Shigley's Mechanical Engineering Design

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Chapter 13

Gears—General

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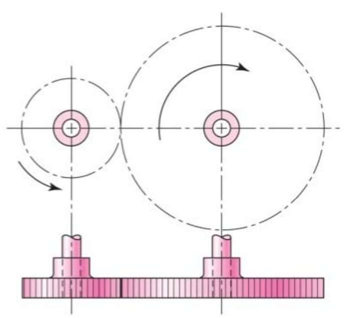
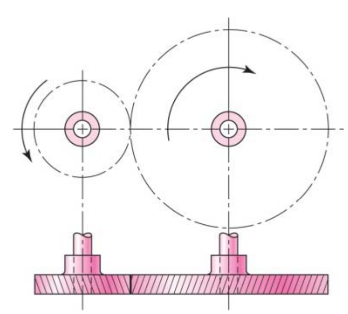
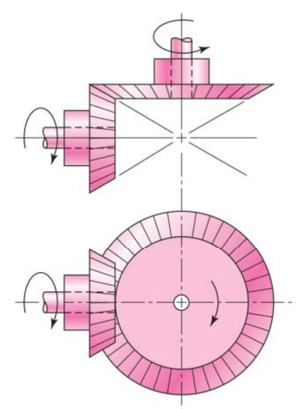
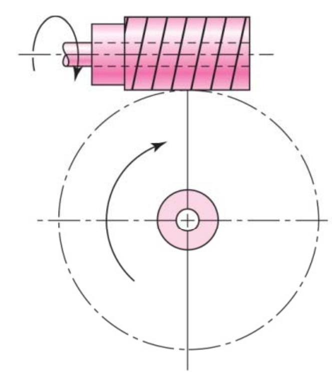
13 Gears—General

Chapter Outline

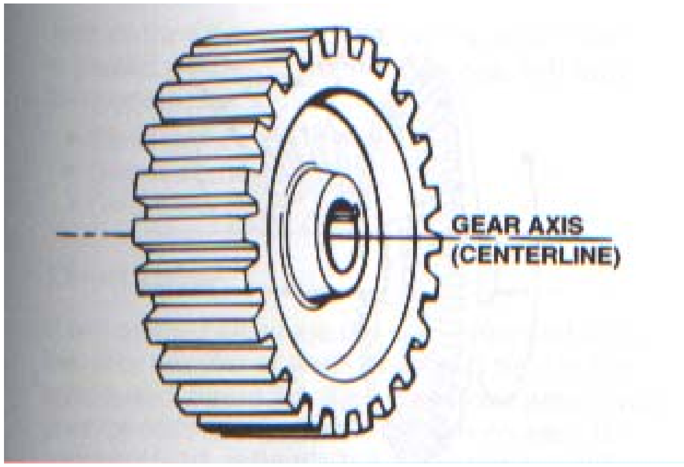
Types of Gears

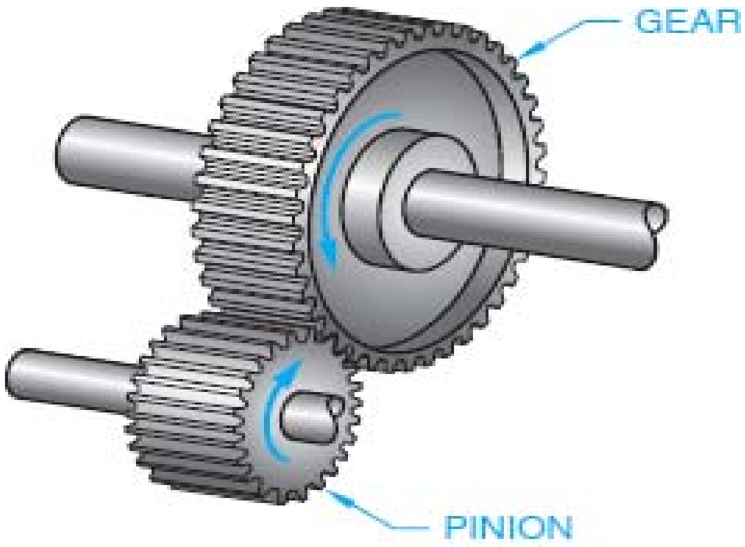
- Spur gears have teeth parallel to the axis of rotation and are used to transmit motion from one shaft to another, parallel, shaft.
- Helical gears have teeth inclined to the axis of rotation. Helical gears are not as noisy, because of the more gradual engagement of the teeth during meshing.
- Bevel gears have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts.
- Worms and worm gears ,The worm resembles a screw. The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand.

SPUR GEAR

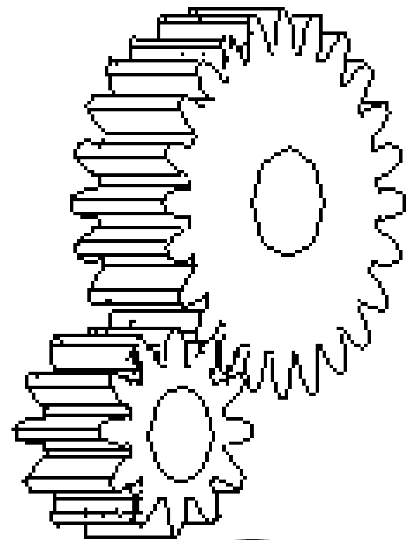
- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer

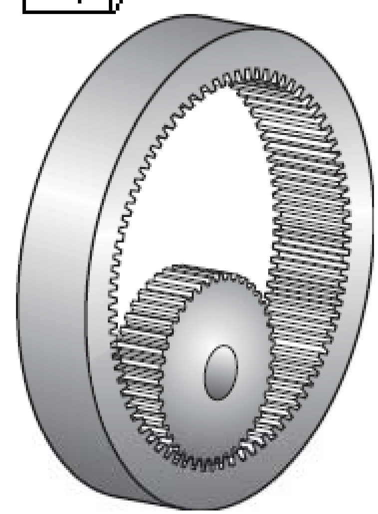




External and Internal spur Gear…

- Advantages:
  - Economical
  - Simple design
  - Ease of maintenance
- Disadvantages:
  - Less load capacity
  - Higher noise levels

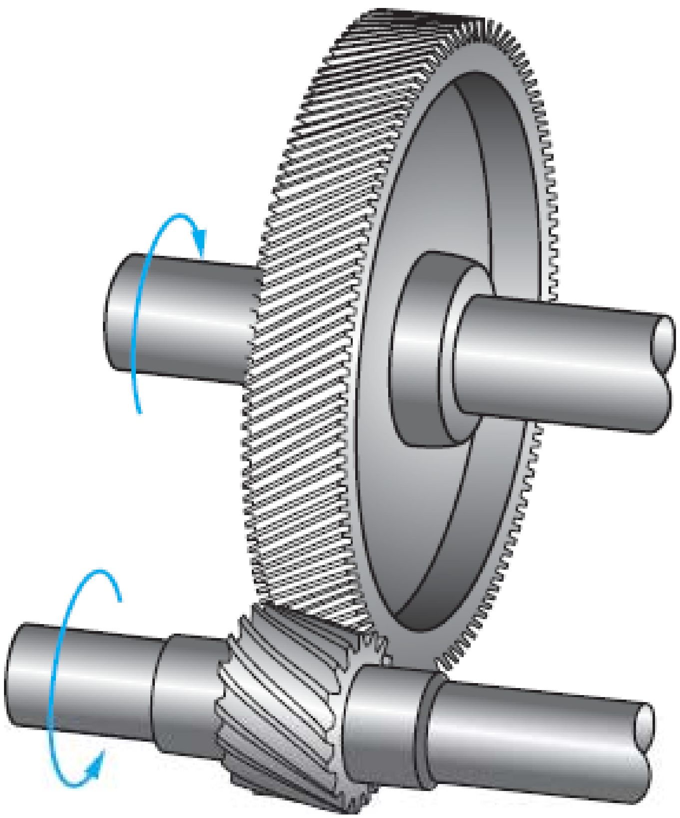




Helical Gear

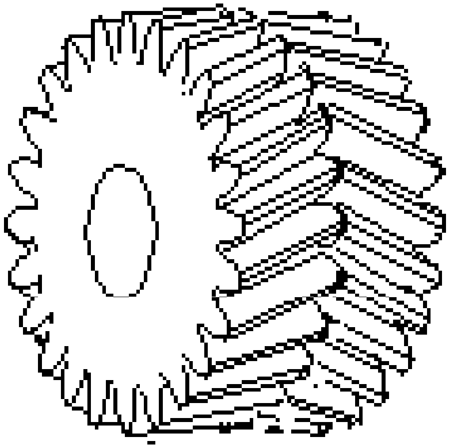
- The teeth on helical gears are cut at an angle to the face of the gear
- This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears
- Carry more load than equivalent-sized spur gears

Helical Gear…



Herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces

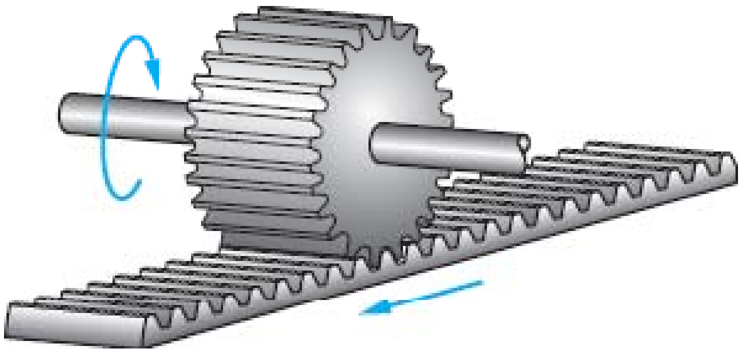


- Herringbone gears are mostly used on heavy machinery.

Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

- A perfect example of this is the steering system on many cars



Bevel gears

- **Bevel gears** are useful when the direction of a shaft's rotation needs to be changed
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral** or **hypoid**
- locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

Straight and Spiral Bevel Gears

WORM AND WORM GEAR

- **Worm gears** are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater
- Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm
- Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc

WORM AND WORM GEAR

Nomenclature

- ✓ The **pitch circle** is a theoretical circle upon which all calculations are usually based; its diameter is the **pitch diameter**.
- ✓ A **pinion** is the smaller of two mating gears. The larger is often called the **gear**.
- ✓ The **circular pitch** p is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. It is equalto the sum of the tooth thickness and width of space.
- ✓ The **module** m is the ratio of the pitch diameter to the number of teeth.
- ✓ The **diametral pitch** P is the ratio of the number of teeth on the gear to the pitch diameter.

$$P = \frac{N}{d}$$

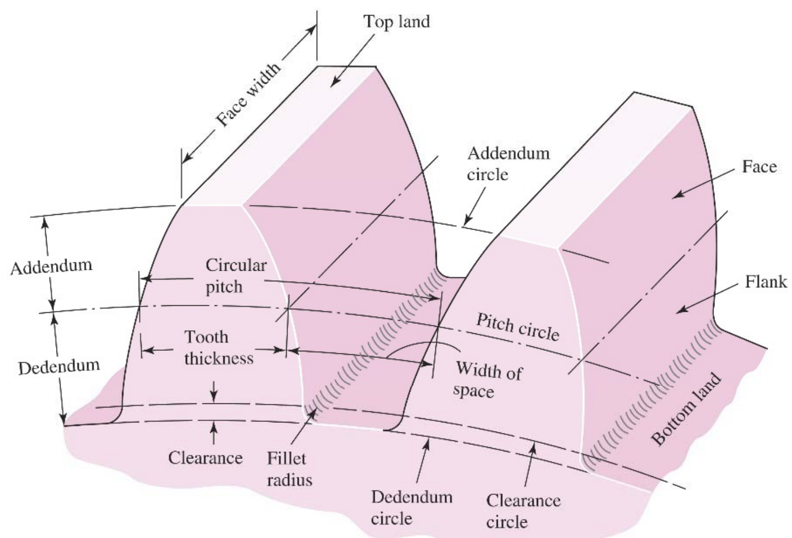
$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

Figure 13–5

Nomenclature of spur-gear teeth.



where

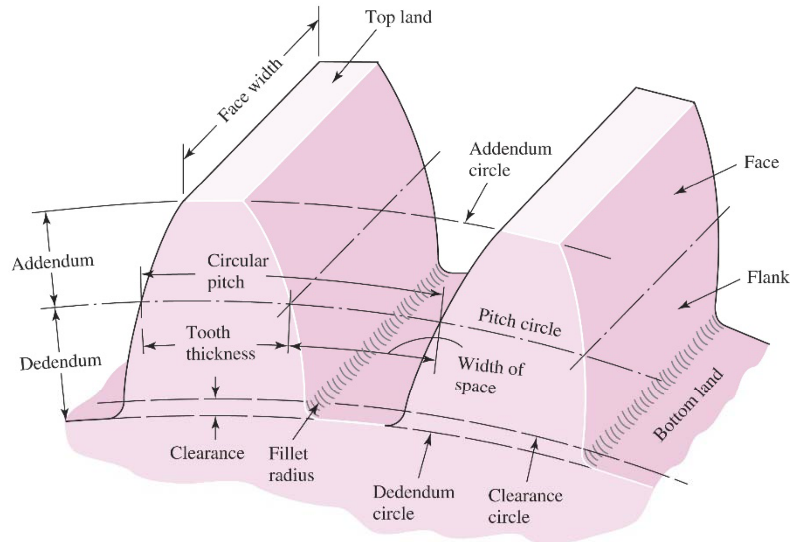
P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

Nomenclature

- ✓ The **addendum** *a* is the radial distance between the **top land** and the pitch circle (1m).
- ✓ The **dedendum** *b* is the radial distance from the **bottom land** to the pitch circle (1.25m). The **whole depth** h_t is the sum of the addendum and the dedendum.
- ✓ The **clearance circle** is a circle that is tangent to the addendum circle of the mating gear.
- ✓ The **clearance** *c* is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.
- ✓ The **backlash** is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.

Figure 13–5

Nomenclature of spur-gear teeth.



$$P = \frac{N}{d}$$

$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

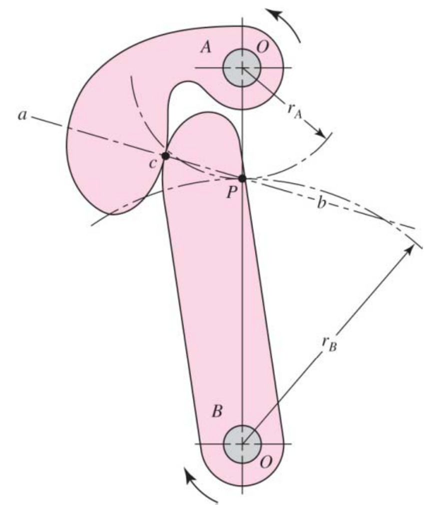
$$pP = \pi$$

where

P = diametral pitch, teeth per inch
N = number of teeth
d = pitch diameter, in
m = module, mm
d = pitch diameter, mm
p = circular pitch

Conjugate Action

- Tooth profiles are designed so as to produce a *constant angular velocity ratio during meshing*, conjugate action.
- When one curved surface pushes against another ,the point of contact occurs where the two surfaces are tangent to each other (point *c*), and the forces at any instant are directed along the common normal *ab (line of action)* to the two curves.
- The angular-velocity ratio between the two arms is inversely proportional to their radii to the point *P*.
- Circles drawn through point P are called pitch circles, and point P is called the pitch point.
- To transmit motion at a constant angular-velocity ratio, the pitch point must remain fixed; that is, all the lines of action for every instantaneous point of contact must pass through the same point *P*.



Mating gear teeth produce rotary motion similar to cams

VELOCITY RATIO OF GEAR DRIVE

- In the case of involute profiles, all points of contact occur on the same straight line *ab*. All normal to the tooth profiles at the point of contact coincide with the line ab, thus these profiles transmit uniform rotary motion.
- *When two gears are in mesh their pitch circles roll on one another without slippage. Then the pitch line velocity is* $V = r_1\omega_1 = r_2\,\omega_2$

d = Diameter of the wheel

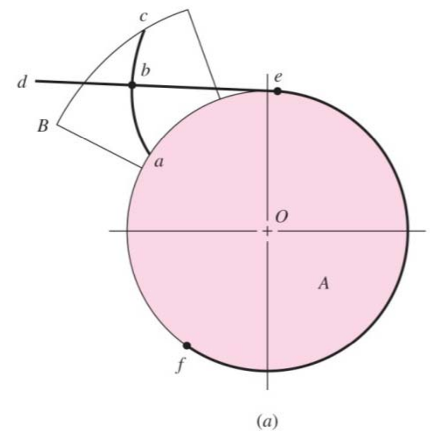
N =Speed of the wheel

ω = Angular speed

velocity ratio (n) = $$\frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Involute Properties (read)

- An involute curve may be generated with a partial flange *B* attached to the cylinder *A*, around which wrapped a cord *def* held tight.
- Point *b* on the cord represents the tracing point, and as the cord is wrapped and unwrapped about the cylinder, point *b* will trace out the involute curve *ac*.
- The generating line *de* is normal to the involute at all points of intersection and, at the same time, is always tangent to the cylinder.
- The point of contact moves along the generating line; the generating line does not change position, because it is always tangent to the base circles; and since the generating line is always normal to the involutes at the point of contact, the requirement for uniform motion is satisfied.



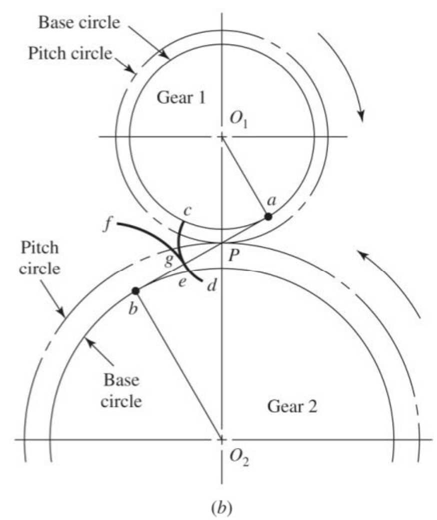
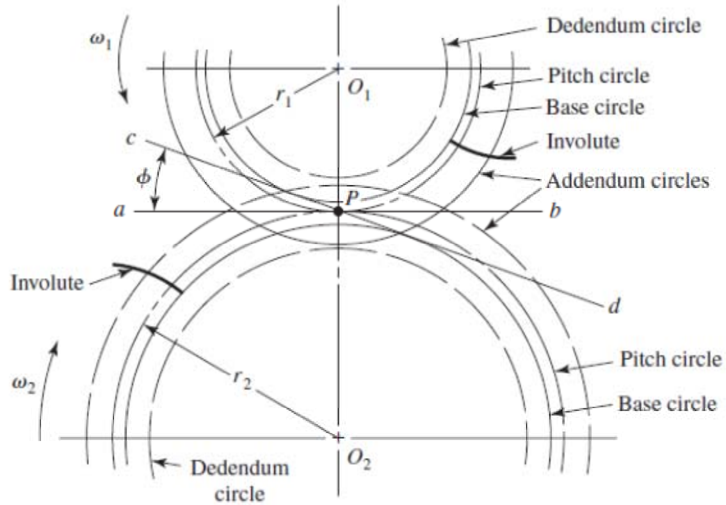


Figure 13–9

Circles of a gear layout.



