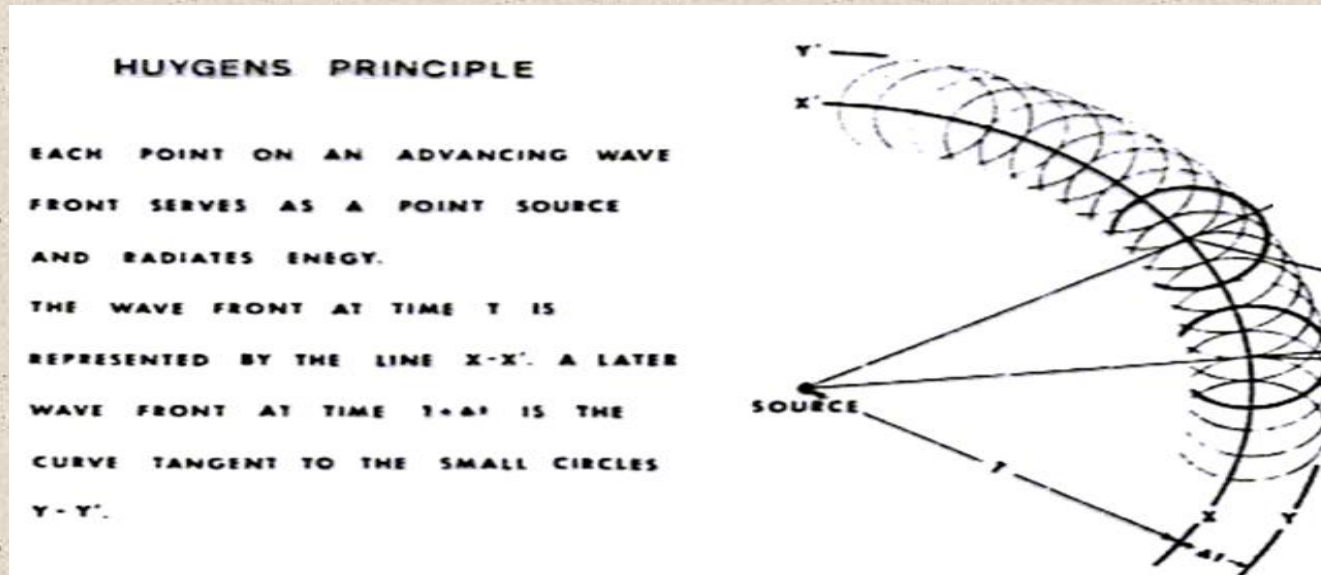


BASIC THEORETICAL PRINCIPLES OF SEISMIC METHODS

I. Huygens Principle:

“Every point on the wave front is a source of a new wave that travels out of it in the form of spherical shells.”

Seismic rays are used instead of the wave front to describe the wave propagation.



☞ Note:

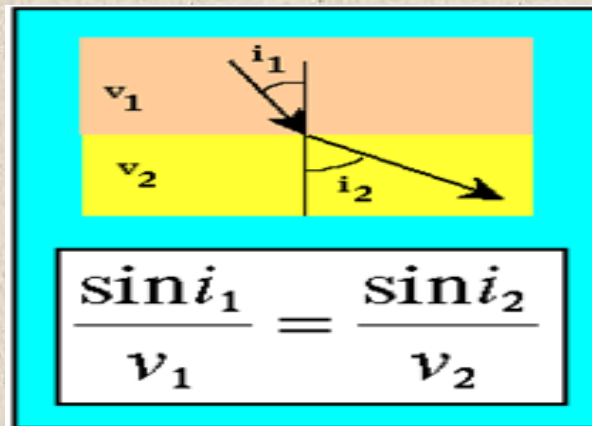
❑ **Raypaths:** *Raypaths are lines that show the direction that the seismic wave is propagating. For any given wave, there are an infinite set of raypaths that could be used.*

❑ **Wavefront:** *Wavefronts connect positions of the seismic wave that are doing the same thing at the same time.*

II. Fermat Principle:

“The wave will travel from the source at a minimum time; *the wave path is not necessary* a straight line.”

III. Snell's Law :

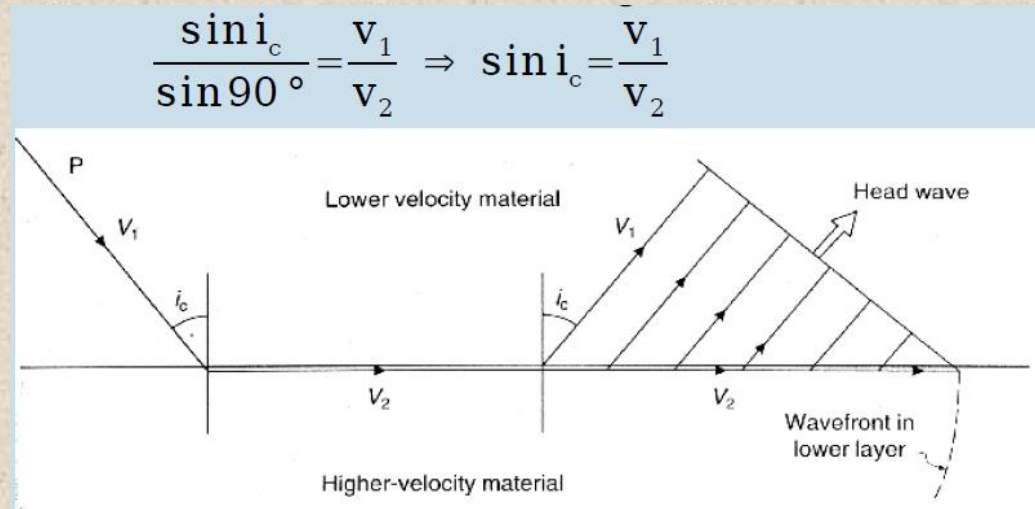


In seismic refraction technique we deal with *Direct* and *refracted* waves. The travel time for the direct waves is calculated simply by dividing the distance by the velocity:

Distance / Velocity

Critical refraction concept:

When the velocity in the upper layer is lower than in the underlying layer, there is a particular angle of incidence, for which the angle of refraction is 90° .



☞ **Critical angle** = the incident angle for which the refraction angle is 90° .

☞ **Note:**

❑ In refraction technique we use the concept of critical angle. In particular, because the wave reflected at the critical angle simply propagates along the refractor about which we would like to obtain information. The waves produced in this way are called *HEAD WAVES*.

