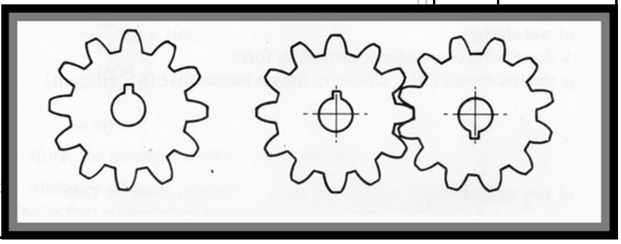
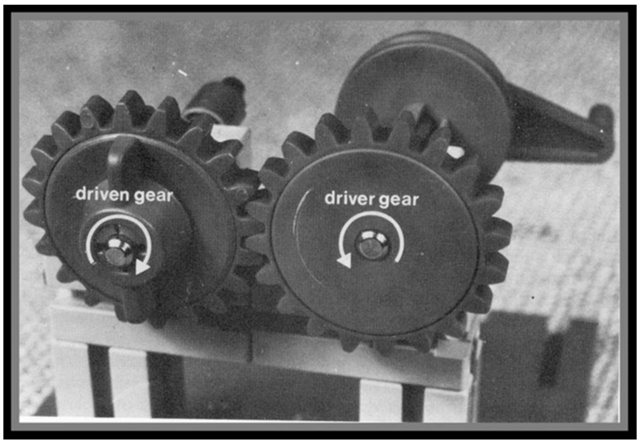
CH#13 Gears-General

- A toothed wheel that engages another toothed mechanism in order to change the speed or direction of transmitted motion
- The gear set transmits rotary motion and force.
- Gears are used in groups of two or more. A group of gears is called a gear train.
- The gears in a train are arranged so that their teeth closely interlock or mesh. The teeth on meshing gears are the same size so that they are of equal strength. Also, the spacing of the teeth is the same on each gear.



ME-305 Machine Design

Drive and Driven Gears



Larger Gear can be called Wheel and Smaller Gear can be Called Pinion

ME-305 Machine Design II

Why Gears?

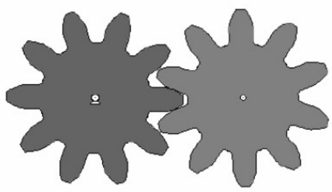
- The designer is frequently confronted with the problem of transferring power from one shaft to another while maintaining a definite ratio between the velocities of rotation of the shafts (to get the required torque).
- Transmission of specified angular motion from one shaft to another

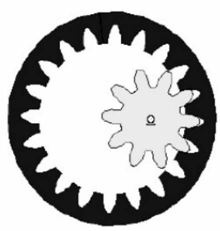
Video

ME-305 Machine Design II

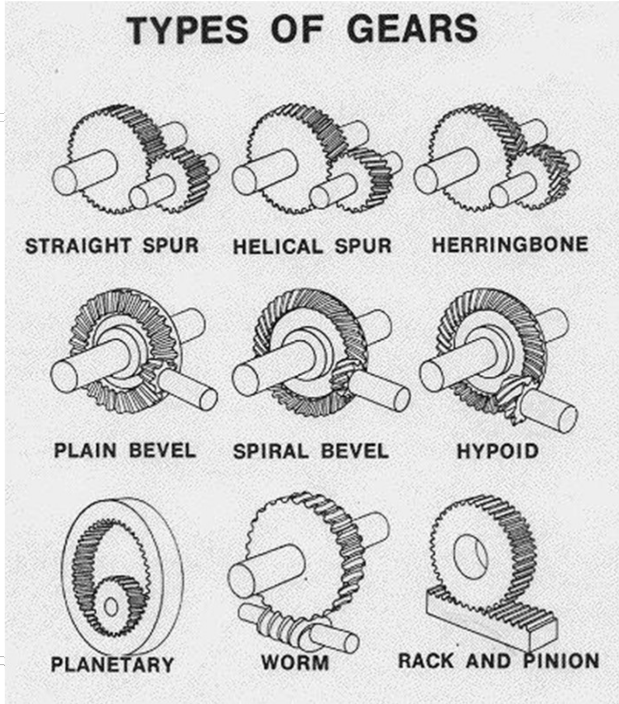
Gear Types

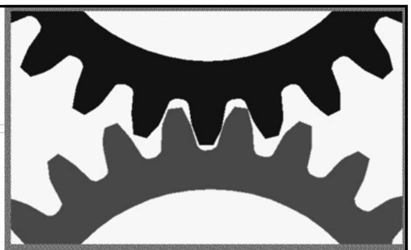
- Internal and External Gears
- Two Gears together are called a gearset





Created for "Design of Machinery, 3rd ed." by R. L. Norton and "The Multimedia Handbook of Mechanical Devices" by S. Wang Software copyright © 2004 by The McGraw-Hill Companies, Inc. All rights reserved.





Spur Gears

- Straight teeth
- Low Cost
- High efficiency (98-99%)

FIGURE 9-14

5 Machine Design II

Helical Gears

- Slanted teeth to smooth contact
- Axis can be parallel or crossed
- Efficiency of 96-98% for parallel and 50-90% for crossed

FIGURE 9-16
Parallel axis helical gears
Courtesy of Martin Sprocket and Gear Co., Arlington, TX

FIGURE 9-17
Crossed axis helical gears
Courtesy of the Boston Gear Division of IMO Industries, Quincy, MA

Crossed Helical Gears

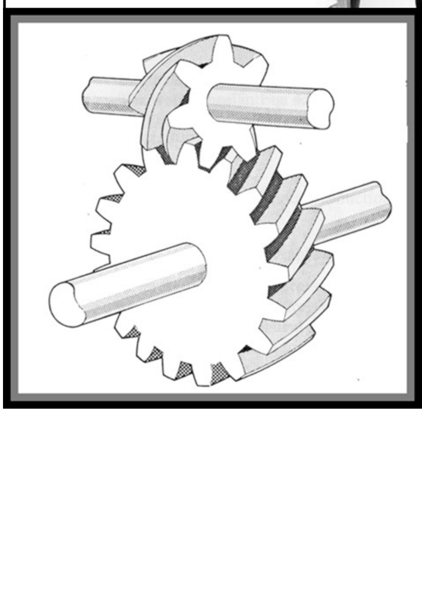


FIGURE 9-17
Crossed axis helical gears
Courtesy of the Boston Gear Division of IMO Industries, Quincy, MA