Next, on each gear draw a circle tangent to the pressure line. These circles are the *base circles*. Since they are tangent to the pressure line, the pressure angle determines their size. As shown in Fig. 13–10, the radius of the base circle is

$$r_b = r \cos \phi \qquad (13\text{–}6)$$

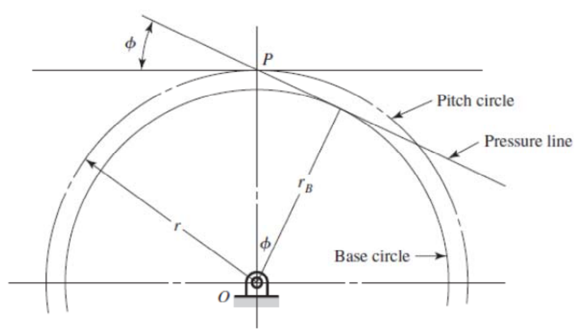
r: radius of the pitch circle

$$p_b = p_c \cos \phi \qquad (13\text{–}7)$$

Base pitch relation to circular pitch

Figure 13–10

Base circle radius can be related to the pressure angle ϕ and the pitch circle radius by $r_b = r \cos \phi$.



Fundamentals

- When two gears are in mesh, their pitch circles roll on one another without slipping. The pitch-line velocity $V = |r_1\omega_1| = |r_2\omega_2|$
- Thus the relation between the radii on the angular velocities is
$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$$
- The addendum and dedendum distances for standard interchangeable teeth are, 1/P and 1.25/P, respectively.
- The pressure line (line of action)

represent the direction in which the

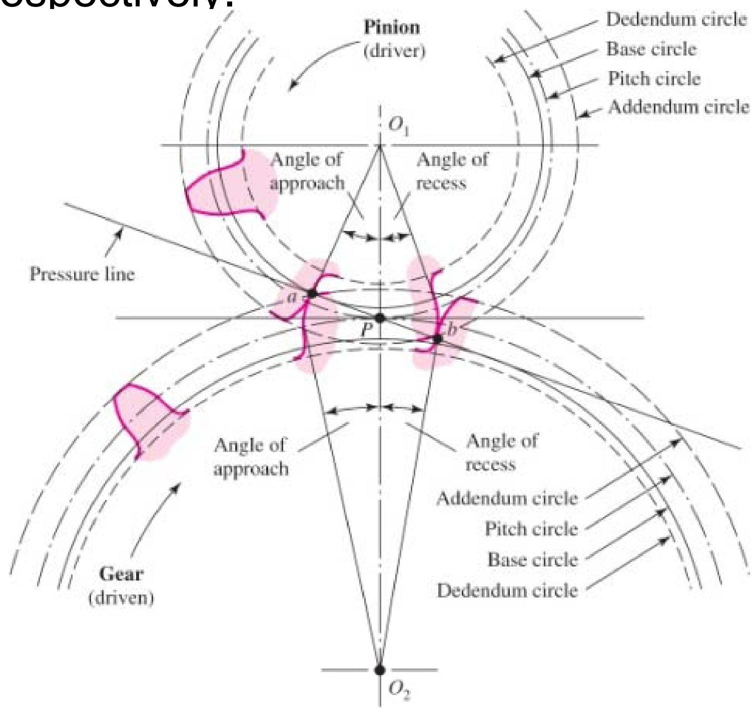
resultant force acts between the gears.

- The angle ϕ is called the pressure angle

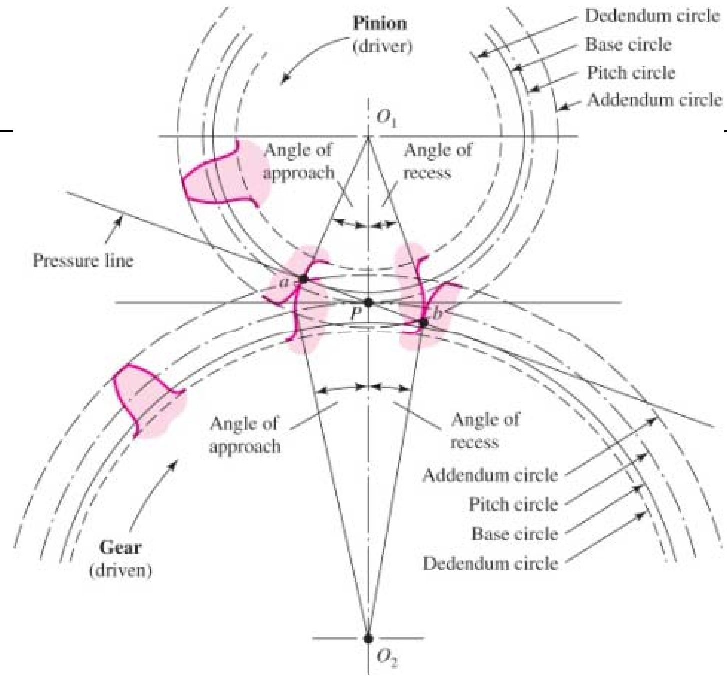
and it usually has values of 20° or 25°

- The involute begins at the base circle and is

undefined below this circle.



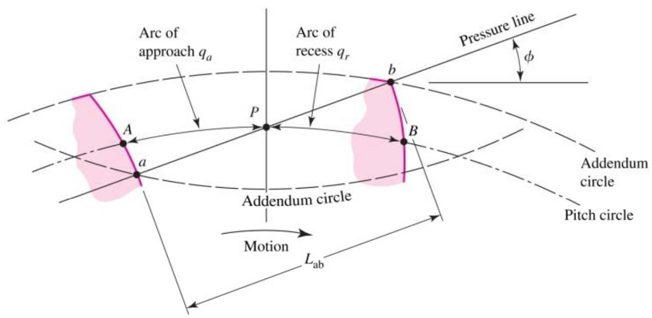
Fundamentals



- If we construct tooth profiles through point *a* and draw radial lines from the intersections of these profiles with the pitch circles to the gear centers, we obtain the angle of approach for each gear.
- The final point of contact will be where the addendum circle of the driver crosses the pressure line. The angle of recess for each gear is obtained in a manner similar to that of finding the angles of approach.
- We may imagine a rack as a spur gear having an infinitely large pitch diameter. Therefore, the rack has an infinite number of teeth and a base circle which is an infinite distance from the pitch point.

Contact Ratio

- The zone of action of meshing gear teeth is shown with the distance *AP* being the **arc of approach** q_a , and the distance *P B* being the **arc of recess** q_r.
- Tooth contact begins and ends at the intersection of the two addendum circles with the pressure line.
- When a tooth is just beginning contact at *a*, the previous tooth is simultaneously ending its contact at *b* for cases when one tooth and its space occupying the entire arc *AB*.
- Because of the nature of this tooth action, either one or two pairs of teeth in contact, it is convenient to define the term contact ratio m_c as



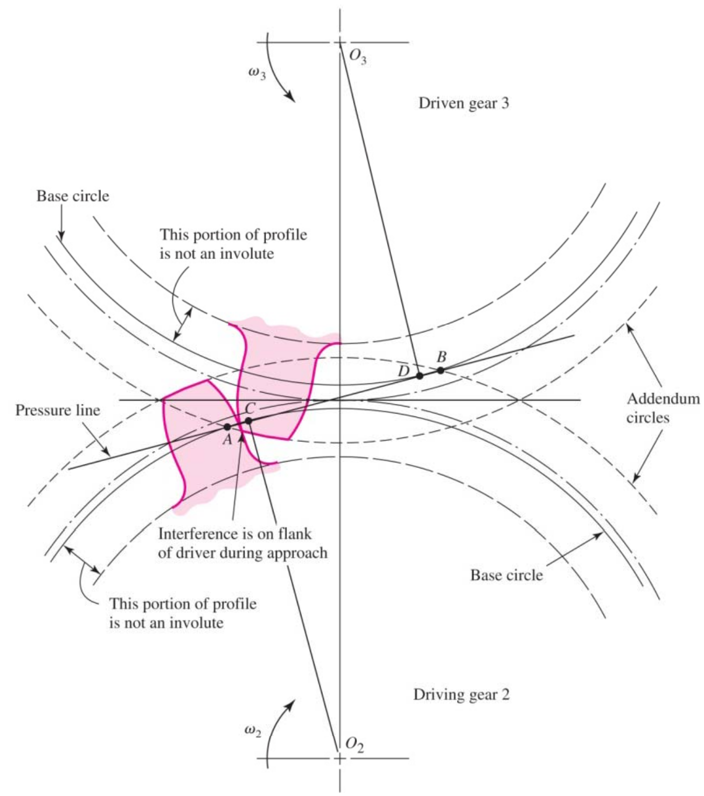
$$m_c = \frac{q_t}{p} = \frac{L_{ab}}{p \cos \phi}$$

a number that indicates the average number of pairs of teeth in contact.

•Gears should not generally be designed having contact ratios less than about 1.20, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level.

Interference

- The contact of portions of tooth profiles that are not conjugate is called interference.
- When the points of tangency of the pressure line with the base circles *C* and *D* are located inside of points *A* and *B* (initial and final points of contact), interference is present.
- The actual effect of interference is that the involute tip or face of the driven gear tends to dig out the noninvolute flank of the driver.
- When gear teeth are produced by a generation process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called undercutting.