❑ Although a Head Wave must travel along a longer path than the direct arrival before it could be recorded at the surface, it travels along the bottom of the layer at a faster speed than the direct arrival. Therefore, Head Waves can be recorded prior to the time of arrival of the direct wave at certain distances.

IV. Law of Reflection:

This law is utilized in the seismic reflection method. It states that “the angle of incidence is equal to the angle of reflection”.

In case of $I=0$, the ratio of the reflected energy of P-wave, E_r , to the incident energy, E_i , is given by:

$$E_r / E_{i|I=0} = \frac{(\rho_2 V_2 - \rho_1 V_1)^2}{(\rho_2 V_2 + \rho_1 V_1)^2}$$

The square root of the above relationship is called **Reflection Coefficient, R** . This coefficient gives the ratio between the amplitudes of the incident (A_i) and reflected waves (A_r). It's given by:

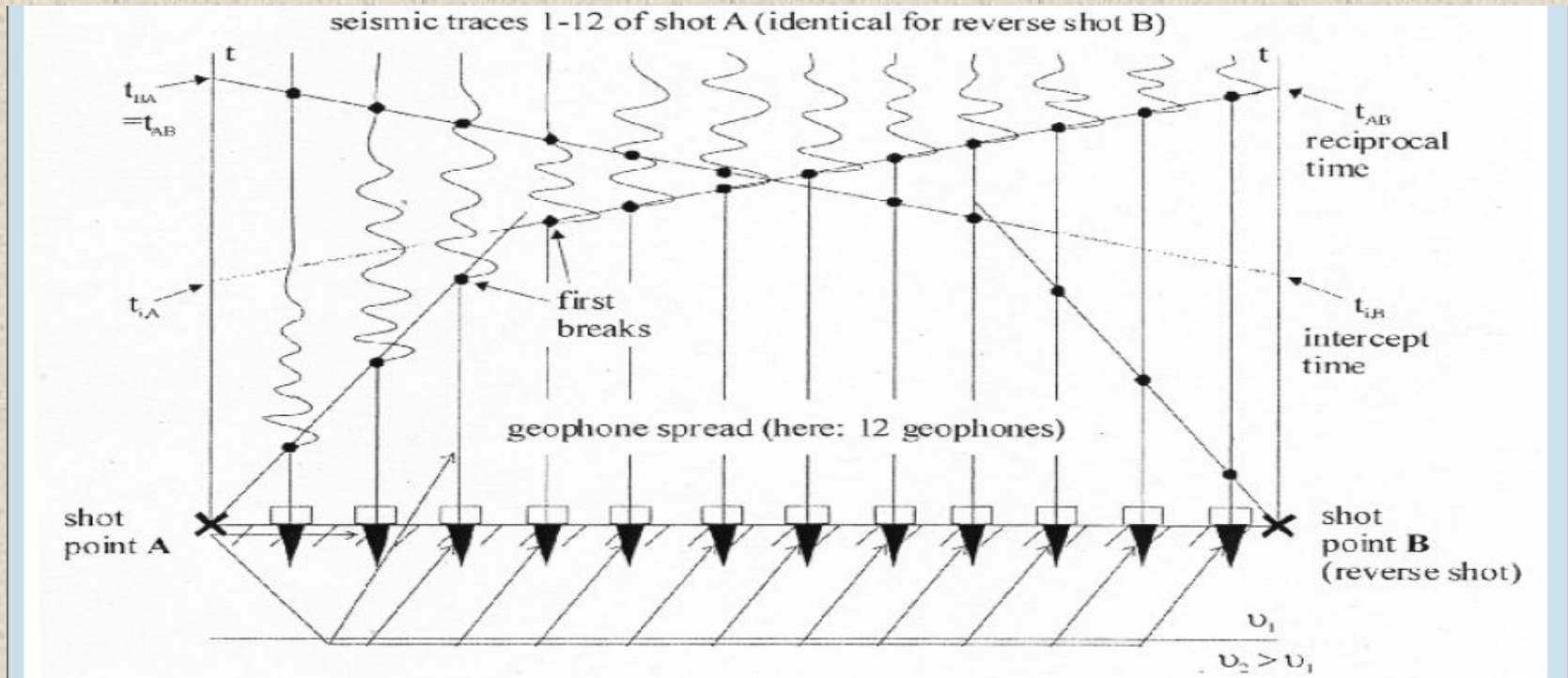
$$R = A_r / A_i = \frac{(\rho_2 V_2 - \rho_1 V_1)}{(\rho_2 V_2 + \rho_1 V_1)}$$

The Reflection Coefficient, R , shows that the quantity of the reflected energy is based on the contrast between the ***acoustic impedance***, defined as *multiplication of velocity by density*, along the opposite side of the reflector surface. In this case, three situations can be recognized:

1. If $\rho_1 \cdot V_1 < \rho_2 \cdot V_2 \rightarrow$ no change in the phase of the reflected wave
2. If $\rho_1 \cdot V_1 > \rho_2 \cdot V_2 \rightarrow$ shift in the phase of the reflected wave with 180°
3. If $\rho_1 \cdot V_1 = \rho_2 \cdot V_2 \rightarrow$ the reflection coefficient is zero: $R = 0$.

☞ **Note:** Since the variation in the density of different types of rocks is relatively small, the reflection coefficient depends mainly on the contrast in velocities at both sides of the reflecting surface.

Seismic Refraction Method



- The first seismic method utilized in the field of exploration.
- It was used in seismology for determining the Mohorovicic discontinuity, and to discover the nuclei of earth.
- The Description of the geometry of refracted waves is more complex than that of reflected ones.

- ❑ The Velocity and thickness of layers are described in terms of TIME. This time is the time required by the refracted wave to travel from the source (at surface) to the receiver (also at the surface), taking in consideration the principle of Fermat.
- ❑ The Distance between the Receiver and the Source must be very much larger than the depth of the investigated discontinuity.
- ❑ Because of this large distance, the frequencies of interest in the refraction is lower than those in reflection.