Herringbone Gears

- Eliminate the thrust force
- 95% efficient
- Very expensive

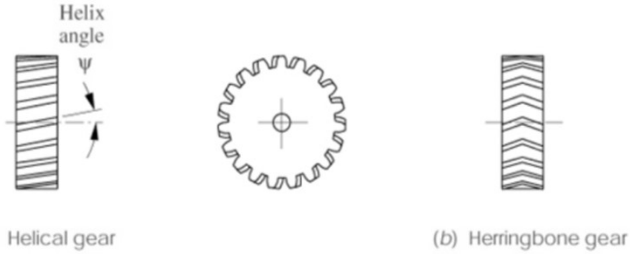
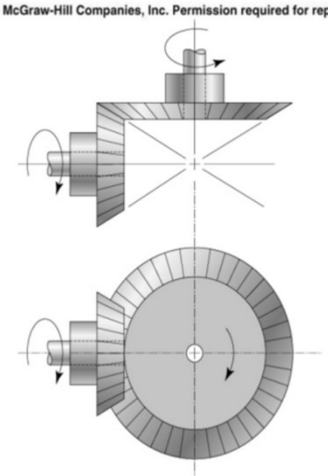
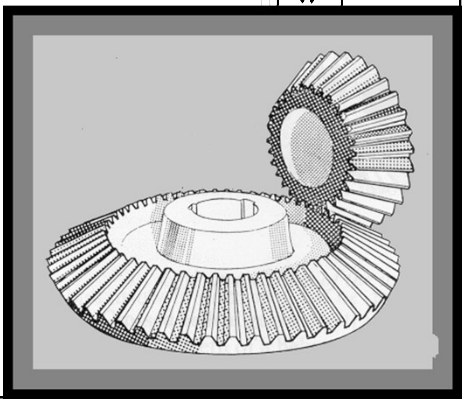


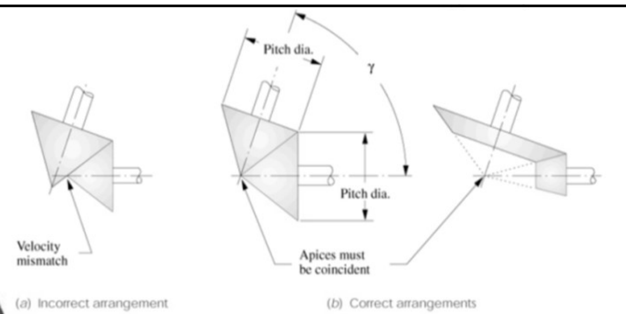
FIGURE 9-15
A helical gear and a herringbone gear

Bevel Gear





ME-305 Mac



E 9-20
jears are based on rolling cones

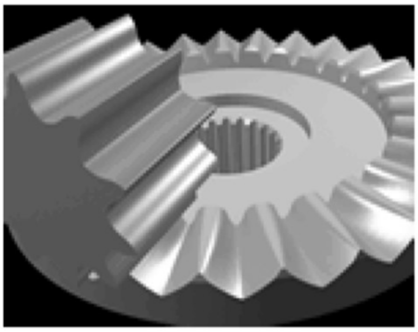


FIGURE 9-21
Straight bevel gears

Spiral Bevel Gears

ME-305 Machine Design II

Rack and Pinion

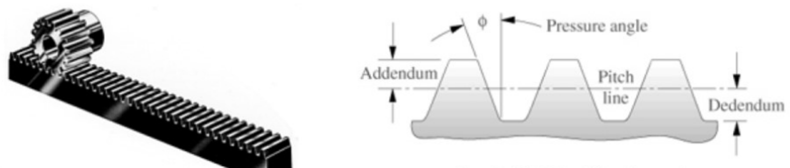
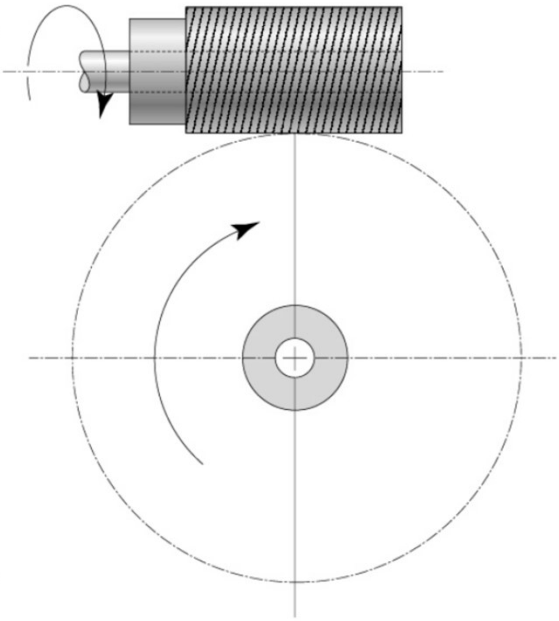
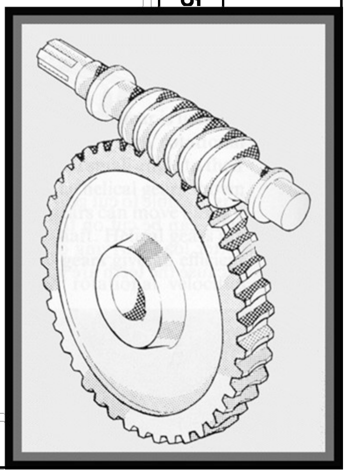


FIGURE 9-19

A rack and pinion *Photo courtesy of Martin Sprocket and Gear Co., Austin, TX*

Worm Gear

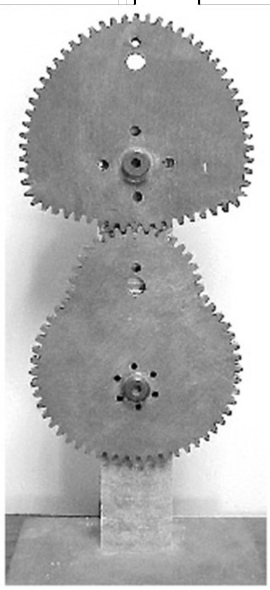




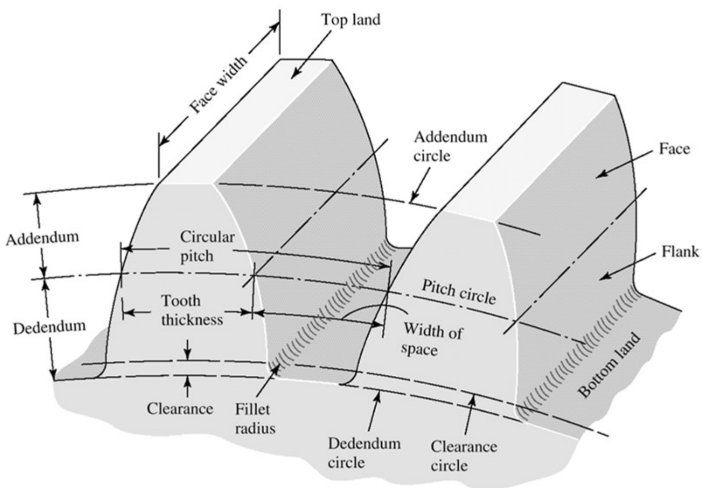
ME-305

Noncircular Gears

- The velocity ratio of non circular gears in not constant
- The purpose is to provide a time-varying output function in response to a constant velocity input