Interference Analysis

- The smallest number of teeth on a spur pinion and gear, one-to-one gear ratio, which can exist without interference is N_P .
- The number of teeth for spur gears is given by

$$N_P = \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right)$$

where k = 1 for full-depth teeth, 0.8 for stub teeth and φ = pressure angle.

- If the mating gear has more teeth than the pinion, that is, $m_G = N_G/N_P = m$ is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

- The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi}$$

- The smallest spur pinion that will operate with a rack without interference is

$$N_P = \frac{2(k)}{\sin^2\phi}$$

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \tag{13–10}$$

where $k = 1$ for full-depth teeth, 0.8 for stub teeth and ϕ = pressure angle.

For a 20° pressure angle, with $k = 1$,

$$N_P = \frac{2(1)}{3\sin^2 20^\circ}\left(1 + \sqrt{1 + 3\sin^2 20^\circ}\right) = 12.3 = 13 \text{ teeth}$$

Thus 13 teeth on pinion and gear are interference-free. Realize that 12.3 teeth is possible in meshing arcs, but for fully rotating gears, 13 teeth represents the least number. For a $14\frac{1}{2}^\circ$ pressure angle, $N_P = 23$ teeth, so one can appreciate why few $14\frac{1}{2}^\circ$-tooth systems are used, as the higher pressure angles can produce a smaller pinion with accompanying smaller center-to-center distances.

If the mating gear has more teeth than the pinion, that is, $m_G = N_G/N_P = m$ is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1 + 2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1 + 2m)\sin^2\phi}\right) \tag{13–11}$$

For example, if $m = 4$, $\phi = 20^\circ$,

$$N_P = \frac{2(1)}{[1 + 2(4)]\sin^2 20^\circ}\left[4 + \sqrt{4^2 + [1 + 2(4)]\sin^2 20^\circ}\right] = 15.4 = 16 \text{ teeth}$$

Thus a 16-tooth pinion will mesh with a 64-tooth gear without interference.

The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \tag{13–12}$$

For example, for a 13-tooth pinion with a pressure angle ϕ of 20°,

$$N_G = \frac{13^2 \sin^2 20^\circ - 4(1)^2}{4(1) - 2(13)\sin^2 20^\circ} = 16.45 = 16 \text{ teeth}$$

For a 13-tooth spur pinion, the maximum number of gear teeth possible without interference is 16.

The smallest spur pinion that will operate with a rack without interference is

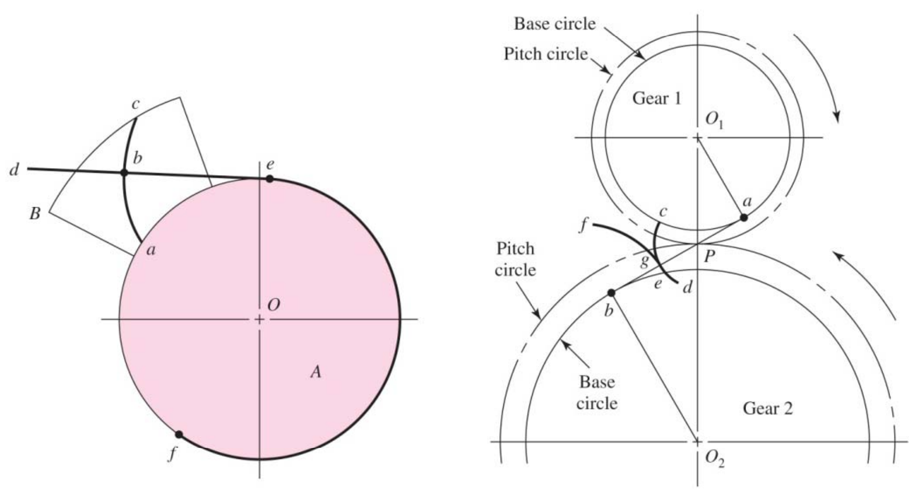
$$N_P = \frac{2(k)}{\sin^2 \phi} \tag{13–13}$$

For a 20° pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is

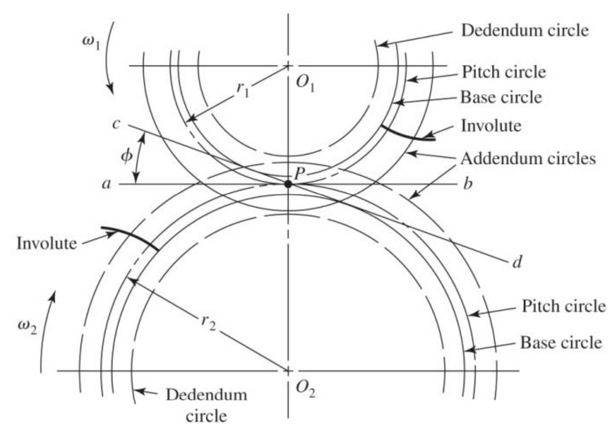
$$N_P = \frac{2(1)}{\sin^2 20^\circ} = 17.1 = 18 \text{ teeth}$$

The Forming of Gear Teeth (read)

- There are a large number of ways of forming the teeth of gears, such as sand casting, shell molding, investment casting, permanent-mold casting, die casting, centrifugal casting, powder-metallurgy process, extrusion.
- The teeth may be finished, after cutting, by either shaving or burnishing. Several shaving machines are available that cut off a minute amount of metal, bringing the accuracy of the tooth profile within the limits of 250 μin.



(a) (b)

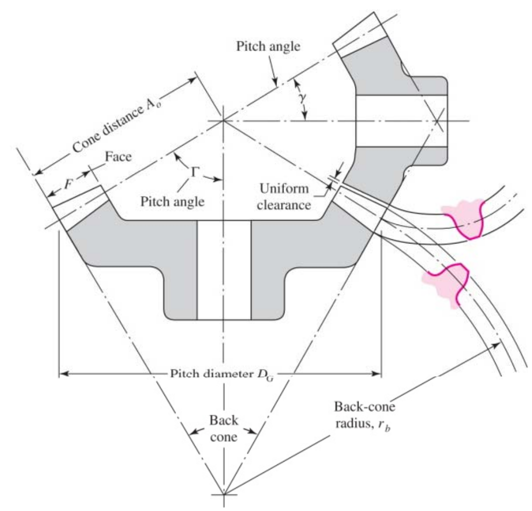


Straight Bevel Gears (read)

- When gears are used to transmit motion between intersecting shafts, some form of bevel gear is required.
- The terminology of bevel gears is illustrated.
- The pitch angles are defined by the pitch cones meeting at the apex, as shown in the figure. They are related to the tooth numbers as follows:

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P}$$

where the subscripts P and G refer to the pinion and gear, respectively, and where γ and Γ are, respectively, the pitch angles of the pinion and gear.



Standard straight tooth bevel gears are cut by using a 20º pressure angle and full depth teeth. This increases contact ratio, avoids undercut, and increases the strength of the pinion.

Parallel Helical Gears

- Helical gears subject the shaft bearings to both radial and thrust loads. When the thrust load become high it maybe desirable to use double helical gears (herringbone) which is equivalent to helical gears of opposite hand, mounted side by side on the same shaft. They develop opposite thrust reactions and thus cancel out.

- When two or more single helical gears are mounted on the same shaft. The hand of the gears should be selected to minimize the thrust load.

Parallel Helical Gears

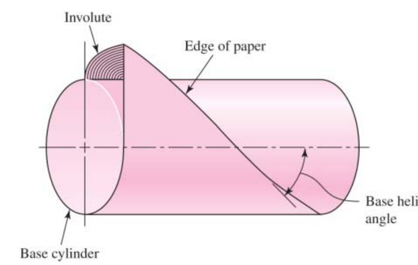
- The shape of the tooth of Helical gears is an involute helicoid.
- The initial contact of helical-gear teeth is a point that extends into a line as the teeth come into more engagement. In spur gears the line of contact is parallel to the axis of rotation; in helical gears the line is diagonal across the face of the tooth.
- The distance *ae* is the normal circular pitch p_n and is related to the transverse circular pitch as follows:

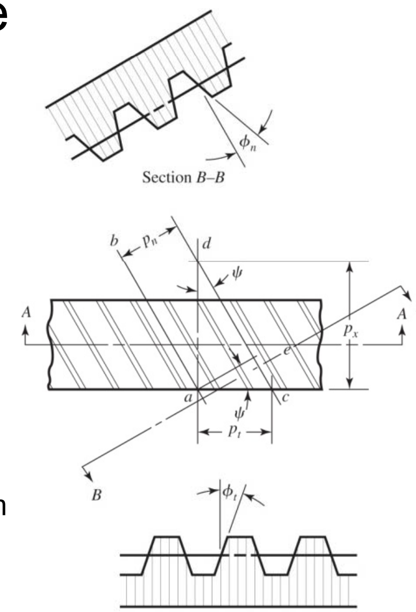
$$p_n = p_t \cos\psi$$

- The distance *ad* is called the axial pitch p_x and is related by the expression

$$p_x = \frac{p_t}{\tan\psi}$$

- The normal diametral pitch $P_n = \frac{P_t}{\cos\psi}$ Transverse diametral pitch
- Normal circular pitch x normal diametral pitch ($p_n x P_n = \pi$)
- The pressure angle φ_n in the normal direction is different from the pressure angle φ_t in the direction of rotation. These angles are related by the equation





$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

Parallel Helical Gears (Cont.)