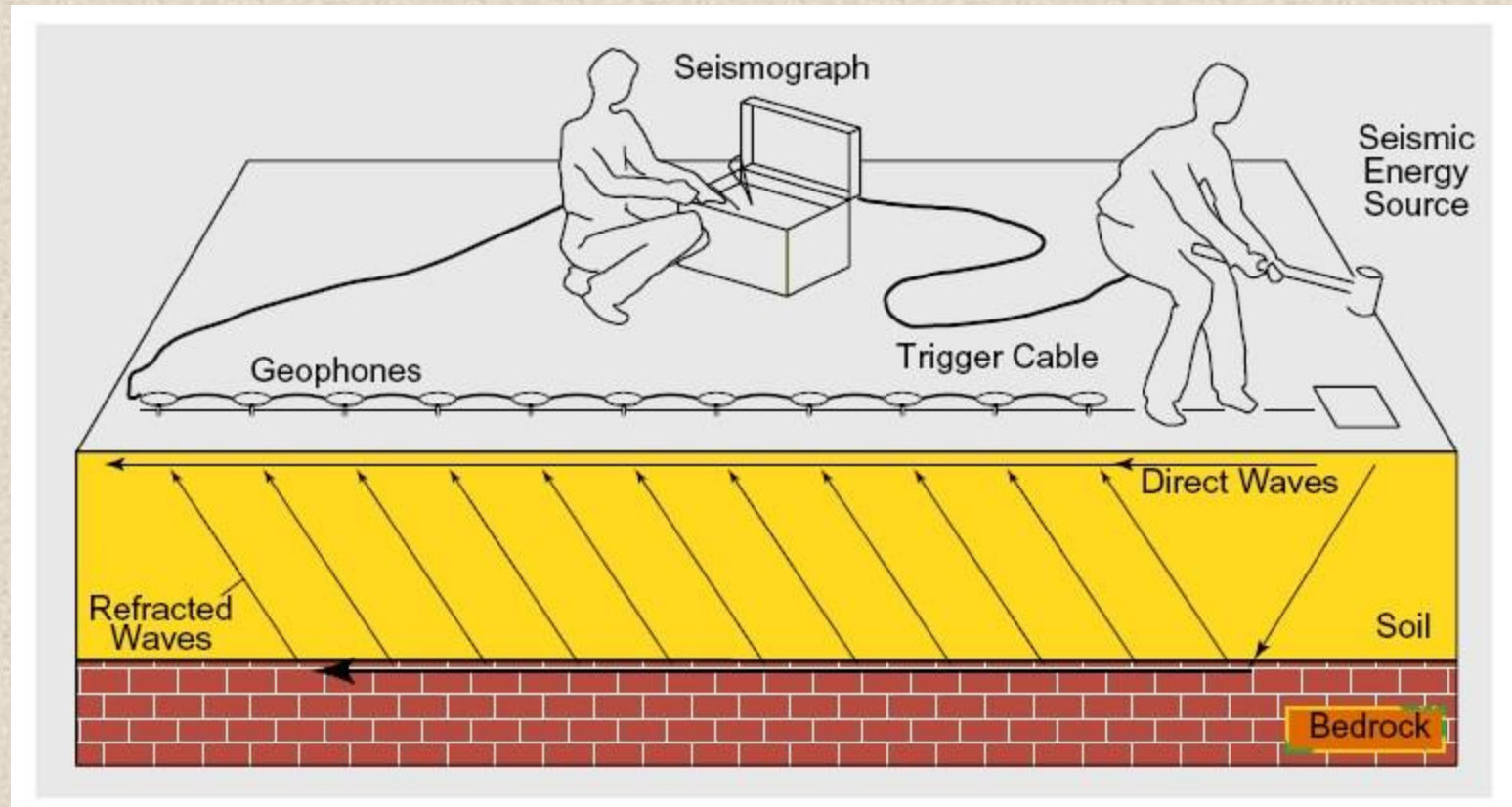
Fields of Applications of Seismic Refraction Method

- ☐ Determining lateral extensions of layers.
- ☐ Mapping of sedimentary basins.
- ☐ Determining the physical properties of the bed rock.
- ☐ Detecting buried structures of small dimensions.
- ☐ Detecting salt domes.

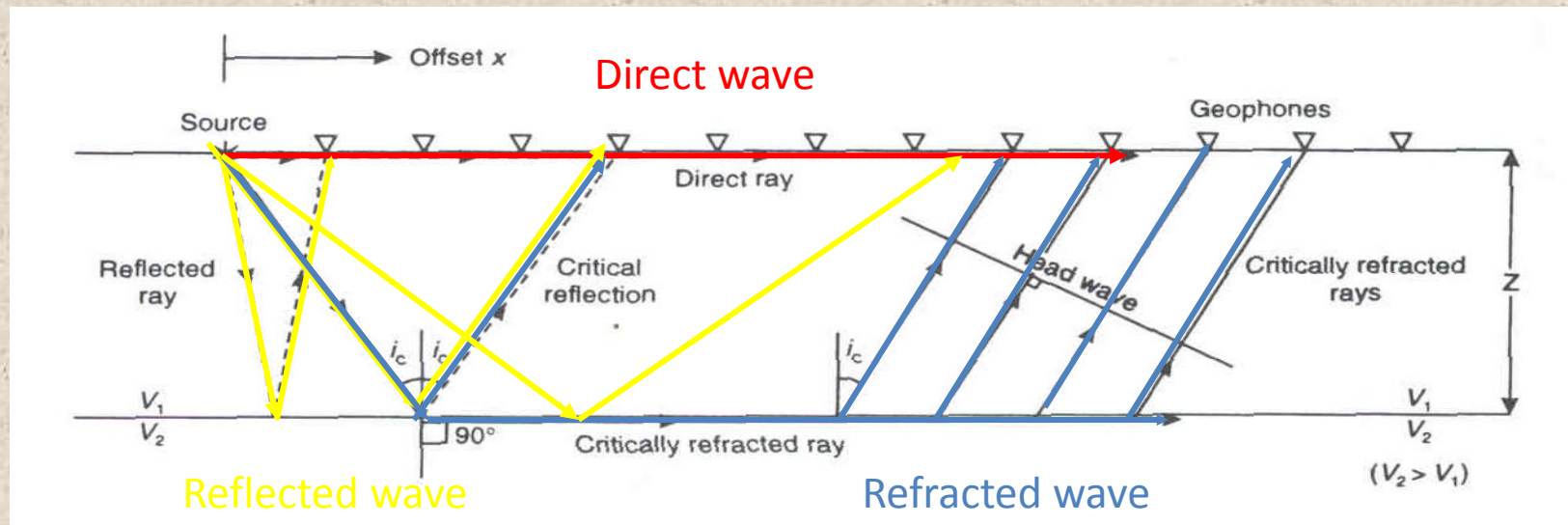
Ideal Conditions of Application

- ☐ Coincidence between seismic interfaces and stratigraphical or lithological ones.
- ☐ Extended interfaces, homogeneous with small dip angles (less than 15° - 20°).
- ☐ Small thickness of layers that are characterized by low velocity.
- ☐ Not complicated topography of the investigated area.

