11.1 Single mean:

Q1 normal distribution

$\sigma^2 = (30)^2 \longrightarrow$ وهنا σ^2 بدلا من s^2 لأنه ذكر لنا نوع التوزيع مباشرة

the sample size $= n = 50$, $\bar{X} = 750$

① $\hat{\mu} = \bar{X} = 750$

② $\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $1-\alpha = .94 \Rightarrow \alpha = .06 \Rightarrow \frac{\alpha}{2} = .03$, $1-\frac{\alpha}{2} = .97$

$\therefore \mu \in (750) \pm z_{.97} \frac{30}{\sqrt{50}} \Rightarrow \mu \in (742.02, 757.4)$

Q2 we have normal distribution

$\sigma^2 = (2)^2$, $n = 49$, $\bar{X} = 4.5$

① $\bar{X} \sim N(\mu, \frac{\sigma^2}{n} = \frac{4}{49} = .0816 = (.2857))$

② $\hat{\mu} = \bar{X} = 4.5$

③ $\sigma_{\bar{X}} = \sqrt{.0816} = .2857$

④ $\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $1-\alpha = .95 \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = .025 = .475$

$\therefore \mu \in (3.94, 5.06)$

⑤ upper C.I $= 5.2 = \bar{X} + z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 4.5 + z_{1-\frac{\alpha}{2}} \frac{2}{\sqrt{49}}$

$\Rightarrow z_{1-\frac{\alpha}{2}} = 2.45$

so the lower C.I $= \bar{X} - z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 4.5 - (2.45) \frac{2}{\sqrt{49}} = 3.8$

⑥ $3.88 = \bar{X} - z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 4.5 - z_{1-\frac{\alpha}{2}} \frac{2}{\sqrt{49}} \Rightarrow z_{1-\frac{\alpha}{2}} = 2.17$

and the same thing if we use

$5.12 = \bar{X} + z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 4.5 + z_{1-\frac{\alpha}{2}} \frac{2}{\sqrt{49}} \Rightarrow z_{1-\frac{\alpha}{2}} = 2.17$

$z_{1-\frac{\alpha}{2}} = 2.17 \Rightarrow P(z < 2.17) = 1-\frac{\alpha}{2} \Rightarrow$

	.07
2.1	.9850

$\Rightarrow .9850 = 1-\frac{\alpha}{2} \Rightarrow \alpha = .03$

so the confidence level $= 1-\alpha = 1-.03 = .97 \Rightarrow 97\%$

⑦ $e = z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $1-\alpha = .95 \Rightarrow \alpha = .05 \Rightarrow z_{.475} = 1.96$

$\Rightarrow e = (1.96) \frac{2}{\sqrt{49}} = .56$

8) $n = \left(\frac{z_{1-\frac{\alpha}{2}}\,\sigma}{e}\right)^2 = \left(\frac{1.96(2)}{.1}\right)^2 = 1536.64 \approx 1537$

Q3 the data we have

3.4 , 4.8 , 3.6 , 5.6 , 3.7 , 4.4 , 5.2 , 4.8 , 3.3 $\Rightarrow n = 9 < 30$

$\bar{X} = 4.311$ ——— باستخدام الآلة الحاسبة فإننا نستطيع إيجاد :

$S = .8417$

and the distribution is normal and σ unknown

$1-\alpha = .99 \Rightarrow \alpha = .1 \Rightarrow \alpha/2 = .05 \Rightarrow 1-\frac{\alpha}{2} = .995 \Rightarrow t_{.995, 8} = 3.3554$

$\therefore \mu \in \bar{X} \pm t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$

$\Rightarrow \mu \in (3.37 , 5.25)$

Q4

the distribution is normal

$\sigma^2 = (5)^2$

1) $e = 2$, $1-\alpha = .9 \Rightarrow \alpha = .1$

$\Rightarrow n = \left(\frac{z_{.95}(5)}{2}\right)^2 = \left(\frac{(1.645)(5)}{2}\right)^2 = 16.912 \approx 17$

2) $n = 49$, $\bar{X} = 390$

(i) $\hat{\mu} = \bar{X} = 390$

(ii) $1-\alpha = .95 \Rightarrow 1-\frac{\alpha}{2} = .975 \Rightarrow z_{.975} = 1.96$

$\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$\Rightarrow \mu \in 390 \pm (1.96)\frac{5}{\sqrt{49}} \Rightarrow \mu \in (388.6 , 391.4)$

Q5 $\sigma^2 = (1.4) \longrightarrow$ اعتبر σ وليس S لأنه ذكر بعبارة "random Variable" والتوزيع بحد ذاته

$e = .3$

$1-\alpha = .96 \Rightarrow \alpha = .04 \Rightarrow z_{.98} = 2.05$

		.05	.06
2.0	.9798	.9800	.9803
		.0002	.0003

$\therefore n = \left(\frac{z_{.98}(1.4)}{e}\right)^2 = 92$

Q6 $n=36$ from normal distribution

$\bar{X}=15.2$, $S^2=9$

ⓐ $\hat{\mu}=\bar{X}=15.2$

ⓑ لأن σ غير معلوم , $n \geq 30$

$$\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \quad , 1-\alpha=.95 \Rightarrow \alpha=.05 \Rightarrow 1-\frac{\alpha}{2}=.975$$

$z_{.975}=1.96$

$$\therefore \mu \in 15.2 \pm (1.96)\frac{3}{\sqrt{36}} = 15.2 \pm (.98)$$

$\Rightarrow \mu \in (14.22, 16.18)$

مقارنة

Q7 $n=10$

the data is 7.25, 8.5, 5, 6.75, 8, 5.25, 10.5, 8.5, 6.75, 9.25

the distribution is normal

من البيانات = نستخرج، ... $S=1.744$, $S^2=3.0435$, $\bar{X}=7.605$ من الآلة الحاسبة