- The pressure angle φ_t in the tangential (rotation) direction is

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right)$$

- The smallest tooth number N_P of a helical-spur pinion that will run without interference with a gear with the same number of teeth is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right)$$

- The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$

- The smallest pinion that can be run with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t}$$

The smallest tooth number N_P of a helical-spur pinion that will run without interference[2] with a gear with the same number of teeth is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right) \tag{13–21}$$

For example, if the normal pressure angle ϕ_n is 20°, the helix angle ψ is 30°, then ϕ_t is

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

$$N_P = \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1+\sqrt{1+3\sin^2 22.80^\circ}\right) = 8.48 = 9 \text{ teeth}$$

For a given gear ratio $m_G = N_G/N_P = m$, the smallest pinion tooth count is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m+\sqrt{m^2+(1+2m)\sin^2\phi_t}\right] \tag{13–22}$$

The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \tag{13–23}$$

For example, for a nine-tooth pinion with a pressure angle ϕ_n of 20°, a helix angle ψ of 30°, and recalling that the tangential pressure angle ϕ_t is 22.80°,

$$N_G = \frac{9^2\sin^2 22.80^\circ - 4(1)^2\cos^2 30^\circ}{4(1)\cos 30^\circ - 2(9)\sin^2 22.80^\circ} = 12.02 = 12$$

The smallest pinion that can be run with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t} \tag{13–24}$$

For a normal pressure angle ϕ_n of 20° and a helix angle ψ of 30°, and $\phi_t = 22.80^\circ$,

$$N_P = \frac{2(1)\cos 30^\circ}{\sin^2 22.80^\circ} = 11.5 = 12 \text{ teeth}$$

EXAMPLE 13–2 A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

(*a*) The pitch diameter

(*b*) The transverse, the normal, and the axial pitches

(*c*) The normal diametral pitch

(*d*) The transverse pressure angle

Solution

Answer (*a*)

$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

Answer (*b*)

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

Answer

$$p_n = p_t \cos\psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

Answer

$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45^\circ} = 1.123 \text{ in}$$

Answer (*c*)

$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

Answer (*d*)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 25^\circ}\right) = 21.88^\circ$$

Worm Gears (read)

- The worm and worm gear of a set have the same hand of helix as for crossed helical gears.
- It is usual to specify the lead angle λ on the worm and helix angle ψ_G on the gear; the two angles are equal for a 90° shaft angle.
- Since it is not related to the number of teeth, the worm may have any pitch diameter; this diameter should, however, be the same as the pitch diameter of the hob used to cut the worm-gear teeth. Generally,

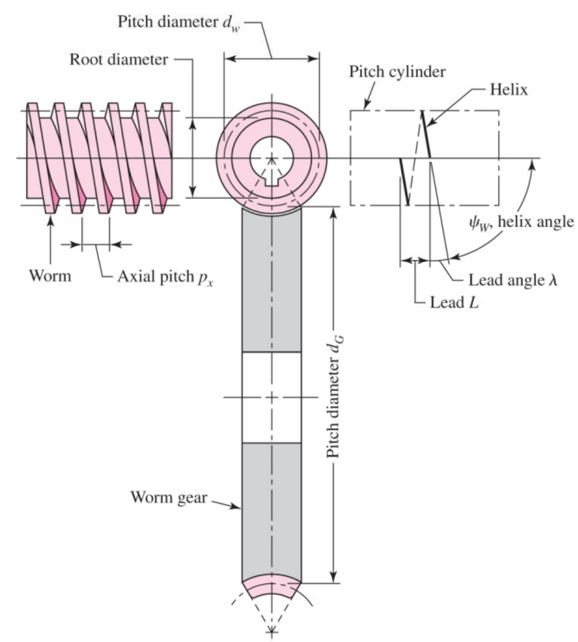
$$\frac{C^{0.875}}{3.0} \leq d_W \leq \frac{C^{0.875}}{1.7}$$

where C is the center distance.

•The lead L and the lead angle λ of the worm have the following relations:

$$L = p_x N_W$$

$$\tan\lambda = \frac{L}{\pi d_W}$$



Tooth Systems

Spur gears

- A tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle.
- Tooth forms for worm gearing have not been highly standardized, perhaps because there has been less need for it.
- The face width F_G of the worm gear should be made equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle.

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Worm gears

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

Standard Tooth Properties

Helical gears

Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$ $[1.0\ m_n]$	*External gears:*	
Dedendum	$\frac{1.25}{P_n}$ $[1.25\ m_n]$	*Standard center distance*	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos\psi}$ $\left[\frac{N_P m_n}{\cos\psi}\right]$	*Gear outside diameter*	$D+2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos\psi}$ $\left[\frac{N_P m_n}{\cos\psi}\right]$	*Pinion outside diameter*	$d+2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$ $\left[\pi m_n - \frac{B_n}{2}\right]$	*Gear root diameter*	$D-2b$
Pinion base diameter	$d \cos\phi_t$	*Pinion root diameter*	$d-2b$
		Internal gears:	
Gear base diameter	$D \cos\phi_t$	*Center distance*	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_t)$	*Inside diameter*	$D-2a$
		Root diameter	$D+2b$

*All dimensions are in inches, and angles are in degrees.

†B_n is the normal backlash.

Corresponding SI units formula in square brackets.

Bevel gears

Item	Formula
Working depth	$h_k = 2.0/P$ [= 2.0 m]
Clearance	$c = (0.188/P) + 0.002$ in [= 0.188 m + 0.05 mm]
Addendum of gear	$a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{90})^2}$ $\left[= 0.54\text{ m} + \frac{0.46\text{ m}}{(m_{90})^2}\right]$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90°$
	$m_{90} = \sqrt{m_G \frac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \frac{10}{P}$, whichever is smaller $\left[F = \frac{A_0}{3} \text{ or } F = 10\text{ m}\right]$
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

Pinion	16	15	14	13
Gear	16	17	20	30

Corresponding SI units formula in square brackets.

EXAMPLE 13–1 A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.

(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.

(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

Solution

Answer (*a*)

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

Answer

$$\frac{d_P + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r \cos \phi$,

Answer

$$r_b \text{ (pinion)} = \frac{8}{2} \cos 20^\circ = 3.76 \text{ in}$$

Answer

$$r_b \text{ (gear)} = \frac{20}{2} \cos 20^\circ = 9.40 \text{ in}$$

(*b*) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \tag{1}$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \tag{2}$$

Solving Eqs. (1) and (2) simultaneously yields

Answer
$$d'_P = 8.143 \text{ in} \qquad d'_G = 20.357 \text{ in}$$

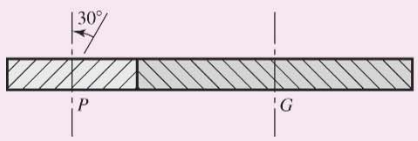
Since $r_b = r\cos\phi$, the new pressure angle is

Answer
$$\phi' = \cos^{-1}\frac{r_b\text{ (pinion)}}{d'_P/2} = \cos^{-1}\frac{3.76}{8.143/2} = 22.56^\circ$$

EXAMPLE 13–1 A parallel helical gearset uses a 17-tooth pinion driving a 34-tooth gear. The pinion has a right-hand helix angle of 30°, a normal pressure angle of 20°, and a normal diametral pitch of 5 teeth/in. Find:

(*a*) The normal, transverse, and axial circular pitches
(*b*) The normal base circular pitch
(*c*) The transverse diametral pitch and the transverse pressure angle
(*d*) The addendum, dedendum, and pitch diameter of each gear

Solution



(*a*)

$$p_n = \pi/5 = 0.6283 \text{ in}$$

$$p_t = p_n/\cos\psi = 0.6283/\cos 30° = 0.7255 \text{ in}$$

$$p_x = p_t/\tan\psi = 0.7255/\tan 30° = 1.25 \text{ in}$$

Answer (*b*) Eq. (13–7): $p_{nb} = p_n \cos\phi_n = 0.6283 \cos 20° = 0.590$ in

Answer (*c*) $P_t = P_n \cos\psi = 5 \cos 30° = 4.33$ teeth/in

$$\phi_t = \tan^{-1}(\tan\phi_n/\cos\psi) = \tan^{-1}(\tan 20°/\cos 30°) = 22.8°$$

Answer (*d*) Table 13–4:

$$a = 1/5 = 0.200 \text{ in}$$

$$b = 1.25/5 = 0.250 \text{ in}$$

$$d_P = \frac{17}{5\cos 30°} = 3.926 \text{ in}$$

$$d_G = \frac{34}{5\cos 30°} = 7.852 \text{ in}$$

EXAMPLE 13–2 A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

(*a*) The pitch diameter

(*b*) The transverse, the normal, and the axial pitches

(*c*) The normal diametral pitch

(*d*) The transverse pressure angle

Solution

Answer (*a*)

$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

Answer (*b*)

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

Answer

$$p_n = p_t \cos\psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

Answer

$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45^\circ} = 1.123 \text{ in}$$

Answer (*c*)

$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

Answer (*d*)

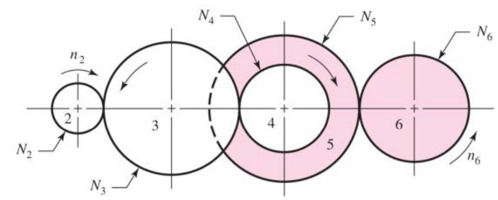
$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 25^\circ}\right) = 21.88^\circ$$

Gear Trains

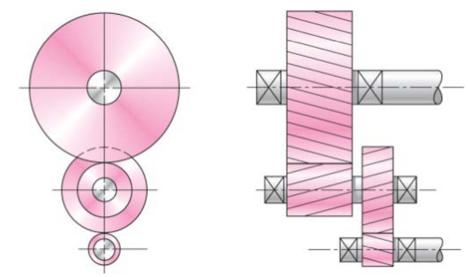
- Consider a pinion 2 driving a gear 3. The speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3}n_2\right| = \left|\frac{d_2}{d_3}n_2\right|$$

where n = revolutions or rev/min N = number of teeth
d = pitch diameter

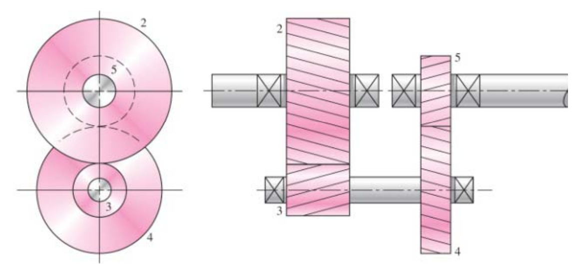


- Gear 3 is an idler that affects only the direction of rotation of gear 6.