Seismic Refraction



❑ Refraction surveys use the process of critical refraction to determine interface depths and layer velocities. Critical refraction requires an increase in velocity with depth. If not, then there is no critical refraction; Hidden layer problem will be faced.



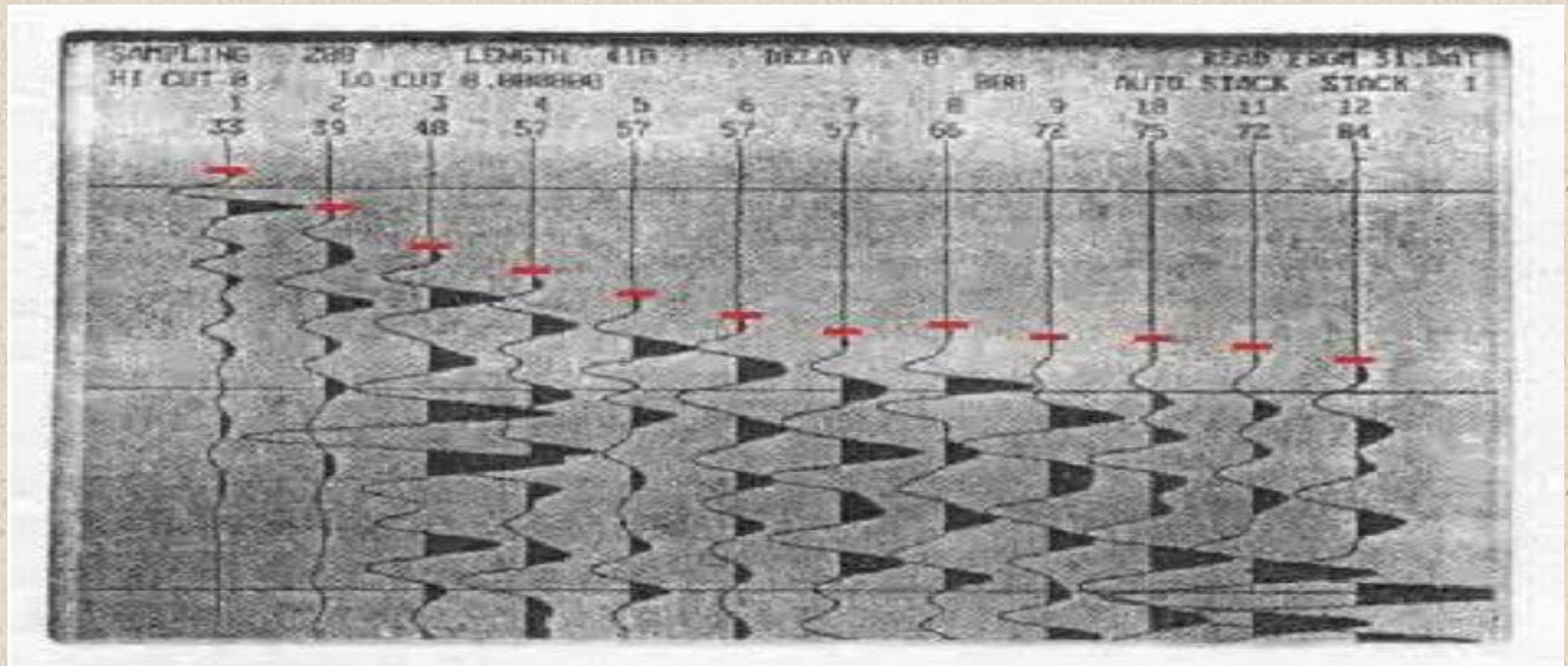
- ❑ Geophones laid out in a line to record arrivals from a shot. Recording at each geophone is a waveform called a **seismogram**.
- ❑ Direct signal from shot travels along top of first layer.
- ❑ Critical refraction is also recorded at distance beyond which angle of incidence becomes critical.

First Arrival Picking

In most refraction analysis, we only use the travel times of the first arrival on each recorded seismogram. As velocity increases at an interface, critical refraction will become first arrival at some source-receiver offset.

First Break Picking

The beginning of the first seismic wave, the first break, on each seismogram is identified and its arrival time is picked. An example of first break picking process is shown in the figure below:



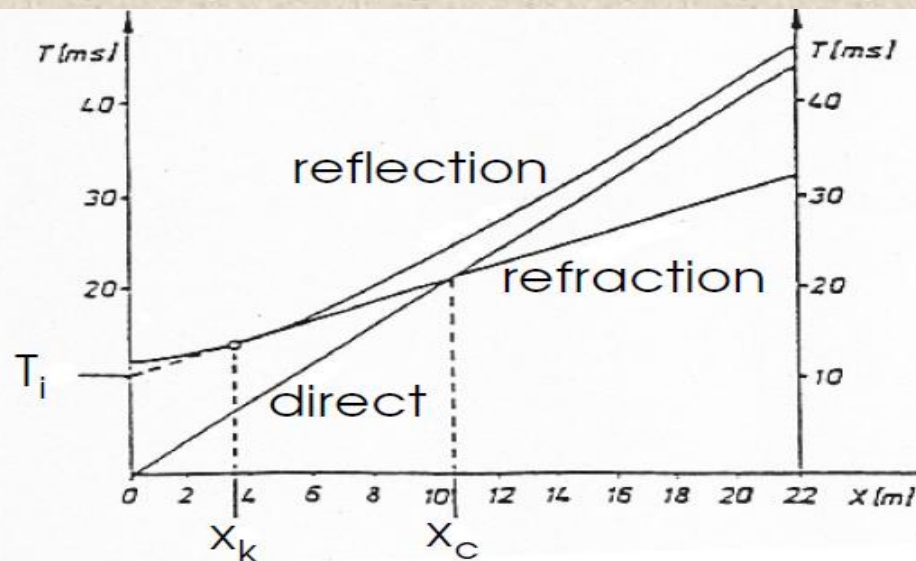
Travel Time Curves

Analysis of seismic refraction data is primarily based on interpretation of critical refraction travel times. Usually we analyze P wave refraction data, but S wave data occasionally recorded.

Plots of seismic arrival times vs. source-receiver offset are called *travel time Curves*.