ⓐ $\bar{X}=7.605$, $S^2=3.0435$

ⓑ $\hat{\mu}=\bar{X}=7.605$

ⓒ لأن σ غير معلوم , $10=n<30$

$$\mu \in \bar{X} \pm t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$$

$1-\alpha=.8 \Rightarrow \alpha=.2 \Rightarrow \frac{\alpha}{2}=.1 \Rightarrow 1-\frac{\alpha}{2}=.9 \Rightarrow t_{.9,9}=1.383$

$\therefore \mu \in (6.8422, 8.3677)$

Q8 the distribution is normal , $\sigma^2=.584$

ⓘ $n=49$, $\bar{X}=5.47$ ⓐ $\hat{\mu}=\bar{X}=5.47$

ⓑ $\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}=\sqrt{\frac{\sigma^2}{n}}=\sqrt{\frac{.584}{49}}=.1091$

ⓒ $\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $1-\frac{\alpha}{2}=.95$

$\Rightarrow \mu \in 5.47 \pm (1.645)(\frac{.7641}{7})$

ⓘⓘ $e=.15$, $1-\alpha=.95 \Rightarrow z_{.975} \Rightarrow n=\left(\frac{z_{.975}\,\sigma}{e}\right)^2=\left(\frac{(1.96)(.7641)}{.15}\right)^2$

Q9 the distribution is normal , $\sigma^2 = (6.8)^2$

$n = 20$, $\bar{X} = 72.8$

ⓐ $\hat{\mu} = \bar{X} = 72.8$

ⓑ $e = 3.4$

$$n = \left(\frac{z_{1-\frac{\alpha}{2}}(\sigma)}{e} \right)^2 = \left(\frac{(1.96)(6.8)}{3.4} \right)^2 = 15.3216 \approx 16$$

ⓒ ⓓ $1-\alpha = .98 \Rightarrow 1 - \frac{\alpha}{2} = .99 \Rightarrow z_{.99} = 2.33$

$$\mu \in \bar{X} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 72.8 \pm (2.33) \frac{6.8}{\sqrt{20}}$$

$\therefore$ the lower bound is $72.8 - (2.33) \frac{6.8}{\sqrt{20}} = 69.26$

$\therefore$ the upper bound is $72.8 + (2.33) \frac{6.8}{\sqrt{20}} = 76.34$

11.2 Two means :

Q1 ⓘ the distribution is normal , $\sigma_1^2 = (6.8)^2$, $n_1 = 20$, $\bar{X}_1 = 72.8$

① $e = 3.4$, $1-\alpha = .95 \Rightarrow 1 - \frac{\alpha}{2} = .975 \Rightarrow z_{.975} = 1.96$

$$n = \left(\frac{(z_{.975})(6.8)}{e} \right)^2 = 15.366 \approx 16$$

② ③ $1-\alpha = .98 \Rightarrow \alpha = .02 \Rightarrow 1 - \frac{\alpha}{2} = .99$

$z_{.99} \Rightarrow p(z < z_{.99}) = .99 \quad \therefore z_{.99} = 2.33$

	.02		.03
2.3	.9898	.9900	.9901
	.0002		.0001

$\therefore \mu \in \bar{X}_1 \pm z_{.99} \frac{\sigma_1}{\sqrt{n_1}} \Rightarrow \mu \in 72.8 \pm (2.33) \frac{6.8}{\sqrt{20}}$

$\therefore$ the lower bound is $72.8 - (2.33) \frac{6.8}{\sqrt{20}} = 72.8 - 3.5428 \approx 69.257$

$\therefore$ the upper bound is $72.8 + (2.33) \frac{6.8}{\sqrt{20}} = 72.8 + 3.5428 = 76.342$

ⓘⓘ the distribution is normal , $\sigma_2^2 = (6.8)^2$, $n_2 = 25$, $\bar{X}_2 = 64.4$

① ② $\mu_1 - \mu_2 \in (\bar{X}_1 - \bar{X}_2) \pm z_{.99} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = (8.4) \pm (2.33)(2.04)$

$\therefore$ the lower bound is $(8.4) - (2.33)(2.04) = 3.6468$

$\therefore$ the upper bound is $(8.4) + (2.33)(2.04) = 13.1532$

Q2 two samples from normal distribution which they are indep.

① $(\widehat{\mu_1-\mu_2}) = \bar{X}_1 - \bar{X}_2 = 10.5 - 10.5$

②

$\mu_1-\mu_2 \in (\bar{X}_1-\bar{X}_2) \pm S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\; t_{n_1+n_2-2,\,1-\frac{\alpha}{2}}$ (استخدمنا هذه الصيغة لان σ_1^2, σ_2^2 غير معلومتان)

$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2} = 4.5416$

$1-\alpha = .95 \Rightarrow 1-\frac{\alpha}{2} = .975 \Rightarrow t_{.975,24} = 2.0639$

$\therefore \mu_1-\mu_2 \in (-10.68, 11.68)$

Q3

the two samples form normal distribution which they are indep.

①

$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2} = 18.494$

② $(\widehat{\mu_1-\mu_2}) = \bar{X}_1-\bar{X}_2 = 82.63 - 80.04 = 2.59$

③④ $1-\alpha = .9 \Rightarrow 1-\frac{\alpha}{2} = .95 \Rightarrow t_{.95,10} = 1.8125$

$\mu_1-\mu_2 \in (\bar{X}_1-\bar{X}_2) \pm t_{.95,10}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = (2.59) \pm (1.8125)(2.5181) = (2.59) \pm (4.564)$

$\therefore$ the lower bound is -1.97

$\therefore$ the upper bound is 7.15

Q4 the two samples from normal distribution which they are indep.