- Gears 2, 3, and 5 are drivers, while 3, 4, and 6 are driven members. We define the **train value** e as

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \qquad n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

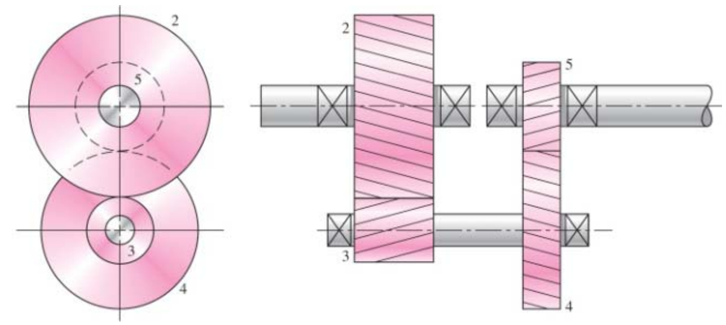


- As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears. A two-stage compound gear train can obtain a train value of up to 100 to 1.

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

- It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line.

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$



The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \tag{13–30}$$

Note that pitch diameters can be used in Eq. (13–30) as well. When Eq. (13–30) is used for spur gears, e is positive if the last gear rotates in the same sense as the first, and negative if the last rotates in the opposite sense.

Now we can write

$$n_L = e n_F \tag{13–31}$$

where n_L is the speed of the last gear in the train and n_F is the speed of the first.

Planetary Gear Train

- Planetary trains always include a sun gear*,* a planet carrier or arm*,* and one or more planet gears*.*
- The figure shows a planetary train composed of a sun gear 2, an arm or carrier 3, and planet gears 4 and 5.
- The angular velocity of gear 2 relative to the arm in rev/min is

$$n_{23} = n_2 - n_3$$

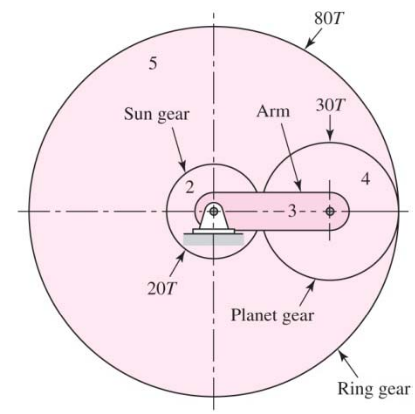
- The ratio of gear 5 to that of gear 2 is the same and is proportional to the tooth numbers, whether the arm is rotating or not. It is the train value.

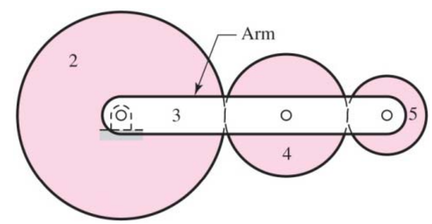
$$e = \frac{n_5 - n_3}{n_2 - n_3} \quad \text{or} \quad e = \frac{n_L - n_A}{n_F - n_A}$$

where n_F = rev/min of first gear in planetary train

n_L = rev/min of last gear in planetary train

n_A = rev/min of arm





EXAMPLE 13–3 A gearbox is needed to provide a 30:1 ($\pm$ 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

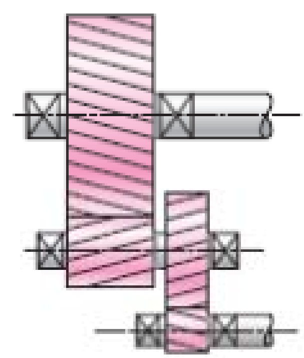
Answer

$$16\sqrt{30} = 87.64 \doteq 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.



EXAMPLE 13–4 A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \quad \text{and} \quad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Then

$$N_2 = 6\,N_3 = 6\,(16) = 96$$

$$N_4 = 5\,N_5 = 5\,(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

EXAMPLE 13–5 A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$. Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$

$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$

$$126 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 126/6 = 21$$

$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

Answer

$$N_2 = 108$$

$$N_3 = 18$$

$$N_4 = 105$$

$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$

$$108 + 18 = 105 + 21$$

$$126 = 126$$

EXAMPLE 13–6 In Fig. 13–30 the sun gear is the input, and it is driven clockwise at 100 rev/min. The ring gear is held stationary by being fastened to the frame. Find the rev/min and direction of rotation of the arm and gear 4.

Solution

Designate $n_F = n_2 = -100$ rev/min, and $n_L = n_5 = 0$. Unlocking gear 5 and holding the arm stationary, in our imagination, we find

$$e = -\left(\frac{20}{30}\right)\left(\frac{30}{80}\right) = -0.25$$

Substituting this value in Eq. (13–32) gives

$$-0.25 = \frac{0 - n_A}{(-100) - n_A}$$

or

Answer

$$n_A = -20 \text{ rev/min}$$

To obtain the speed of gear 4, we follow the procedure outlined by Eqs. (*b*), (*c*), and (*d*). Thus

$$n_{43} = n_4 - n_3 \qquad n_{23} = n_2 - n_3$$

and so

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} \tag{1}$$

But

$$\frac{n_{43}}{n_{23}} = -\frac{20}{30} = -\frac{2}{3} \tag{2}$$

Substituting the known values in Eq. (1) gives

$$-\frac{2}{3} = \frac{n_4 - (-20)}{(-100) - (-20)}$$

Solving gives

Answer

$$n_4 = 33\tfrac{1}{3} \text{ rev/min}$$

Force Analysis : Spur Gearing

- Free-body diagrams of the forces and moments acting upon two gears of a simple gear train are shown.
- The power *H* transmitted through a rotating gear can be obtained from the standard relationship of the product of torque *T* and angular velocity .

$$H = T\omega = (W_t d/2)\,\omega$$

- Gear data is often tabulated using pitch-line velocity, $V = (d/2)\,\omega$.

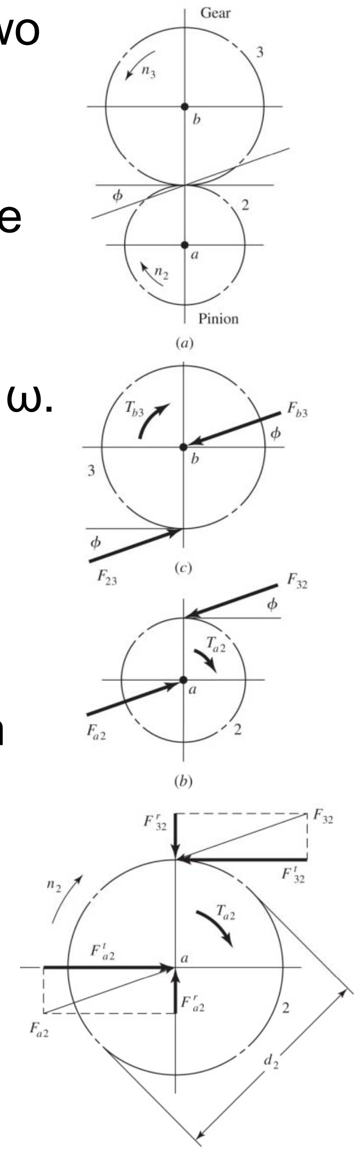
$$V = \pi d n/12$$

where
V =pitch-line velocity ft/min; d =gear diameter,in; n =gear speed, rev/min

- With the pitch-line velocity and appropriate conversion factors incorporated, Eq. (13–33) can be rearranged and expressed in customary units as

$$W_t = 33\,000\frac{H}{V}$$

where W_t=transmitted load, lbf; H =power,hp; V =pitch-line velocity,ft/min



The corresponding equation in SI is

$$W_t = \frac{60\,000H}{\pi d n}$$

where W_t = transmitted load, kN

H = power, kW

d = gear diameter, mm

n = speed, rev/min

Force Analysis : Bevel Gearing (read)

- In determining shaft and bearing loads for bevel-gear applications, the usual practice is to use the tangential or transmitted load that would occur if all the forces were concentrated at the midpoint of the tooth.
- The transmitted load

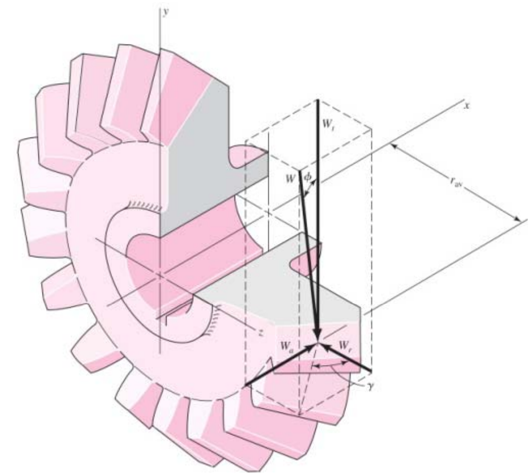
$$W_t = \frac{T}{r_{av}}$$

where T is the torque and r_{av} is the pitch radius at the midpoint of the tooth for the gear under consideration.