In the figure below, travel time curves for three arrivals can be noted:

- Direct arrival from source to receiver in top layer
- Critical refraction along top of second layer
- Reflection from top of second layer



Traveltime curves for reflected, refracted and direct waves.

Refraction for $x > x_k$. At a distance x_k called critical distance the reflected arrival is coincident with the first critically refracted arrival and the travel times of the two are identical.

Critical Distance is:

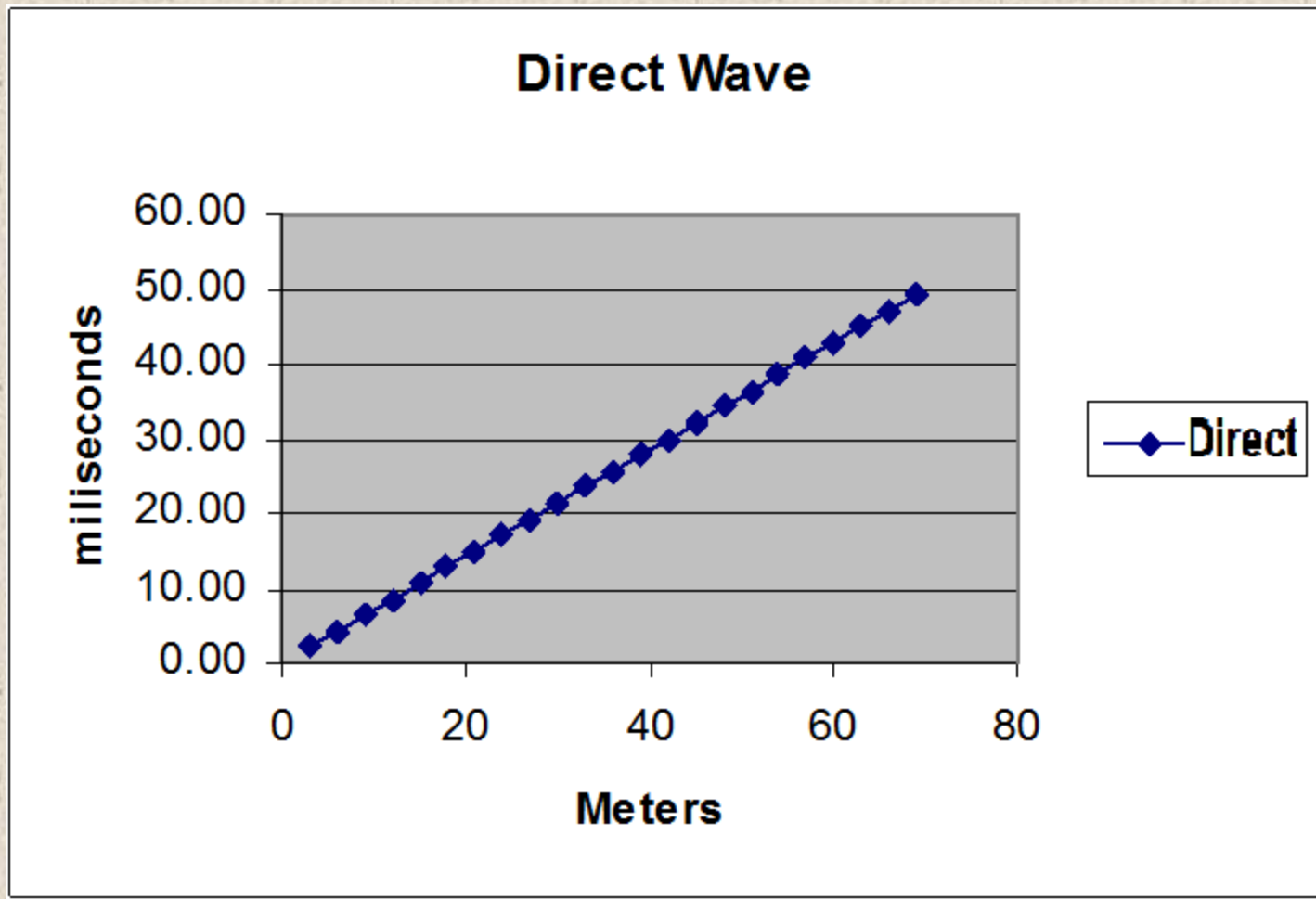
the offset at which critical refraction first appears. In this case:

- Critical refraction has same travel time as reflection
- Angle of reflection same as critical angle

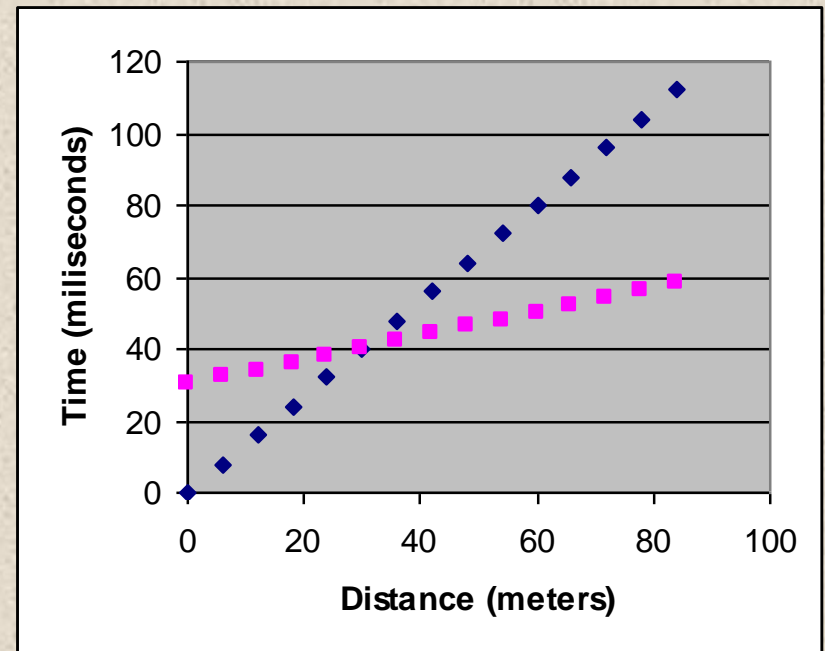
Crossover Distance is

The offset at which critical refraction becomes first arrival.