$n_1 = 10$, $n_2 = 12$

① $(\widehat{\mu_1-\mu_2}) = \bar{X}_1-\bar{X}_2 = 37000 - 38000 = -1000$

②

$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2} = 5595 \Rightarrow S_p = 74.8$

$\therefore \mu_1-\mu_2 \in (\bar{X}_1-\bar{X}_2) \pm S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\; t_{n_1+n_2-2,\,1-\frac{\alpha}{2}}$, $t_{.95,20} = 2.086$

$\in (-1000) \pm (74.8)(.4282)(2.086) = (-1000) \pm (66.813)$

$\therefore \mu_1-\mu_2 \in (-1066.813, -933.186)$

Q5 ⓐ $\widehat{(\mu_1-\mu_2)} = \bar{X}_1 - \bar{X}_2 = 3$

ⓑ $\mu_1-\mu_2 \in (-2.46, 8.46)$

Q6 $\bar{X}_A = 5.7$, $S_A^2 = 1.324$, $n_A = 6$

$\bar{X}_B = 5.4$, $S_B^2 = 1.6$, $n_B = 6$

يُظهر هذه النتائج باستخدام العينة لابد
من استخدام الآلة الحاسبة

ⓐ $\mu_A \in \bar{X}_A \pm t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$

$\therefore \mu_A \in (4.462, 6.938)$

ⓑ

$\mu_A - \mu_B \in (\bar{X}_A - \bar{X}_B) \pm S_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \; t_{n_A+n_B-2, 1-\frac{\alpha}{2}}$, $S_p^2 = 1.446$

$\therefore \mu_A - \mu_B \in (-1.939, 2.539)$

Q7

$n_A = 8$, $\bar{X}_A = 1.25$, $S_A = .1509$

$n_B = 8$, $\bar{X}_B = 1.38$, $S_B = .1432$

where the two samples from normal distribution

ⓐ $\widehat{(\mu_A - \mu_B)} = \bar{X}_A - \bar{X}_B = 1.25 - 1.38 = -.13$

ⓑ

$\mu_A - \mu_B \in (\bar{X}_A - \bar{X}_B) \pm S_p \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \; t_{.95, 14}$

11.3 Single proportion

Q1

$n = 200$, $n_1 = 15$ (which they are smoke)

① $\hat{p}_{smoke} = \frac{n_1}{n} = \frac{15}{200} = .075 \Rightarrow 1-\hat{p} = .925$

② $p \in \hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (.075) \pm (1.96) \sqrt{\frac{(.075)(.925)}{200}}$

$\therefore p \in (.0563, .0936)$

Q2 $n=500$, $n_1=150$ (which are female)

① $\hat{p} = \frac{n_1}{n} = \frac{150}{500} = .3$

②③ $p \in \hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\therefore p \in (.2663 , .3337)$

Q3 $n=500$, $n_1=114$ (heated by oil)

① $\hat{p} = \frac{114}{500} = .228$

② $p \in \hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, $1-\alpha = .98 \Rightarrow 1-\frac{\alpha}{2} = .99 \Rightarrow z_{.99} = 2.33$

Q4 $n=1200$, $n_1=50$

① $\hat{p} = \frac{50}{1200} = .0417$

②③ $p \in (.0304 , .0530)$

Q5 $n=250$, $n_1=120$

ⓐ $\hat{p} = \frac{120}{250} = .48$

ⓑ $p \in (.4181 , .5419)$

Q6 تكرار السؤال ٤

11.4 two proportion : ملاحظة

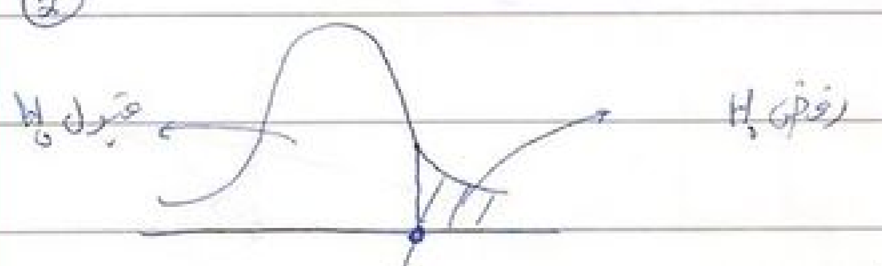
"hypotheses testing"

12.1 Single mean:

Q1 we have normal distribution, $\sigma^2 = (2)^2$, $n = 49$, $\bar{X} = 4.5$, $\alpha = .05$

① $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{4.5 - 5}{2/7} = -1.75$

②



$z_{1-.05} = z_{.95} = \dfrac{1.64 + 1.65}{2} = 1.645$

reject H_0 in $(1.645, \infty)$

③ as $Z = -1.75 < 1.645 \Rightarrow$ so we accept H_0

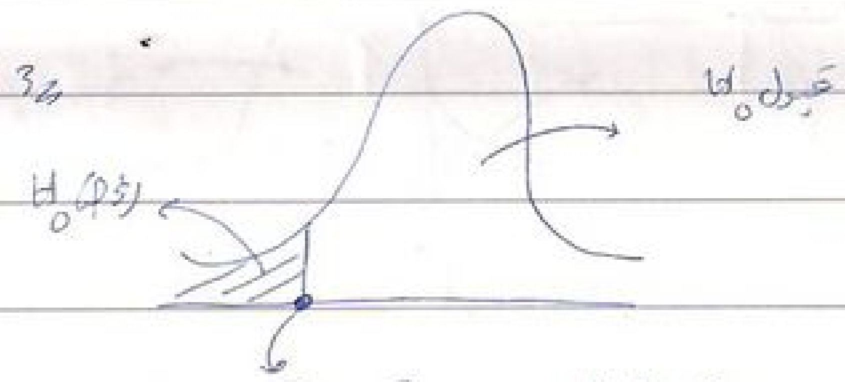
Q2

we have a normal dis. with $\sigma^2 = (30)^2$, $n = 50$, $\bar{X} = 750$, $\alpha = .05$