- The forces acting at the center of the tooth are shown

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$



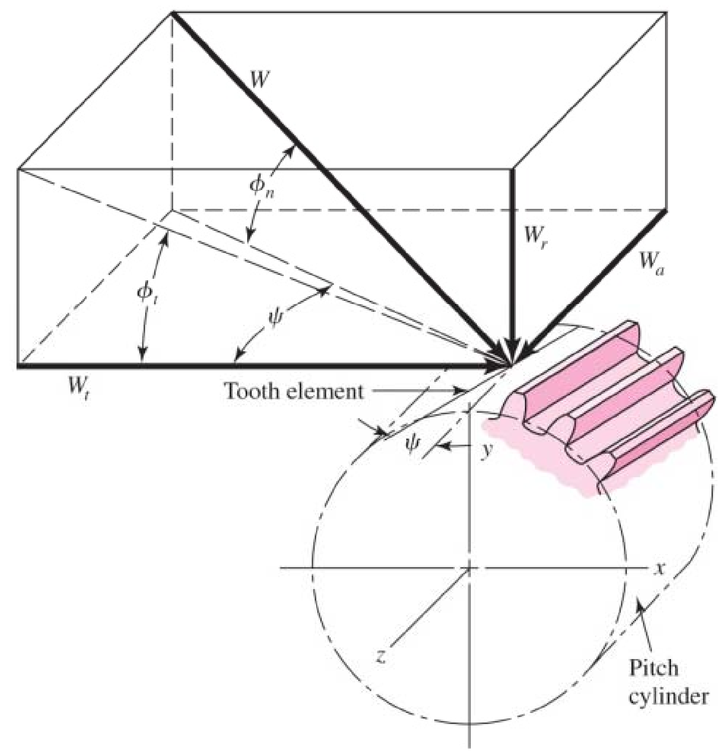
Force Analysis : Helical Gearing

- A three-dimensional view of the forces acting against a helical-gear tooth is shown.
- The three components of the total (normal) tooth force W are

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

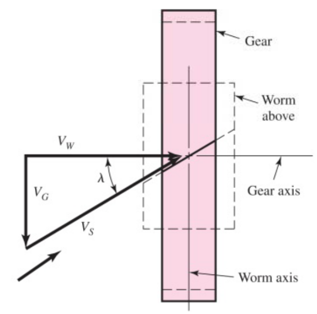


where W = total force
W_r = radial component
W_t = tangential component,
also called transmitted load
W_a = axial component,
also called thrust load

Force Analysis : Worm Gearing (read)

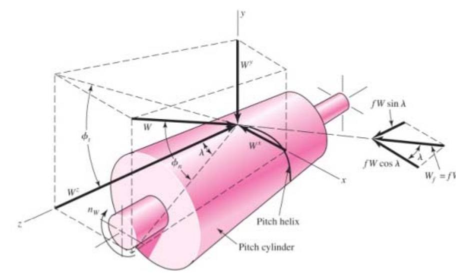
- If friction is neglected, then the only force exerted by the gear will be the force *W* as shown.

$$W^x = W\cos\phi_n \sin\lambda$$
$$W^y = W\sin\phi_n$$
$$W^z = W\cos\phi_n \cos\lambda$$



- Since the gear forces are opposite to the worm forces

$$W_{Wt} = -W_{Ga} = W^x$$
$$W_{Wr} = -W_{Gr} = W^y$$
$$W_{Wa} = -W_{Gt} = W^z$$



- By introducing a coefficient of friction *f*

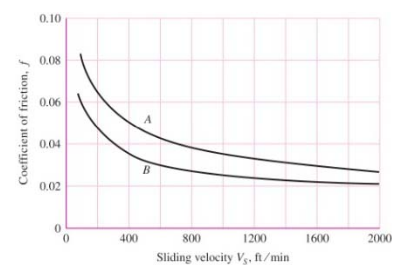
$$W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$$
$$W^y = W\sin\phi_n$$
$$W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$$

- Efficiency η can be defined by using the equation

$$\eta = \frac{W_{Wt}(\text{without friction})}{W_{Wt}(\text{with friction})}$$ when $$W_{Wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

After some rearranging

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$



- Many experiments have shown that the coefficient of friction is dependent on the relative or sliding velocity.

$$V_S = \frac{V_W}{\cos\lambda}$$

EXAMPLE 13–7 Pinion 2 in Fig. 13–34*a* runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

Solution The pitch diameters of gears 2 and 3 are

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

From Eq. (13–36) we find the transmitted load to be

$$W_t = \frac{60\,000H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is $F^t_{23} = 0.546$ kN, as shown in Fig. 13–34*b*. Therefore

$$F^r_{23} = F^t_{23} \tan 20° = (0.546) \tan 20° = 0.199 \text{ kN}$$

and so

$$F_{23} = \frac{F^t_{23}}{\cos 20°} = \frac{0.546}{\cos 20°} = 0.581 \text{ kN}$$

Since gear 3 is an idler, it transmits no power (torque) to its shaft, and so the tangential reaction of gear 4 on gear 3 is also equal to W_t. Therefore

$$F^t_{43} = 0.546 \text{ kN} \qquad F^r_{43} = 0.199 \text{ kN} \qquad F_{43} = 0.581 \text{ kN}$$

and the directions are shown in Fig. 13–34*b*.

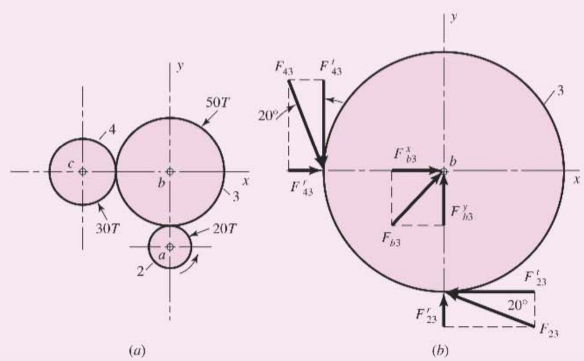
The shaft reactions in the x and y directions are

$$F^x_{b3} = -\left(F^t_{23} + F^r_{43}\right) = -(-0.546 + 0.199) = 0.347 \text{ kN}$$

$$F^y_{b3} = -\left(F^r_{23} + F^t_{43}\right) = -(0.199 - 0.546) = 0.347 \text{ kN}$$

Figure 13–34

A gear train containing an idler gear. (*a*) The gear train. (*b*) Free-body of the idler gear.



The resultant shaft reaction is

$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491 \text{ kN}$$

These are shown on the figure.

EXAMPLE 13–9 In Fig. 13–38 a 750-W electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive x axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal module of 3.0 mm. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at A and B. The thrust should be taken out at A.

Solution From Eq. (13–19) we find

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

Also, $m_t = m_n/\cos\Psi = 3/\cos 30^\circ = 3.46$ mm. Therefore the pitch diameter of the pinion is $d_p = 18(3.46) = 62.3$ mm. The pitch-line velocity is

$$V = \pi dn = \frac{\pi(62.3)(1800)}{60} = 5871.6 \text{ mm/s} = 5.87 \text{ m/s}$$

The transmitted load is

$$W_t = \frac{H}{V} = \frac{750}{5.87} = 128 \text{ N}$$

From Eq. (13–40) we find

$$W_r = W_t \tan\phi_t = (128)\tan 22.8^\circ = 54 \text{ N}$$

$$W_a = W_t \tan\psi = (128)\tan 30^\circ = 74 \text{ N}$$

$$W = \frac{W_t}{\cos\phi_n\cos\psi} = \frac{128}{\cos 20^\circ\cos 30^\circ} = 157 \text{ N}$$

Figure 13–38

The motor and gear train of Ex. 13–9.

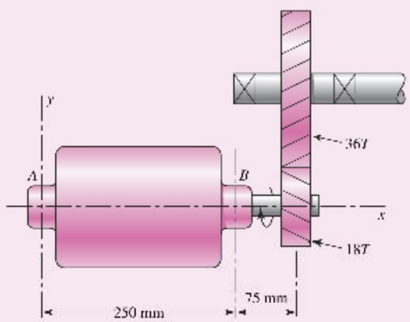


Figure 13–39

Free-body diagram of motor shaft of Ex. 13–9.

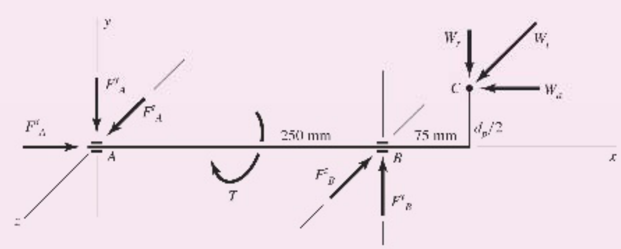
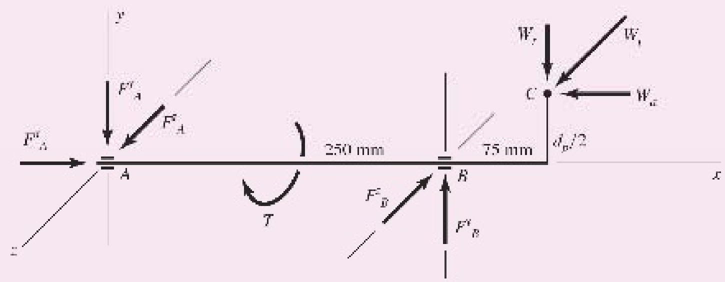


Figure 13–39

Free-body diagram of motor shaft of Ex. 13–9.