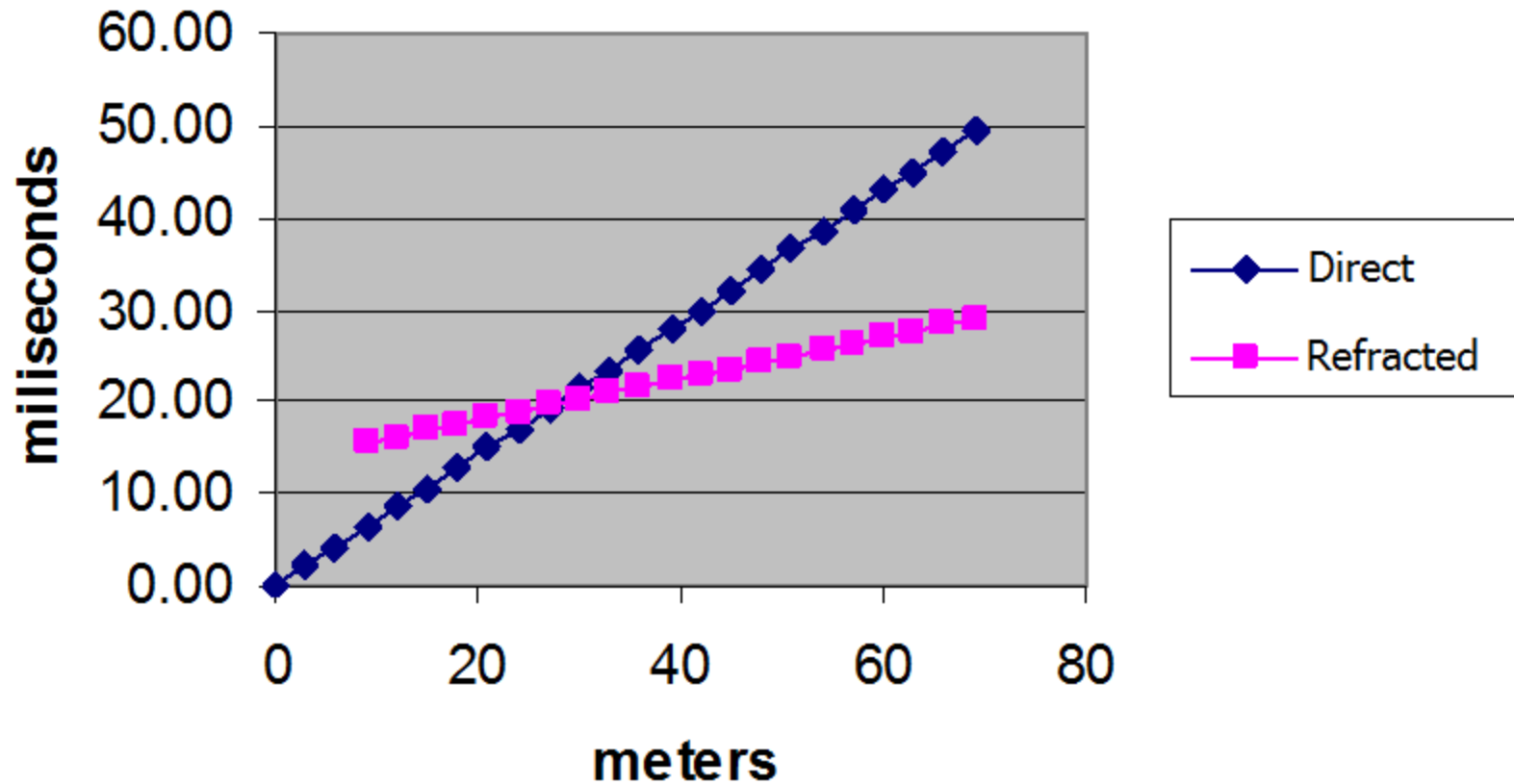
Plotting Travel Time Curves



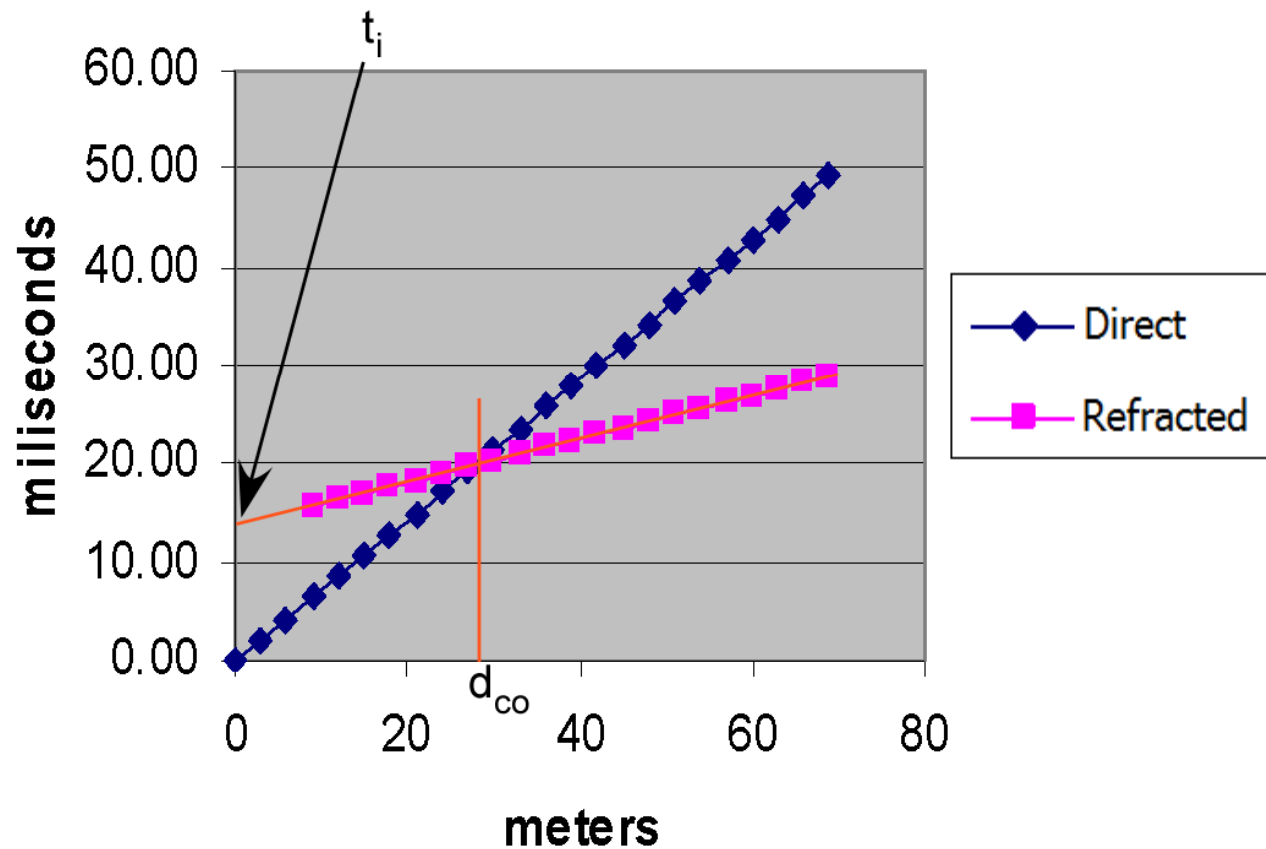
Distance	TimeD	Time R
0	0	30
6	8	32
12	16	34
18	24	36
24	32	38
30	40	40
36	48	42
42	56	44
48	64	46
54	72	48
60	80	50
66	88	52
72	96	54
78	104	56
84	112	58