1/ $H_0: \mu = 740$ vs $H_1: \mu < 740$

2/ $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{750 - 740}{30 / \sqrt{50}} = 2.357$

3/



$z_\alpha = z_{.05} = -1.645$

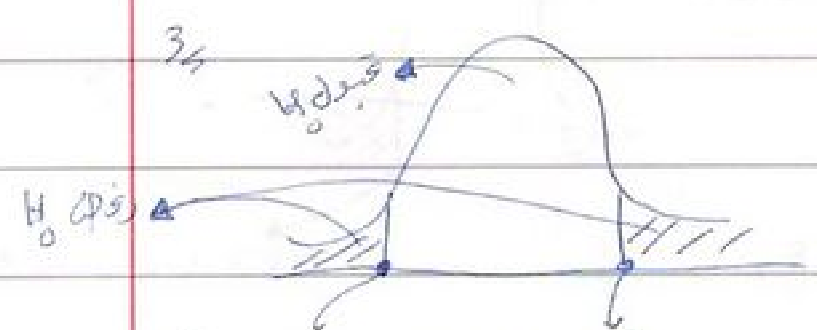
4/ as $Z = 2.357 > -1.645$ so we accept H_0

Q3 normal dis., $n = 20$, $\bar{X} = 655$, $s = 27$

1/ $H_0: \mu = 660$ vs $H_1: \mu \neq 660$ at $\alpha = .02$

2/ $T = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}} = \dfrac{655 - 660}{27 / \sqrt{20}} = -.828$

3/



$t_{.01, 19} = -2.539$ $t_{.99, 19} = 2.539$

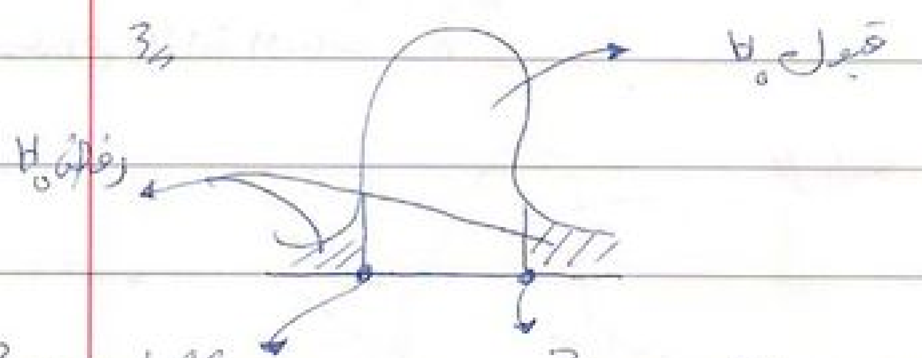
4/ as $T = -.828 > -2.539$ so we accept H_0

Q4 normal dis. , $n=36$, $\bar{X}=15.2$, $S^2=(3)^2$, $\alpha=.05$

1/ $H_0: \mu=15$ Vs $H_1: \mu\neq15$

2/ $Z=\dfrac{\bar{X}-\mu_0}{S/\sqrt{n}}=\dfrac{15.2-15}{3/\sqrt{36}}=.40$

3/



$Z_{\frac{\alpha}{2}}=-1.96$ $Z_{1-\frac{\alpha}{2}}=Z_{.975}=1.96$

4/ as $Z=.4<1.96$ so we accept H_0

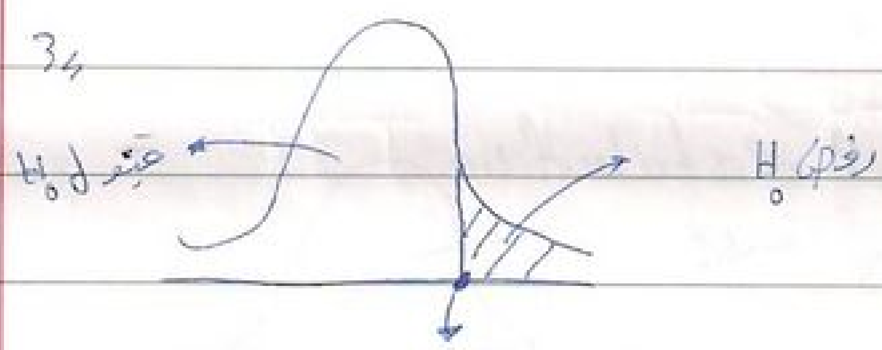
Q5 $n=10$, normal dis.