13-2 Nomenclature



ME-305 Machine Design II

13-2 Nomenclature

- Pitch Circle diameter (d)
- Circular pitch (p)= distance between two adjacent teeth = $\pi d/N = \pi m$ (N is the number of teeth)
- Module (m)=d/N
- Diameteral pitch (P) = N/d
- Addendum = a = 1/P
- Dedendum = b = 1.25/P
- Tooth thickness = t = p/2 = $\pi d/2N = \pi m/2$

ME-305 Machine Design II

13-3 Conjugate Action

Mating gear teeth acting against each other to produce rotary motion are similar to cams. When the tooth profiles, or cams, are designed so as to produce a constant angular-velocity ratio during meshing, these are said to have *conjugate action.*

Involute Profile is one of the solutions to achieve Conjugate Action.

Machine D

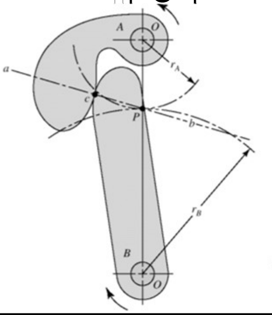
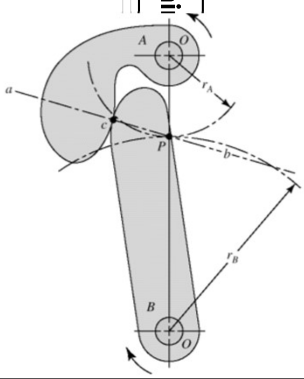


Figure 13–7

Cam *A* and follower *B* in contact. When the contacting surfaces are involute profiles, the ensuing conjugate action produces a constant angular-velocity ratio.

13-4 Involute Properties

- The forces at any instant are directed along the common normal ab to the two curves
- The line ab, representing the direction of action of the forces, is called *the line of action*
- The line of action will intersect the line of centres O-O at some point P
- *The angular velocity ratio between the two arms is inversely proportional to their radii to the point P* ***(Fundamental law of Gearing)***
- Circles drawn through point P from each centres are called *pitch circles*, and the radius of each circle is called the *pitch radius*
- Point P is called the *pitch point*



ME-305 Machi

13-5 Fundamentals

- Why gear?
- Suppose we wish to design a speed reducer such that the input speed is 1800 rev/min and output speed is 1200 rev/min.
- Fundamental law of gearing

$$V = |r_1\omega_1| = |r_2\omega_2| \qquad \left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$$

- This is a ratio of 2:3; the gears pitch diameters would be in the same ratio, for example a 200mm pinion driving a 300mm gear (or 400mm pinion with 600mm gear and so on)
- Various dimensions found in gearing are always based on the pitch circles

ME-305 Machine Design II

13-5 Fundamentals

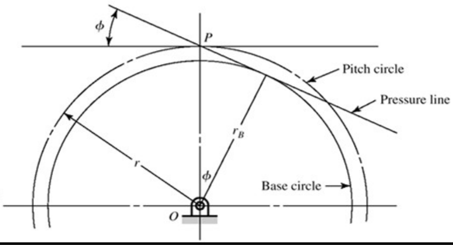
ME-305 Machine Desigr

- Suppose we specify that an 18-tooth pinion is to mesh with a 30-tooth gear and that the diametral pitch of the gearset is to be 2 teeth per inch

$$\frac{N_1}{d_1} = P, \qquad \frac{N_2}{d_2} = P$$

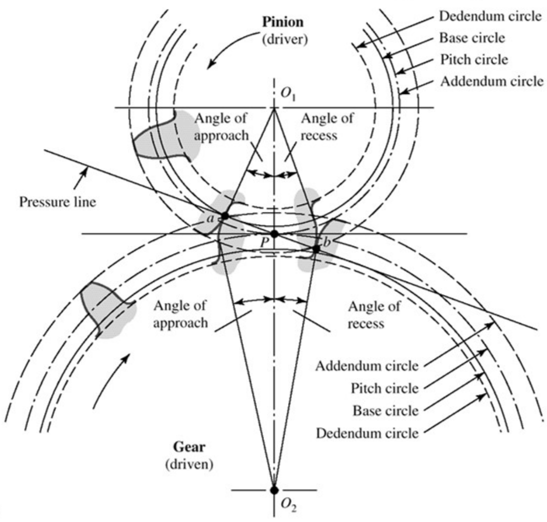
- Pressure angle ϕ represents the line of action (direction of force on the tooth) and is equal to 20°, 25° and rarely $14\frac{1}{2}^o$.
- Base circle "r_b" is related to the pressure angle as;

$$r_b = r\cos\emptyset$$



13-5 Fundamentals

- Angle of approach
- Angle of recess



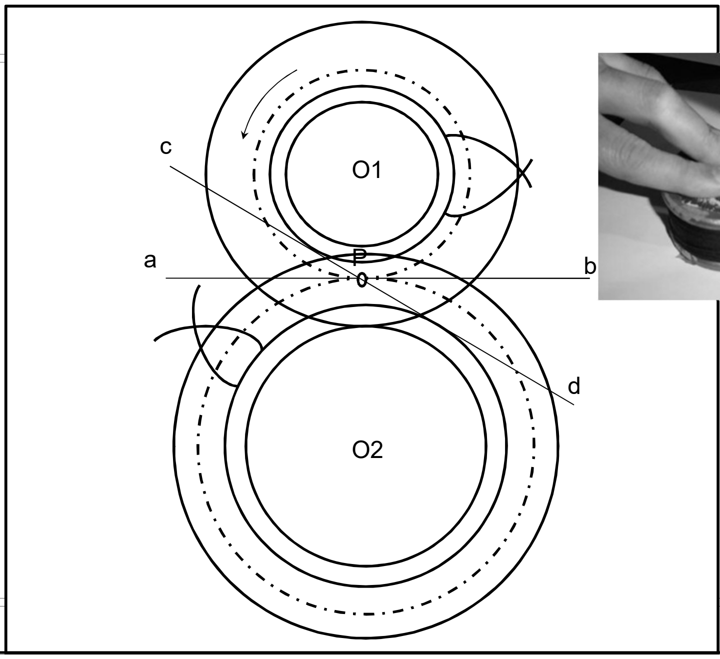
Example 13-1

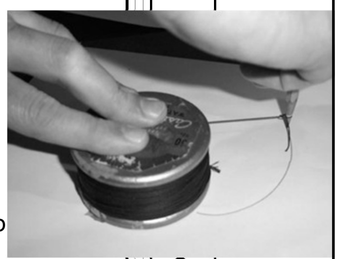
A gearset of a 16-tooth driving a 40-tooth gear. The module is 5mm and the addendum and dedendum are 1/P and 1.25/P respectively. The gears are cut using 20^o pressure angle

a) Compute
   i. the circular pitch, $(p = \pi m)$
   ii. the centre distance, and $(m = \frac{d}{N})$
   iii. the radii of the base circles. $(r_b = rcos\emptyset)$

b) In mounting the gears, the centre distance was incorrectly made 6mm larger. Compute the new values of pitch-circle radii and pressure angle and comment.