Direct and Refracted Waves



Direct and Refracted Waves



Interpretation of Refraction Travel Time Curves

Interpretation objective is to determine interface depths and layer velocities. Data interpretation requires making **assumptions** about layering in subsurface: shape and number of different first arrivals.

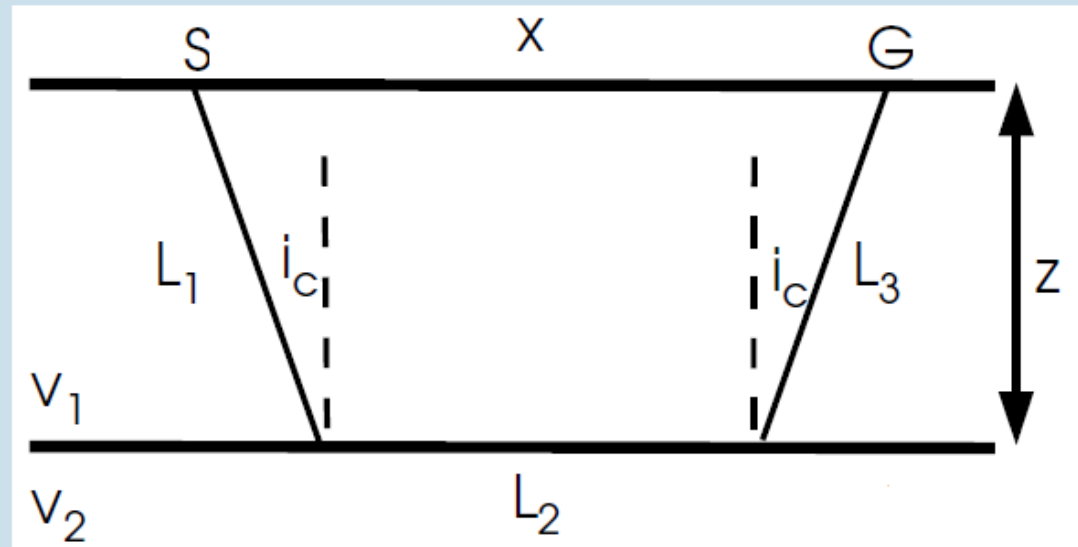
The Assumptions are:

- Subsurface is composed of stack of layers, usually separated by plane interfaces
- Seismic velocity is uniform in each layer
- Layer velocities increase in depth
- All ray paths are located in vertical plane, i.e. no 3-D effects with layers dipping out of plane of profile.

Two layer case

Travel way of
the refracted wave

$$L_R = L_1 + L_2 + L_3$$



Travel time of the refracted wave

$$T_R = \frac{L_1}{v_1} + \frac{L_2}{v_2} + \frac{L_3}{v_1}$$

$$\Rightarrow T_R = \frac{x}{v_2} + \frac{2z}{v_1} \frac{\sqrt{v_2^2 - v_1^2}}{v_2}$$

Travel time of the direct wave

$$T_D = \frac{L_D}{v_1}$$

From travel times of direct arrival and critical refraction, we can find *velocities of two layers* and *depth to interface* :

1. Velocity of layer 1 given by slope of direct arrival
2. Velocity of layer 2 given by slope of critical refraction
3. Depth of the refractor

Derivation of the travel time equation for the case of two horizontal layers

Starting with: $T_R = \frac{L_1}{v_1} + \frac{L_2}{v_2} + \frac{L_3}{v_1}$

$$T_R = Z / V_1 \cos i_c + (X - 2Z \tan i_c) / V_2 + Z / V_1 \cos i_c$$

$$T_R = X/V_2 + 2Z / V_1 \cos i_c - 2Z \tan i_c / V_2$$

Substituting for V_2 and for $\tan i_c$ according to relations (4) and (5) above, we obtain:

$$T_R = X/V_2 + 2Z / V_1 \cos i_c - 2Z \sin^2 i_c / V_1 \cos i_c$$

$$T_R = X/V_2 + 2Z (1 - \sin^2 i_c) / V_1 \cos i_c$$

Using relation (6)

$$T_R = X/V_2 + 2Z \cos i_c / V_1,$$

Or:

$$T_R = X/V_2 + 2Z \sqrt{(1 - (V_1/V_2)^2)} / V_1$$

Finally, we obtain:

$$T_R = X/V_2 + 2Z \sqrt{V_2^2 - V_1^2} / V_1 V_2$$

The following relations need to be taken in consideration :

$$L_1 = L_3 \dots\dots\dots(1)$$

$$\cos i_c = Z/L_1 \rightarrow L_1 = Z / \cos i_c \dots\dots\dots(2)$$

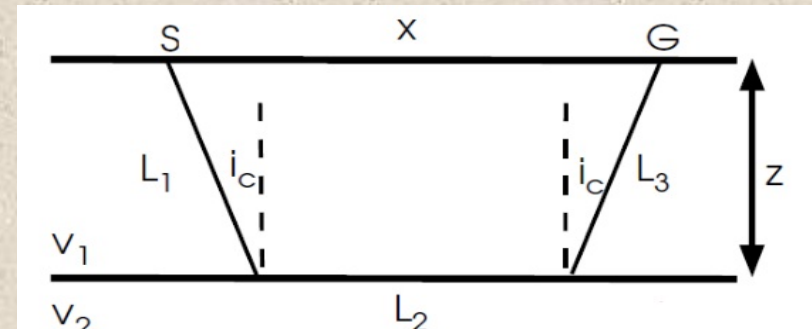
$$L_2 = X - 2(Z \tan i_c) \dots\dots\dots(3)$$