7.25 , 8.5 , 5 , 6.75 , 8 , 5.25 , 10.5 , 8.5 , 6.75 , 9.25 $\Rightarrow$ $\bar{X}=7.575$ $S=1.7242$

1/ $H_0: \mu=7.5$ Vs $H_1: \mu>7.5$, $\alpha=.02$

2/ $T=\dfrac{\bar{X}-\mu_0}{S/\sqrt{n}}=\dfrac{7.575-7.5}{1.7242/\sqrt{10}}=.1375$

3/



$t_{1-\alpha}=t_{.80}$ $\longrightarrow$ هذه القيمة ليست موجودة في جدول T

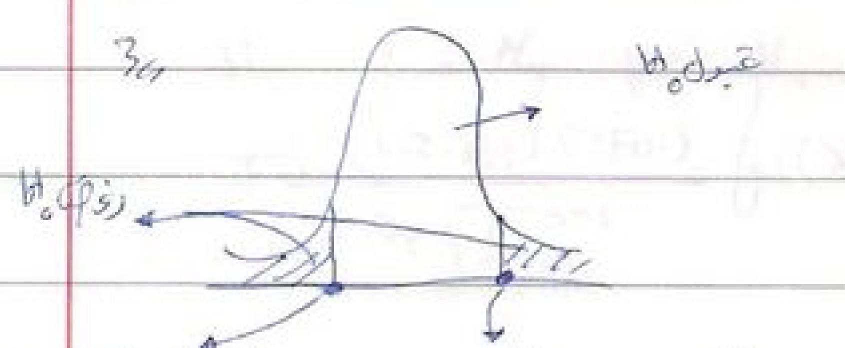
4/ if $T<t_{.80}$ so we accept H_0

Q6 normal dis. $\sigma^2=.584$, $n=49$, $\bar{X}=5.47$

1/ $H_0: \mu=5.5$ Vs $H_1: \mu\neq5.5$, $\alpha=.01$

2/ $Z=\dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}=\dfrac{5.47-5.5}{\sqrt{\frac{.584}{49}}}=-.2748$

3/



$Z_{\frac{\alpha}{2}}=-2.575$ $Z_{1-\frac{\alpha}{2}}=Z_{.995}=\dfrac{2.57+2.58}{2}=2.575$

4/ as $Z=-.2748>-2.575$ so we accept H_0

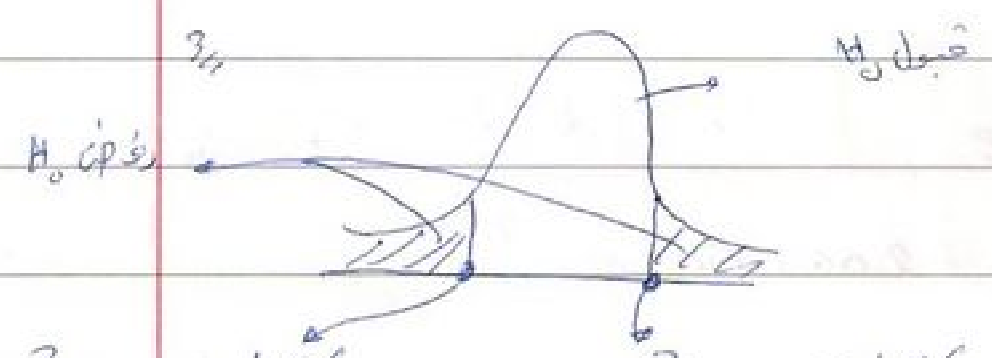
Q7 normal dis. $\sigma^2 = (6.8)^2$, $\bar{X} = 70$, $n = 20$

1// $H_0 : \mu = 72.8$ vs $H_1 : \mu \neq 72.8$ $\alpha = .05$

2//

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = 1.84$$

3//



$z_{.025} = -1.96$ $\qquad$ $z_{.975} = 1.96$

4// as $Z = 1.84 < 1.96$ so we accept H_0

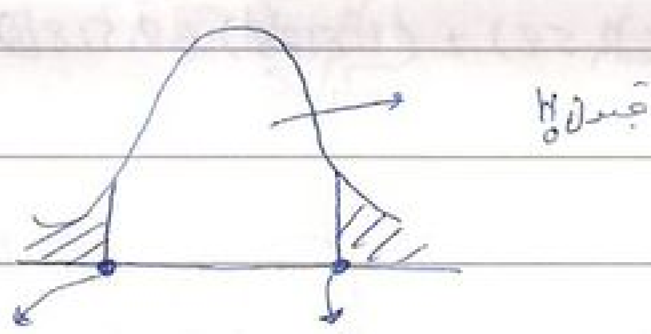
12.2 two means

Q1 1// $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \neq \mu_2$

2// $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ where $\mu_1 - \mu_2 = 0$, $S_p^2 = 4.542$

$= .896$

3// , $\alpha = .05$



$t_{.025, 24}$ $\qquad$ $t_{.975, 24}$

4// we reject H_0 if $T > t_{.975, 24}$ or $T < t_{.025, 24}$

So as $T < t_{.975, 24}$ so we accept H_0

Q2 ① $S_p^2 = 18.494$

② ③ ④

1// $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 \neq \mu_2$

2// $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = +1.024$

3// $\alpha = .1$

so we accept H_0 in $(-1.812, 1.812)$

$t_{\frac{\alpha}{2}, n_1+n_2-2}$ $\qquad$ $t_{1-\frac{\alpha}{2}, n_1+n_2-2} = 1.812$

$= -1.812$

4// as $T < 1.812$ so we accept H_0

Q3 $n_1 = 10$, $n_2 = 12$

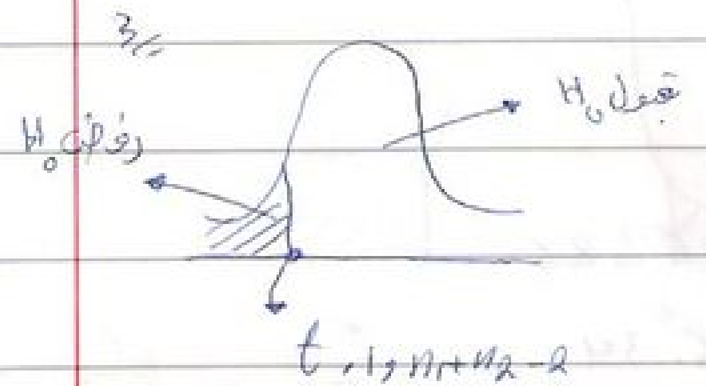
$\overline{X_1} = 37000$, $S_1 = 5100$

$\overline{X_2} = 38000$, $S_2 = 6000$

1, $H_0 : \mu_1 = \mu_2$ Vs $H_1 : \mu_1 < \mu_2$, $\alpha = .1$

2, $T = \dfrac{(\overline{X_1} - \overline{X_2}) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, $S_p = 79.8$

