(X\leq 8)=P(X=8)+P(X=7)+\cdots+P(X=1)+P(X=0)$

or

$=1-P(X>8)$

$=1-P(X\geq 9)$

$=1-[P(X=9)+P(X=10)]$

$=1-[f(9)+f(10)]$

(4) $n=3$, $p=q=.5$

$f(x)=\binom{3}{x}(.5)^x(.5)^{3-x}$, $x=0,1,2,3$

[a] $P(X=0)=f(0)=\binom{3}{0}(.5)^0(.5)^3=(.5)^3$

[b] $P(X\leq 2)=P(2)+P(1)+P(0)$

or

$=1-P(X>2)$

$=1-P(X\geq 3)$

$=1-P(X=3)$

$=1-f(3)=1-\binom{3}{3}(.5)^3(.5)^0=1-.125=.875$

[c] $E(X)=np=3(.5)=1.5$

[d] $Var(X)=npq=n(.5)(.5)=.75$

(5) $n=5, p=.2, q=1-.2=.8$

[i] $f(x)=\binom{5}{x}(.2)^x(.8)^{5-x}$, $x=0,1,2,3,4,5$

[ii] $P(X=5)=f(5)=\binom{5}{5}(.2)^5(.8)^0=(.2)^5$

[iii]

$P(X\geq 1)=P(X=1)+P(X=2)+\cdots+P(X=5)$

or $=1-P(X<1)$

$=1-P(X\leq 0)$

$=1-f(0)=1-\binom{5}{0}(.2)^0(.8)^5=1-(.8)^5$

[iv] $E(X)=np=5(.2)=1$

[v] $Var(X)=5(.2)(.8)=.8$

(6) $n=7, p=.2, q=1-.2=.8$

$f(x)=\binom{7}{x}(.2)^x(.8)^{7-x}$, $x=0,1,2,3,4,5,6,7$

[a] $P(X\leq 2)=1-P(X>2)=1-P(X\geq 3)=1-[P(X=3)+P(X=4)+\cdots+P(X=7)]$

or $=P(X=2)+P(X=1)+P(X=0)$

$=f(2)+f(1)+f(0)$

[b] $P(X\geq 3)=P(X=3)+P(X=4)+\cdots+P(X=7)$

or $=1-P(X<3)$

$=1-P(X\leq 2)$

$=1-[P(X=2)+P(X=1)+P(X=0)]$

$=1-[f(2)+f(1)+f(0)]$

[c] $E(X)=np=7(.2)=1.4$

(7) السؤال مكرر

وهو موجود في صفحة 4 من مذكرة الاحتمالات لسؤال 13

(8) $p=.1, n=3$

$f(x)=\binom{x}{3}(.1)^x(.9)^{3-x}$, $x=0,1,2,3$

[a] $P(X=0)=f(0)=\binom{3}{0}(.1)^0(.9)^3=(.9)^3=.729$

[b] $P(X=1)=f(1)=\binom{3}{1}(.1)^1(.9)^2=.243$

(9) $X \sim Binomial(n,p)$, $E(X)=1$, $Var(X)=.75$

$\therefore P(X=1) = \binom{n}{x} p^x (1-p)^{n-x} = f(1)$

we want to find n and p as:

Find p and q : $E(X)=1 \Rightarrow np=1 \longrightarrow [1]$

$Var(X)=.75 \Rightarrow npq=.75 \longrightarrow [2]$

by put [1] in [2] we get:

$npq=.75 \Rightarrow (1)q=.75 \Rightarrow \boxed{q=.75}$

$\Rightarrow p=1-.75=.25 \Rightarrow \boxed{p=.25}$

Find n : $E(X)=1$

$\Rightarrow np=1 \Rightarrow (.25)n=1 \Rightarrow \frac{1}{4}n=1 \Rightarrow \boxed{n=4}$

$\therefore P(X=1) = f(1) = \binom{4}{1}(.25)^1(.75)^{4-1}$

(10) ولا حاجة

(11) $n=5$, $p=.75$, $q=.25$

$f(x) = \binom{5}{x}(.75)^x(.25)^{5-x}$, $x=0,1,2,3,4,5$

[1] $P(X=0) = f(0) = \binom{5}{0}(.75)^0(.25)^5 = .00098$

[2] $P(X=4) = f(4) = \binom{5}{4}(.75)^4(.25)^{5-4} = .3955$

[3] $P(X \geq 4) = 1 - P(X<4) = 1 - P(X \leq 5) = 1 - [P(X=5) + P(X=4) + \dots + P(X=0)]$

or $= P(X=4) + P(X=5)$

$= f(4) + f(5)$

$= .6328$

[4] $E(X) = np = 5(.75) = 3.75$