These three forces, W_r in the $-y$ direction, W_a in the $-x$ direction, and W_t in the $+z$ direction, are shown acting at point C in Fig. 13–39. We assume bearing reactions at A and B as shown. Then $F_A^x = W_a = 74$ N. Taking moments about the z axis,

$$-(54)(325) + (74)\left(\frac{62.3}{2}\right) + 250F_B^y = 0$$

or $F_B^y = 61$ N. Summing forces in the y direction then gives $F_A^y = 7$ N. Taking moments about the y axis, next

$$250F_B^z - 128(325) = 0$$

or $F_B^z = 166$ N. Summing forces in the z direction and solving gives $F_A^z = 38$ N. Also, the torque is $T = W_t d_p/2 = 128(62.3/2) = 3982$ N · mm.

For comparison, solve the problem again using vectors. The force at C is

$$\mathbf{W} = -74\mathbf{i} - 54\mathbf{j} + 128\mathbf{k} \text{ N}$$

Position vectors to B and C from origin A are

$$\mathbf{R}_B = 250\mathbf{i} \qquad \mathbf{R}_C = 325\mathbf{i} + 31.15\mathbf{j}$$

Taking moments about A, we have

$$\mathbf{R}_B \times \mathbf{F}_B + \mathbf{T} + \mathbf{R}_C \times \mathbf{W} = \mathbf{0}$$

Using the directions assumed in Fig. 13–39 and substituting values gives

$$250\mathbf{i} \times (F_B^y\,\mathbf{j} - F_B^z\,\mathbf{k}) - T\mathbf{i} + (325\mathbf{i} + 31.15\mathbf{j}) \times (-74\mathbf{i} - 54\mathbf{j} + 128\mathbf{k}) = \mathbf{0}$$

When the cross products are formed, we get

$$\left(250F_B^y\,\mathbf{k} + 250F_B^z\,\mathbf{j}\right) - T\mathbf{i} + (3987\mathbf{i} - 41\,600\mathbf{j} - 15\,245\mathbf{k}) = \mathbf{0}$$

whence $T = 4$ kN · mm, $F_B^y = 61$ N, and $F_B^z = 166$ N.

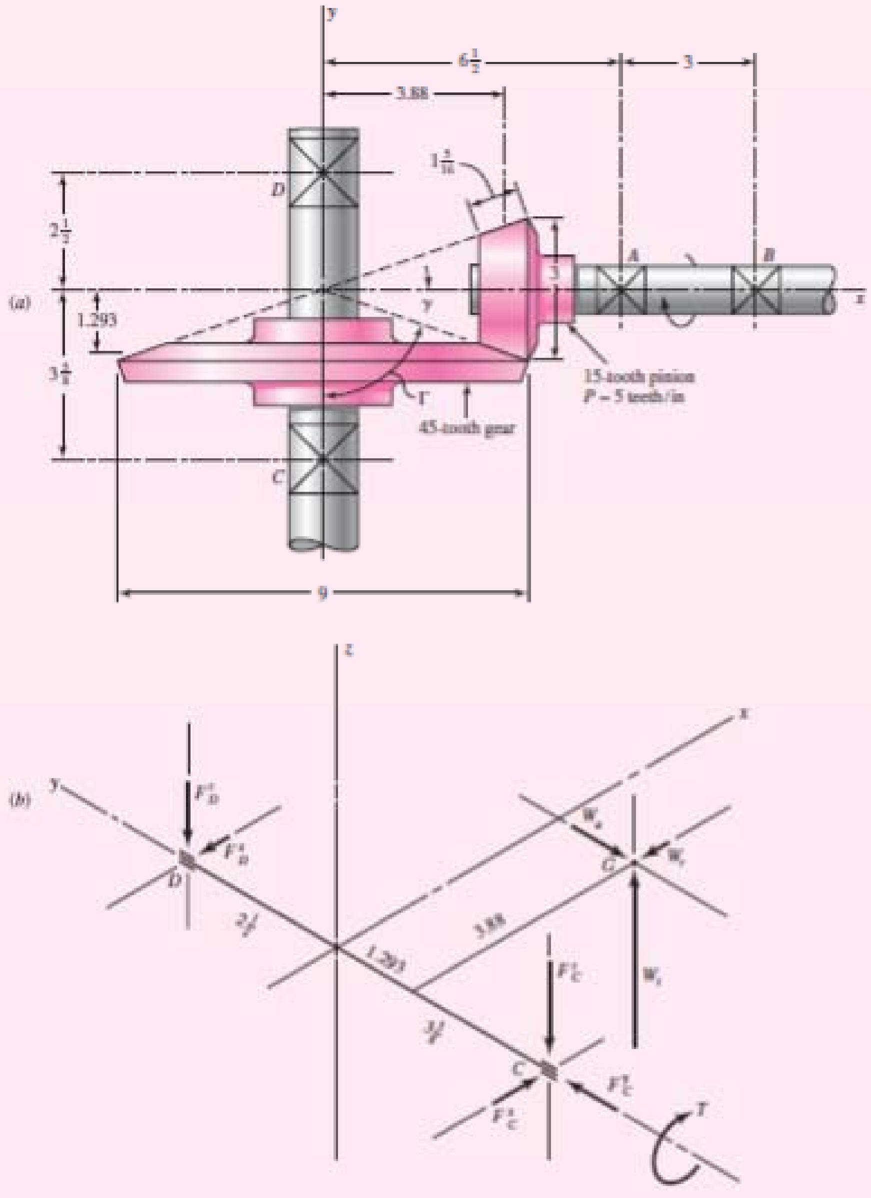
Next,

$$\mathbf{F}_A = -\mathbf{F}_B - \mathbf{W}, \text{ and so } \mathbf{F}_A = 74\mathbf{i} - 7\mathbf{j} + 38\mathbf{k} \text{ N}.$$

EXAMPLE 13–8 The bevel pinion in Fig. 13–36*a* rotates at 600 rev/min in the direction shown and transmits 5 hp to the gear. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by pitch cones. Bearings *A* and *C* should take the thrust loads. Find the bearing forces on the gearshaft.

Figure 13–36

(*a*) Bevel gearset of Ex. 13–8. (*b*) Free-body diagram of shaft *CD*. Dimensions in inches.



Solution The pitch angles are

$$\gamma = \tan^{-1}\left(\frac{3}{9}\right) = 18.4^\circ \qquad \Gamma = \tan^{-1}\left(\frac{9}{3}\right) = 71.6^\circ$$

The pitch-line velocity corresponding to the average pitch radius is

$$V = \frac{2\pi r_P n}{12} = \frac{2\pi(1.293)(600)}{12} = 406 \text{ ft/min}$$

Therefore the transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(5)}{406} = 406 \text{ lbf}$$

which acts in the positive z direction, as shown in Fig. 13–36b. We next have

$$W_r = W_t \tan\phi \cos\Gamma = 406 \tan 20^\circ \cos 71.6^\circ = 46.6 \text{ lbf}$$

$$W_a = W_t \tan\phi \sin\Gamma = 406 \tan 20^\circ \sin 71.6^\circ = 140 \text{ lbf}$$

where W_r is in the $-x$ direction and W_a is in the $-y$ direction, as illustrated in the isometric sketch of Fig. 13–36b.

In preparing to take a sum of the moments about bearing D, define the position vector from D to G as

$$\mathbf{R}_G = 3.88\mathbf{i} - (2.5 + 1.293)\mathbf{j} = 3.88\mathbf{i} - 3.793\mathbf{j}$$

We shall also require a vector from D to C:

$$\mathbf{R}_C = -(2.5 + 3.625)\mathbf{j} = -6.125\mathbf{j}$$

Then, summing moments about D gives

$$\mathbf{R}_G \times \mathbf{W} + \mathbf{R}_C \times \mathbf{F}_C + \mathbf{T} = \mathbf{0} \tag{1}$$

When we place the details in Eq. (1), we get

$$\begin{aligned}(3.88\mathbf{i} - 3.793\mathbf{j}) \times (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k})& \\ + (-6.125\mathbf{j}) \times (F_C^x\mathbf{i} + F_C^y\mathbf{j} + F_C^z\mathbf{k}) + T\mathbf{j} = \mathbf{0}&\end{aligned} \tag{2}$$

After the two cross products are taken, the equation becomes

$$(-1540\mathbf{i} - 1575\mathbf{j} - 720\mathbf{k}) + (-6.125F_C^z\mathbf{i} + 6.125F_C^x\mathbf{k}) + T\mathbf{j} = \mathbf{0}$$

from which

$$\mathbf{T} = 1575\mathbf{j}\ \text{lbf} \cdot \text{in} \qquad F_C^x = 118\ \text{lbf} \qquad F_C^z = -251\ \text{lbf} \tag{3}$$

Now sum the forces to zero. Thus

$$\mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0} \tag{4}$$

When the details are inserted, Eq. (4) becomes

$$(F_D^x\mathbf{i} + F_D^z\mathbf{k}) + (118\mathbf{i} + F_C^y\mathbf{j} - 251\mathbf{k}) + (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k}) = \mathbf{0} \tag{5}$$

First we see that $F_C^y = 140$ lbf, and so

Answer

$$\mathbf{F}_C = 118\mathbf{i} + 140\mathbf{j} - 251\mathbf{k}\ \text{lbf}$$

Then, from Eq. (5),

Answer

$$\mathbf{F}_D = -71.4\mathbf{i} - 155\mathbf{k}\ \text{lbf}$$

These are all shown in Fig. 13–36*b* in the proper directions. The analysis for the pinion shaft is quite similar.