3,



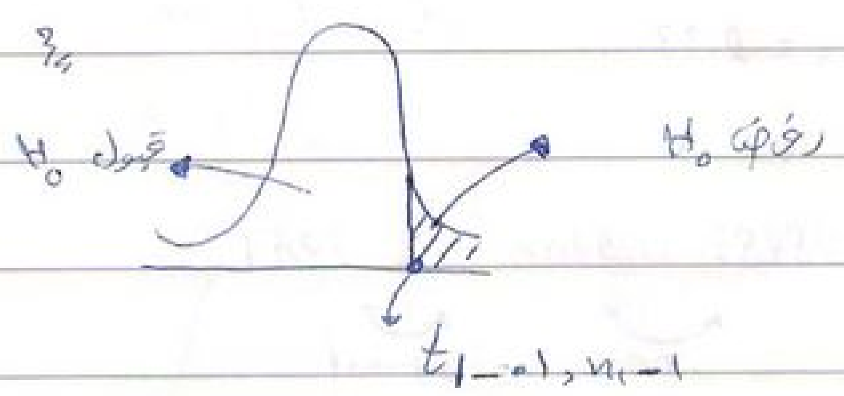
4, if $T < t_{.1, n_1+n_2-2}$ we reject H_0
other wise we accept H_0

Q4

(a) 1, $H_0 : \mu_1 = 5.6$ Vs $H_1 : \mu > 5.6$ $\alpha = .1$

2, $T = \dfrac{\overline{X_1} - \mu_0}{S_1 / \sqrt{n_1}}$, $\mu_0 = 5.6$, $\underline{\overline{X_1}, S_1}$ $\longrightarrow$ يحسبان عن طريق الآلة الحاسبة لـ type A

3,



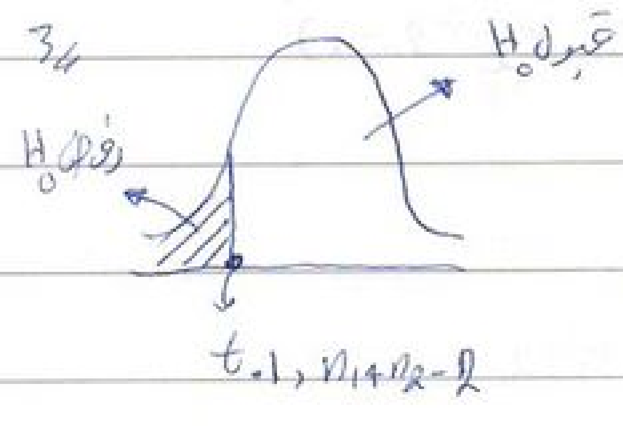
4, if $T > t_{1-.1, n_1-1}$ then we reject H_0
other wise we accept H_0

(b) 1, $H_0 : \mu_1 = \mu_2$ Vs $H_1 : \mu_1 < \mu_2$, $\alpha = .1$

2,

$T = \dfrac{(\overline{X_1} - \overline{X_2}) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, S_p , $\underline{S_2, \overline{X_2}}$ $\longrightarrow$ type B يحسبان عن طريق الآلة الحاسبة لـ

3,



4, if $T < t_{.1, n_1+n_2-2}$ then we reject H_0
other wise we accept H_0.

Q5 normal dis. $n_1 = 100$, $\bar{X}_1 = 2.7$, $\sigma_1^2 = .36$ $\alpha = .05$

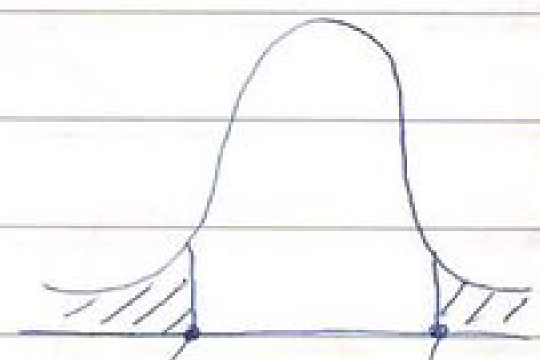
$n_2 = 100$, $\bar{X}_2 = 2.57$, $\sigma_2^2 = .4$

1/ $H_0 : \mu_1 = \mu_2$ VS $H_1 : \mu_1 \neq \mu_2$

2/

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 1.835$$

3/



$z_{\frac{\alpha}{2}} = z_{.025}$ $z_{1-\frac{\alpha}{2}} = z_{.975}$

4/ we reject H_0 if $Z > z_{.975}$ or $Z < z_{.025}$

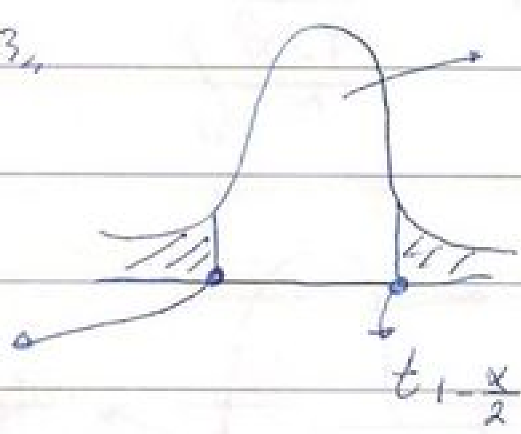
So as $Z < z_{.975}$ so we accept H_0

Q6 1/ $H_0 : \mu_1 = \mu_2$ VS $H_1 : \mu_1 \neq \mu_2$ $\alpha = .05$

2/

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

3/



$t_{\frac{\alpha}{2}, n_1+n_2-2}$ $t_{1-\frac{\alpha}{2}, n_1+n_2-2}$

4/ if $T > t_{1-\frac{\alpha}{2}, n_1+n_2-2}$ or $T < t_{\frac{\alpha}{2}, n_1+n_2-2}$