How to draw (Graphical method)



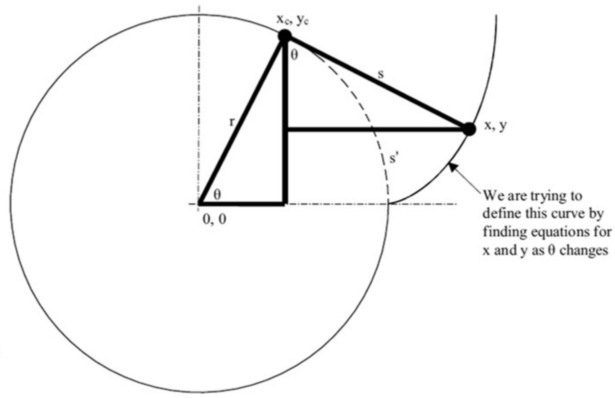


e Design II

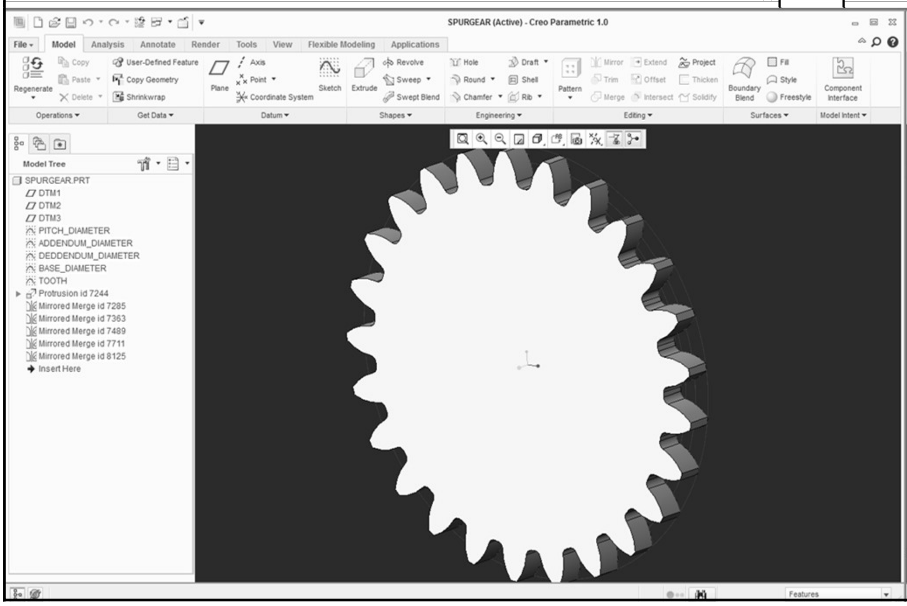
How to draw (Analytical method)

$$x = r\cos\theta + (c \times \sin\theta)$$
$$y = r\sin\theta - (c \times \cos\theta$$

Where “c” is the circumference of the circle



How to draw (Analytical method)



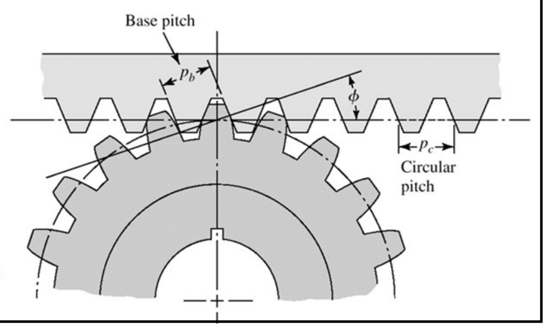
Fundamentals of internal gears

- A rack can be imagined as a spur with an infinitely large pitch diameter
- The sides of the involute teeth on the rack are straight lines making an angle to the line of centres equal to the pressure angle

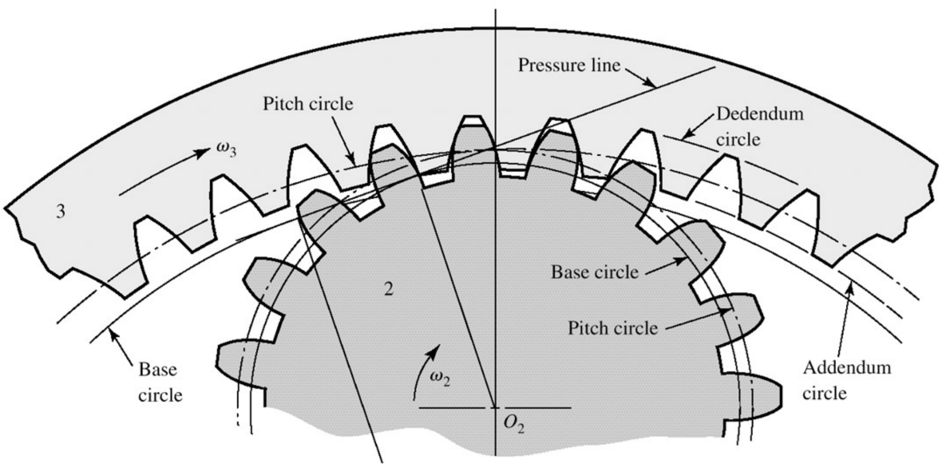
ME-305 Machine

$$p_b = p_c \cos \phi$$

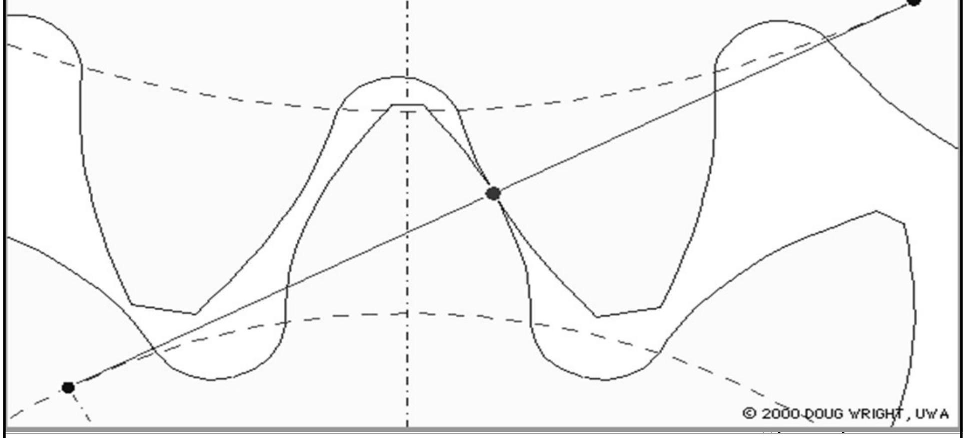
where p_b is the base pitch.