then we reject H_0 ,

other wise we accept H_0

12.3 Single proportion:

Q1 $n = 500$, $n_1 = 150$

① ② ③

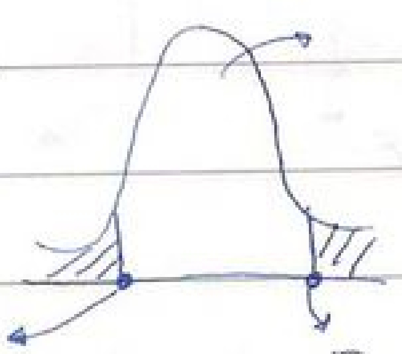
1, $H_0 : p = .25$ vs $H_1 : p \neq .25$ $\alpha = .1$

2,

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}}, \quad P_0 = .25, \quad \hat{P} = \frac{150}{500}$$

$= 2.43475 \simeq 2.4348$

3,



$z_{\frac{\alpha}{2}} = -1.645$ $z_{1-\frac{\alpha}{2}} = 1.645$

So we accept H_0 in $(-1.645, 1.645)$

4, as $Z > z_{1-\frac{\alpha}{2}} = 1.645$ so we reject H_0

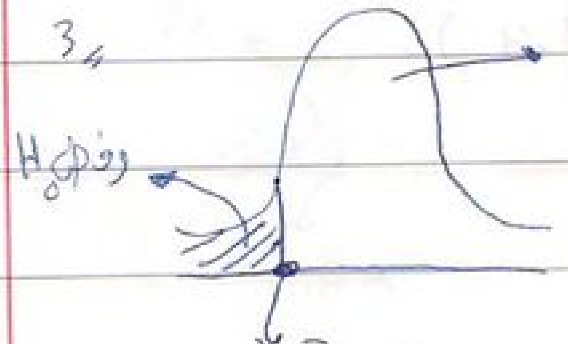
Q2 $n = 500$, $n_1 = 114$, $\alpha = .01$

1, $H_0 : p = .2$ vs $H_1 : p < .2$

2,

$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}}, \quad \hat{P} = \frac{114}{500}, \quad P_0 = .2$$

3,



$z_{.01}$

4, if $Z < z_{.01}$ then we reject H_0
other wise we accept if H_0

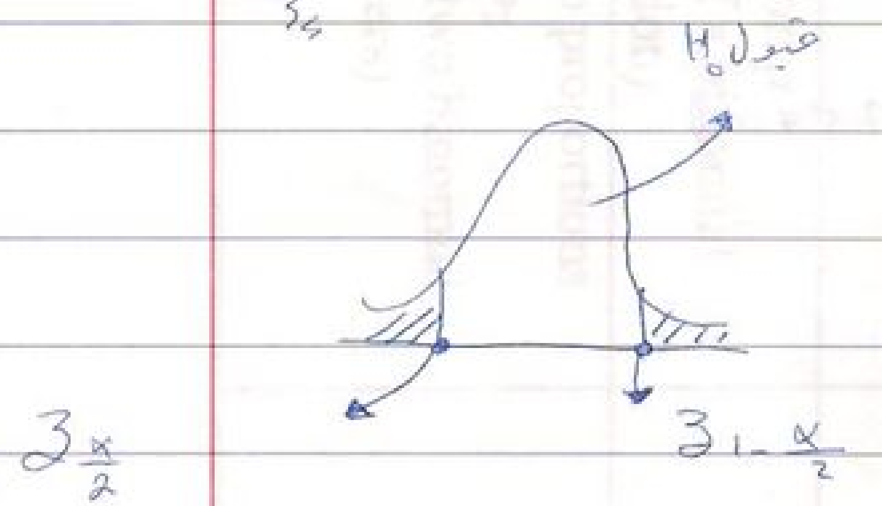
Q3 ملغي

Q4 $n = 250$, $n_1 = 120$

1" $H_0: p = .5$ Vs $H_1: p \neq .5$, $\alpha = .1$

2" $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$, $p_0 = .5$

3"



4" if $Z > z_{1-\frac{\alpha}{2}}$ or $Z < z_{\frac{\alpha}{2}}$
so we reject H_0, other wise we accept it H_0