Fundamentals of internal gears



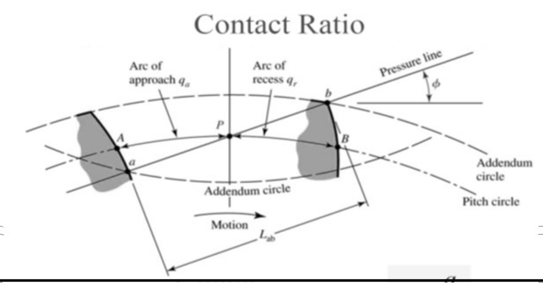
Contact Ratio



The animation shows clearly :
1. The contact point marching along the line of action
2. Guaranteed tooth tip clearance due to the addendum exceeding the base circle
3. A significant gap between the non-drive face of a pinion tooth and the adjacent wheel tooth
4. How load is transferred from one pair of contacting teeth to the next as rotation proceeds

13-6 Contact Ratio

- Initial contact occurs at “a”
- Addendum circle intersects the pressure line
- Final contact is at “b”
- Arc of approach = q_a and Arc of recess = q_r
- If q_t=p, then only one tooth and its space will occupy the entire arc



ME-305 Machine Design II

13-6 Contact Ratio

ME-305 Machine Design II

- i.e. as soon as one tooth starts contact another ends.
- When q_t = 1.2p means one tooth start contact at "a" before the other end contact.
- Two teeth are in contact for short period of time.
- This is define as $m_c = \frac{L_{ab}}{p\cos\emptyset}$ and should not be less than 1.2
- m_c can be represented in terms of pitch circle radii as;

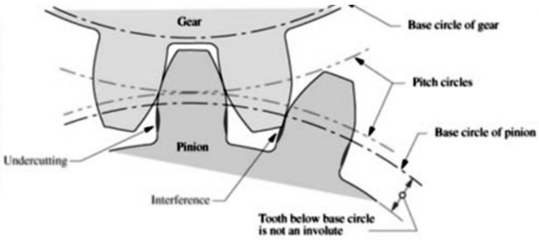
$$m_c = \frac{\sqrt{(r_{a1})^2 - (r_{b1})^2} + \sqrt{(r_{a2})^2 - (r_{b2})^2} - C\sin\emptyset}{p\cos\emptyset}$$

13-7 Interference and Undercutting

ME-305 Machine Design II

- If there are too few pinion teeth, then the gear cannot turn
- Also if the contact starts bellow the base circle where the profile is non-involute
- Interference is automatically removed by the generation process by introducing *Undercutting*

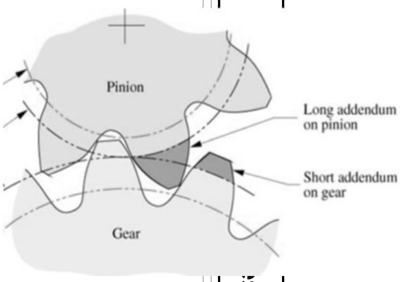
For no undercutting	
ϕ (deg)	Min # teeth
14.5	23
20	13
25	9



13-7 Interference and Undercutting

- Undercutting should be avoided
- Two ways to avoid undercutting
  1. Increase addendum on pinion and decrease on gear
  2. Use minimum number of teeth such that;

$$N_P = \frac{2k}{(1+2m)sin^2\emptyset}\left(m + \sqrt{m^2 + (1+2m)sin^2\emptyset}\right)^*$$



- Where k=1 for full depth, 0.8 for stub teeth (Refer to topic 13-12 for different depths) and *m* is module
- If *m=4mm* and ϕ *=20º* , then

$$N_P = 15.4 \approx 16\ teeth$$

* Robert Lipp, *"Avoiding tooth interference in Gears,"* Machine Design, Vol. 54, No. 1, 1982, pp. 122-124

sign II

13-8 The Forming of Gear teeth

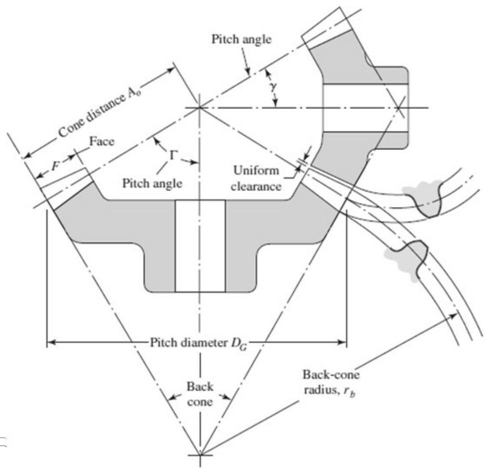
Reading Assignment

- Gear teeth can be manufactured with large number of ways
  1. Sand casting
  2. Shell moulding
  3. Investment casting
  4. Centrifugal casting
  5. Permanent-mould casting
  6. Powder metallurgy process
  7. Extrusion
  8. Milling
  9. Shaping
  10. Hobbing *(Video)*

ME-305 Machine Design II

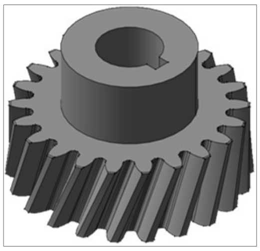
13-9 Straight Bevel Gears

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P}$$



13-10 Parallel Helical Gears

in Fig. 13–22. If a piece of paper cut in the shape of a parallelogram is wrapped around a cylinder, the angular edge of the paper becomes a helix. If we unwind this paper, each point on the angular edge generates an involute curve. This surface obtained when every point on the edge generates an involute is called an *involute helicoid.*



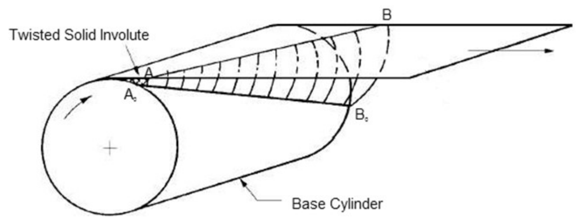


Fig. 6-2 Generation of the Helical Tooth Profile

13-10 Parallel Helical

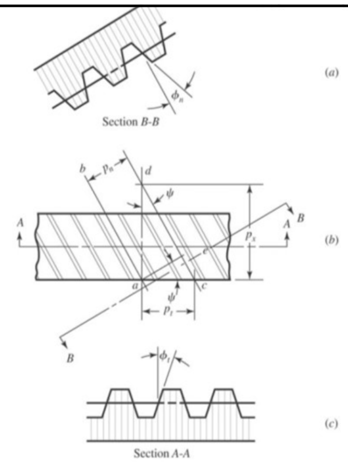
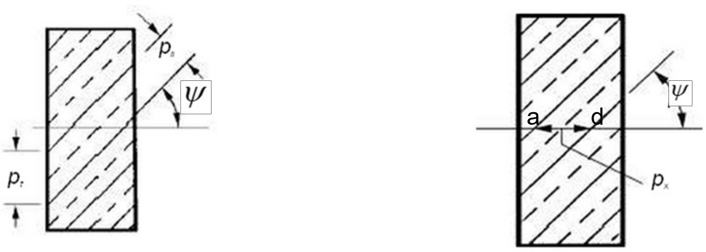


Figure 13–23 represents a portion of the top view of a helical rack. Lines *ab* and *cd* are the centerlines of two adjacent helical teeth taken on the same pitch plane. The angle ψ is the *helix angle*. The distance *ac* is the *transverse circular pitch* p_t in the plane of rotation (usually called the *circular pitch*). The distance *ae* is the *normal circular pitch* p_n and is related to the transverse circular pitch as follows:

$$p_n = p_t \cos \psi \qquad (13\text{–}12)$$

13-10 Parallel Helical Gears



The distance *ad* is called the *axial pitch* p_x and is related by the expression

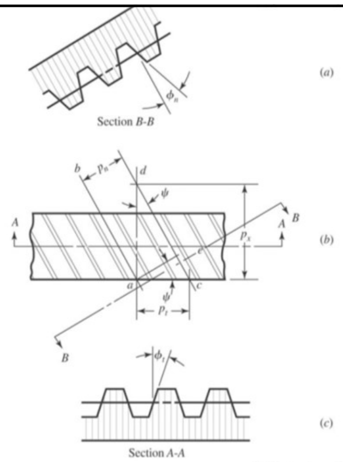
$$p_x = \frac{p_t}{\tan \psi}$$

Since $p_n P_n = \pi$, the *normal diametral pitch* is

$$P_n = \frac{P_t}{\cos \psi}$$

ME-305 Machine Design II

13-10 Parallel Helical (



The pressure angle ϕ_n in the normal direction is different from the pressure angle ϕ_t in the direction of rotation, because of the angularity of the teeth. These angles are related by the equation

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t}$$

The helix angle is the same on each gear, but one gear must have a right-hand helix and left-hand helix.

Interference in Helical Gear

Just as with spur gears, helical-gear teeth can interfere. Equation (13–15) can be solved for the pressure angle ϕ_t in the tangential (rotation) direction to give

$$\phi_t = \tan^{-1}\left(\frac{\tan \phi_n}{\cos \psi}\right)$$

The smallest tooth number N_P of a helical-spur pinion that will run without interference[3] with a gear with the same number of teeth is

$$N_P = \frac{4k \cos \psi}{6 \sin^2 \phi_t}\left(1 + \sqrt{1 + 3 \sin^2 \phi_t}\right) \quad (a)$$

Example 13-2

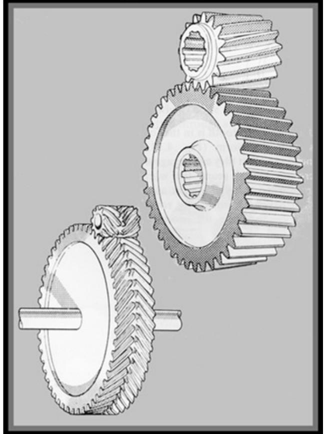
A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse module of 5.0mm, and has 18 teeth. Find

(a) The pitch diameter $(d = Nm_t)$

(b) The transverse, normal and axial pitches $(p_t = \pi m_t,\ p_n = p_t cos\psi,\ p_x = \frac{p_t}{tan\psi})$

(c) Normal module $(m_n = m_t cos\psi)$

(d) The transverse pressure angle $(cos\psi \quad \frac{tan\phi_n}{tan\phi_t})$



13-12 Tooth Systems (Reading Assignment)

Table 13–1

Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

[3]Standardized by the American Gear Manufacturers Association (AGMA). Write AGMA for a complete list of standards, because changes are made from time to time. The address is: 1500 King Street, Suite 201, Alexandria, VA 22314; or, www.agma.org.

ME-305 Machine Design II

13-13 Gear trains

ME-305 Machine Design II

- Consider a pinion 2 driving a gear 3. The speed of the driven gear is

$$n_3 = \left| \frac{N_2}{N_3} n_2 \right| = \left| \frac{d_2}{d_3} n_2 \right|$$

Where n = revolution or rev/min
N = number of teeth
d = pitch diameter

- Equation applies to any gearset no matter whether the gears are spur, helical, bevel, or worm
- The absolute-value signs are used to permit complete freedom in choosing positive and negative directions
- In the case of spur and parallel helical gears, the directions ordinarily corresponds to the right-hand rule and are positive for counter clockwise rotation

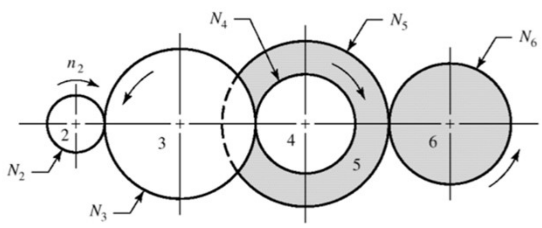
13-13 Gear trains…

ME-305 Machine Design II

- The gear train shown is made up of five gears. The speed of gear 6 is

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6} n_2$$

- Gear 3 is an idler, that its tooth numbers cancel in equation, and hence its effect is only changing the direction of rotation of gear 6
- Gear 2, 3, and 5 are drivers, while 3, 4, and 6 are driven members



13-13 Gear trains…

- Train value "*e*" can thus be defined as;

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

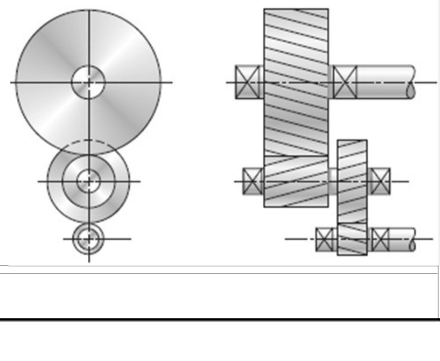
- Pitch diameters can also be used in the above equation
- When the above equation is used for spur gears, positive "*e*" shows that the last gear rotates in the same sense as the first.
- Also it can be written that

$$n_L = e n_F$$

- Where n_L is the speed of the last gear in the train and n_F is the speed of the first

13-13 Gear trains…

- As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears.
- Greater ratios can be obtained in less space and with fewer dynamic problems by compounding additional pairs of gears.
- A two-stage compound gear train, such as shown in figure, can obtain a train value of up to 100 to 1.