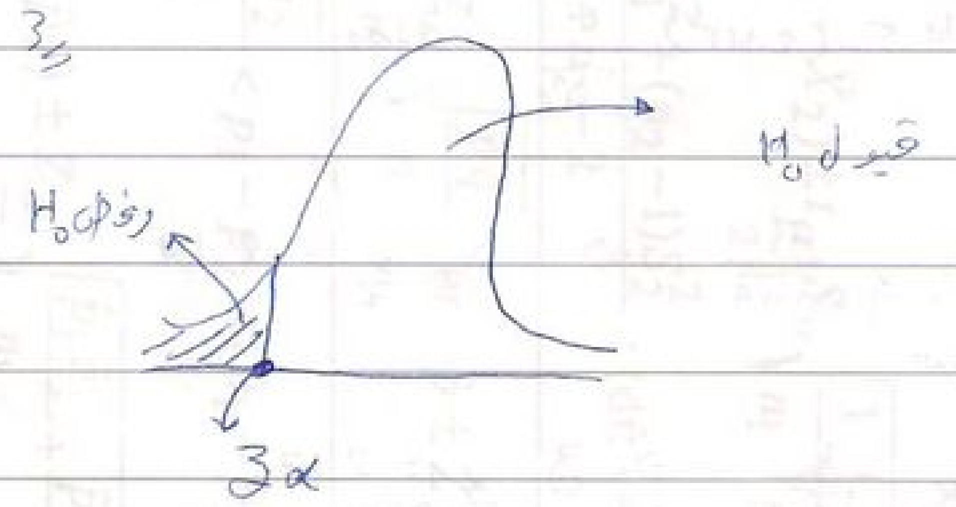
Q5 $n = 1200$, $n_1 = 50$

1" $H_0: P = .05$ Vs $H_1: p < .05$, $\alpha = .1$

2"

$Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$, $p_0 = .05$

3"



4"

if $Z < z_\alpha$ then we reject H_0
other wise we accept H_0

12.4 Two proportions: ملغي

Summary of Confidence Interval Procedures

Problem Type	Point Estimate	Two-Sided 100(1-α)% Confidence Interval
Mean μ variance σ^2 known, normal distribution, or any distribution with n>30	$\overline{X}$	$\overline{X}-Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}<\mu<\overline{X}+Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$ or $\overline{X}\pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
Mean μ normal distribution, variance σ^2 unknown	$\overline{X}$	$\overline{X}-t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}<\mu<\overline{X}+t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ or $\overline{X}\pm t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ (df: $\nu=n-1$)
Difference in two means μ_1 and μ_2 variances σ_1^2 and σ_2^2 are known, normal distributions, or any distributions with n_1, n_2>30	$\overline{X}_1-\overline{X}_2$	$(\overline{X}_1-\overline{X}_2)-Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}<\mu_1-\mu_2<(\overline{X}_1-\overline{X}_2)+Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$ or $(\overline{X}_1-\overline{X}_2)\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$
Difference in means μ_1 and μ_2 normal distributions, variances $\sigma_1^2=\sigma_2^2$ and unknown	$\overline{X}_1-\overline{X}_2$	$(\overline{X}_1-\overline{X}_2)-t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}<\mu_1-\mu_2<(\overline{X}_1-\overline{X}_2)+t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ or $(\overline{X}_1-\overline{X}_2)\pm t_{\frac{\alpha}{2}}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$; (df: $\nu=n_1+n_2-2$)
Proportion p (or parameter of a binomial distribution)	$\hat{p}$	$\hat{p}-Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}<p<\hat{p}+Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ or $\hat{p}\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; $\hat{q}=1-\hat{p}$
Difference in two proportions p_1-p_2 (or difference in two binomial parameters)	$\hat{p}_1-\hat{p}_2$	$(\hat{p}_1-\hat{p}_2)-Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}<p_1-p_2<(\hat{p}_1-\hat{p}_2)+Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$ or $(\hat{p}_1-\hat{p}_2)\pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$

Summary of Hypotheses Testing Procedures

Null Hypothesis	Test Statistic	Alternative Hypothesis	Critical Region (Rejection Region)
$H_o: \mu = \mu_o$ variance σ^2 is known, Normal distribution, or any distribution with n>30	$Z = \dfrac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$	$H_1: \mu \neq \mu_o$	$\|Z\| > Z_{\alpha/2}$
		$H_1: \mu > \mu_o$	$Z > Z_\alpha$
		$H_1: \mu < \mu_o$	$Z < -Z_\alpha$
$H_o: \mu = \mu_o$ Normal distribution, variance σ^2 is unknown	$T = \dfrac{\overline{X} - \mu_o}{S / \sqrt{n}}$; df: $\nu = n-1$	$H_1: \mu \neq \mu_o$	$\|T\| > t_{\alpha/2}$
		$H_1: \mu > \mu_o$	$T > t_\alpha$
		$H_1: \mu < \mu_o$	$T < -t_\alpha$
$H_o: \mu_1 = \mu_2$ Variances σ_1^2 and σ_2^2 are known, Normal distributions, or any distributions with n_1, n_2 >30	$Z = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$H_1: \mu_1 \neq \mu_2$	$\|Z\| > Z_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$Z > Z_\alpha$
		$H_1: \mu_1 < \mu_2$	$Z < -Z_\alpha$
$H_o: \mu_1 = \mu_2$ Normal distributions, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$T = \dfrac{\overline{X}_1 - \overline{X}_2}{S_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$; df: $\nu = n_1 + n_2 - 2$ $S_p^2 = [(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2]/(n_1 + n_2 - 2)$	$H_1: \mu_1 \neq \mu_2$	$\|T\| > t_{\alpha/2}$
		$H_1: \mu_1 > \mu_2$	$T > t_\alpha$
		$H_1: \mu_1 < \mu_2$	$T < -t_\alpha$
$H_o: p = p_o$ Proportion or parameter of a binomial distribution p ($q = 1 - p$)	$Z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}} = \dfrac{X - np_o}{\sqrt{np_o q_o}}$	$H_1: p \neq p_o$	$\|Z\| > Z_{\alpha/2}$
		$H_1: p > p_o$	$Z > Z_\alpha$
		$H_1: p < p_o$	$Z < -Z_\alpha$
$H_o: p_1 = p_2$ Difference in two proportions or two binomial parameters $p_1 - p_2$	$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ $\hat{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$	$H_1: p_1 \neq p_2$	$\|Z\| > Z_{\alpha/2}$
		$H_1: p_1 > p_2$	$Z > Z_\alpha$
		$H_1: p_1 < p_2$	$Z < -Z_\alpha$