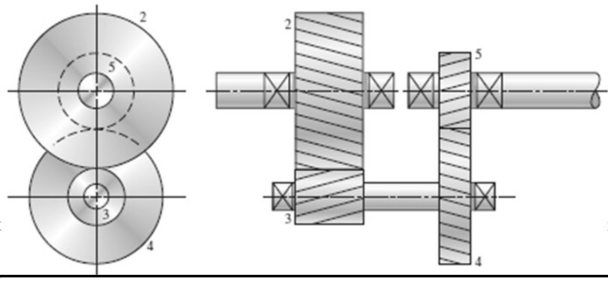


13-13 Gear trains…

- It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line.
- This configuration is called a ***compound reverted gear train.***
- Reference to figure, the distance constraint is

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

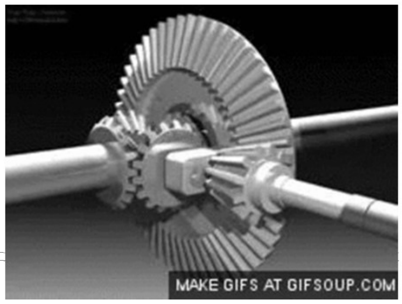
$$N_2 + N_3 = N_4 + N_5$$

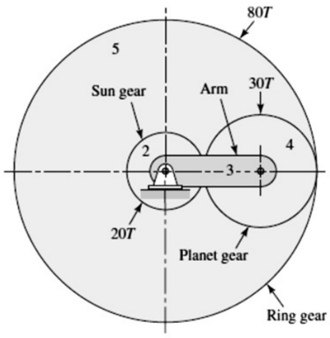


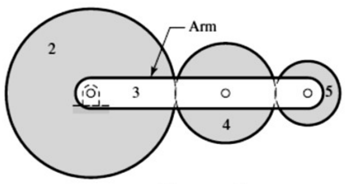
13-13 Gear trains…

Planetary or epicyclic gear train

- A gear train in which one gear axis rotates about other.
- Consist of
  - Sun gear
  - An arm and
  - Planet gear
- 2 dof







Example 14-3

- A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

$$16\sqrt{30} = 87.64 \doteq 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

Example 14-4

- A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Example 13-5

- A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

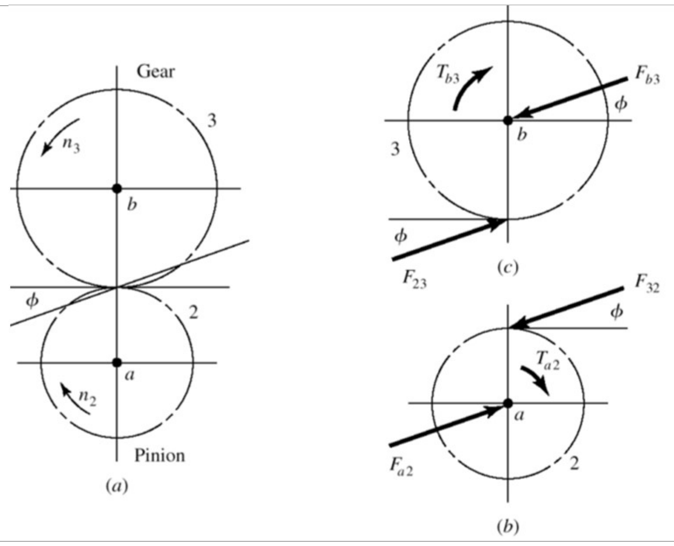
13-14 Force Analysis

ME-305 Machine Design II

Notation to be used

- Input gear will be designated as 2 and then gears will be numbered successively 3,4 etc. until the last gear in the train is arrived
- There may be several shafts involved, and usually one or two gears are mounted on each shaft as well as other elements; designates the shafts using lowercase letters of the alphabet, a, b, c, etc.
- Forces exerted by gear 2 on 3 will be F_{23}
- Superscript notations will be used to indicate radial and tangential directions and coordinate directions

13-14 Force Analysis-Spur Gear



ME-305 Machine Design II

13-14 Force Analysis-Spur Gear…

- We define

$$W_t = F_{32}^t$$

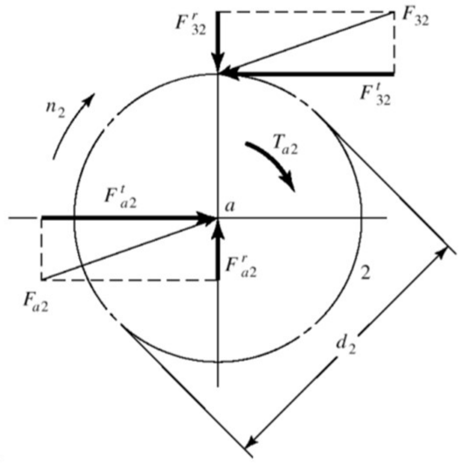
- W_t is the transmitted load
- The torque is

$$T = \frac{d}{2} W_t \text{ (N-mm)}$$

- Where $T = T_{a2}$ and $d = d_2$
- If $V = \pi d n$ is the pitch line velocity, then power is defined as;

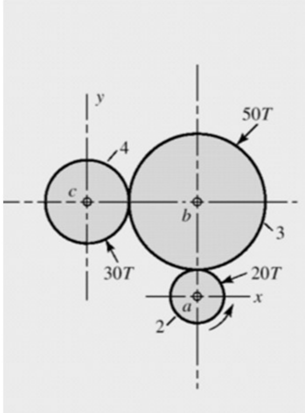
$$H = \frac{W_t V}{60000} \Rightarrow W_t = \frac{60000H}{\pi d n}$$

  - W_t is in kN
  - H is in kW
  - d is in mm
  - n is in rev/min



Example 13.6

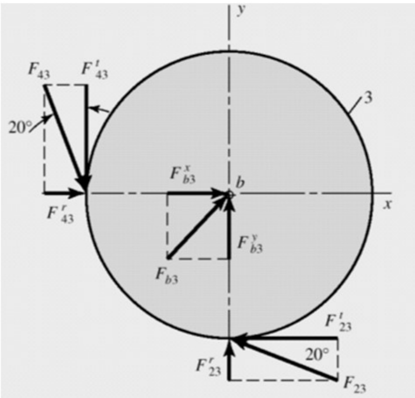
Pinion 2 in figure runs at 1750 rpm and transmits 2.5kW to idler gear 3. The teeth are cut on 20° full-depth system and have module of m=2.5mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.



(a)

ME-305 Machine Design II

Example 13.6



(b)

ME-305 Machine Design II

Solution

<u>Solution procedure</u>

1. Calculate d
2. Calculate $W_t = \frac{60000H}{\pi d n}$
3. $W_t = F^t_{23}$
4. $F^r_{23} = F^t_{23} \cos\phi$
5. Find F_{23}
6. Repeat step 1 to 5 for other gear
7. Calculate shaft reactions by $\Sigma F=0$

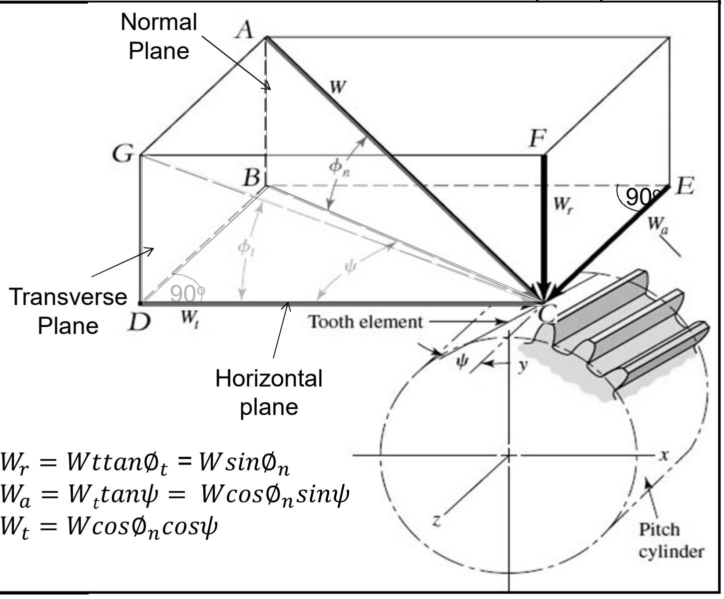
ME-305 Machine Design II

13-16 Force Analysis-Helical Gear

Horizontal plane= *BCD*
Transverse plane= *CDG*
Normal plane= *ABC*

From which;

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

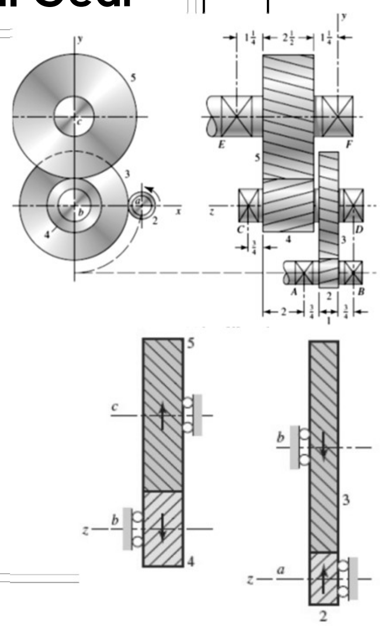


$W_r = Wt\tan\emptyset_t = W\sin\emptyset_n$
$W_a = W_t \tan\psi = W\cos\emptyset_n \sin\psi$
$W_t = W\cos\emptyset_n \cos\psi$

13-16 Force Analysis-Helical Gear

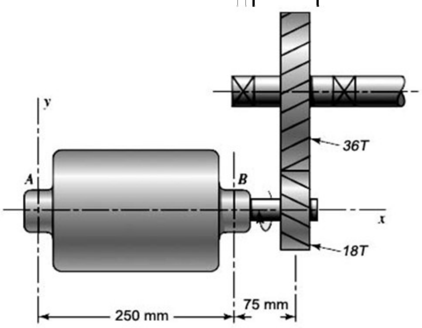
Direction of thrust force

- Direction of the thrust force exerted by each gear upon its shaft are shown in figure
- The axial force of gear 2 on shaft *a* is in the negative *z*-direction.
- The axial force of gear 3 on shaft *b* is in the positive *z*-direction.
- The axial force of gear 4 on shaft *b* is in the positive *z*-direction.
- The axial force of gear 5 on shaft *c* is in the negative *z*-direction.
- *To cancel thrust load on shaft, mount both gears (gear 3 & 4 in fig.) with same hand.*

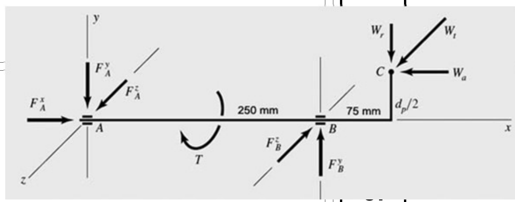


Example 13.9

- In Fig. a 750 W electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive *x* axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal module of 3 mm. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at *A* and *B*. The thrust should be taken out at *A*.



Solution



Solution procedure

1. Calculate $m_t = \frac{m_n}{cos\psi}$
2. Calculate $d_p = m_t N_p$
3. Calculate $W_t = \frac{60000\mathrm{H}}{\pi d_p n}$ kN (*H* in kW, *d* in mm, *n* in rpm)
4. Calculate $\phi_t = \tan^{-1}\frac{\tan \quad n}{\cos}$
5. Calculate $W_r = W_t \tan\phi_t$
6. Calculate $W_a = W_t \tan\psi$
7. and $W = \frac{W_t}{\cos\phi_n \cos\psi}$
8. $\sum_A^z M = 0$ (moment about *A* in the x-y plane, about z-axis) to get F_B^y
9. $\sum_y F = 0$ to get F_A^y
10. $\sum_A^y M = 0$ (moment about *A* in the x-z plane, about y-axis) to get F_B^z
11. $\sum_z F = 0$ to get F_A^z
12. Torque on the shaft is $T = \frac{W_t d_p}{2}$

Machine Design II

Sample problems

- 13-1 to13-4, 13-6, 13-8, 13-10, 13-15, 13-16, 13-20, 13-24,
- 13-31, 13-33, 13-34, 13-45, 13-47, 13-50

From

Shigley's Mechanical Engineering Design, 9th Ed.
(SI Units)

ME-305 Machine Design II