$$\sin i_c = V_1/V_2 \rightarrow V_2 = V_1 / \sin i_c \dots\dots\dots(4)$$

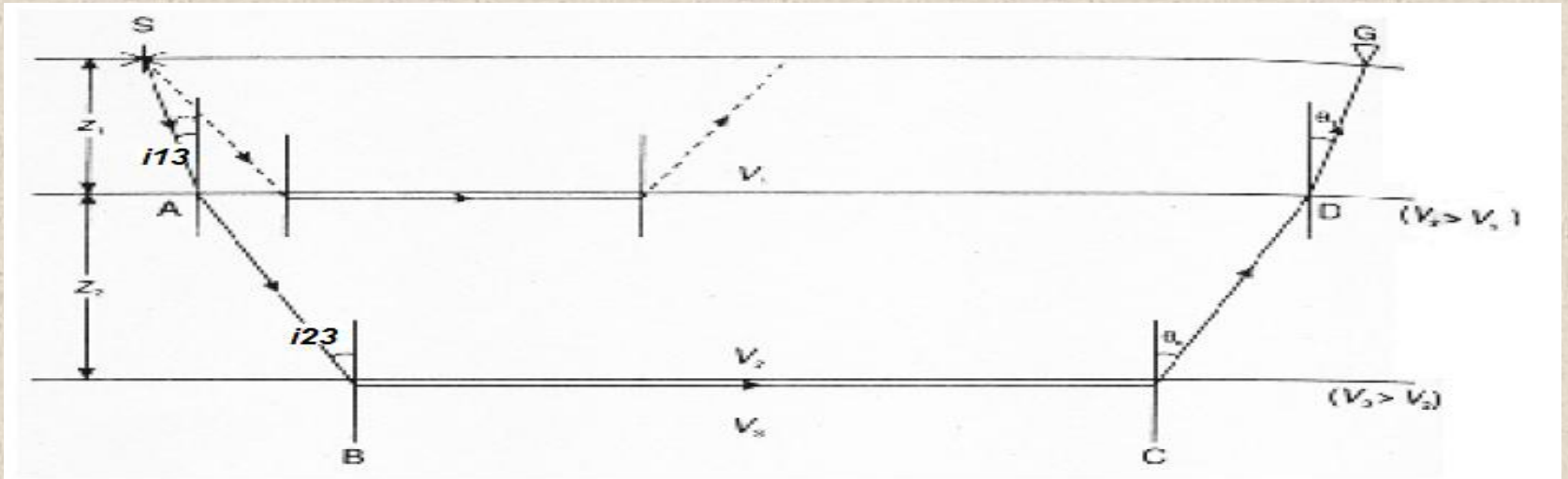
$$\tan i_c = \sin i_c / \cos i_c \dots\dots\dots(5)$$

$$\sin^2 i_c + \cos^2 i_c = 1 \rightarrow \sin^2 i_c = 1 - \cos^2 i_c \dots\dots(6)$$

$$\rightarrow \cos i_c = \sqrt{1 - \sin^2 i_c} = \sqrt{V_2^2 - V_1^2} / V_2$$



Three Horizontal Layers Case



Based on the figure above, the refracted travel time can be written as:

$$T_R = 2SA/V_1 + 2AB/V_2 + BC/V_3$$

$$T_R = 2Z_1 / V_1 \cos i_{13} + 2Z_2 / V_2 \cos i_{23} + (X - 2Z_1 \tan i_{13} - 2Z_2 \tan i_{23}) / V_3$$

$$T_R = X/V_3 + (2Z_2/V_2)^* (1/\cos i_{23} - V_2 \tan i_{23}/V_3) + (2Z_1/V_1)^* (1/\cos i_{13} - V_1 \tan i_{13}/V_3)$$

Noting that: $V_2/V_3 = \sin i_{23}$, and $V_1/V_3 = \sin i_{13}$, we obtain:

$$T_R = X/V_3 + 2Z_2 \cos i_{23}/V_2 + 2Z_1 \cos i_{13}/V_1 \quad \text{Or:}$$

$$T_R = X/V_3 + 2Z_2\sqrt{V_3^2 - V_2^2}/V_3^*V_2 + 2Z_1\sqrt{V_3^2 - V_1^2}/V_3^*V_1$$

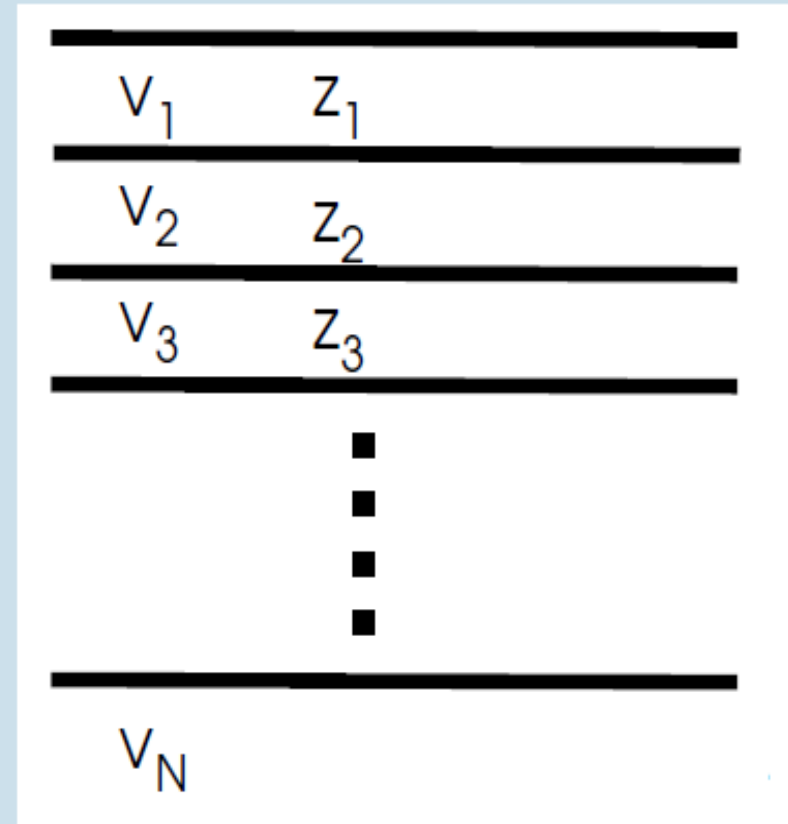
Multilayer case

N layers

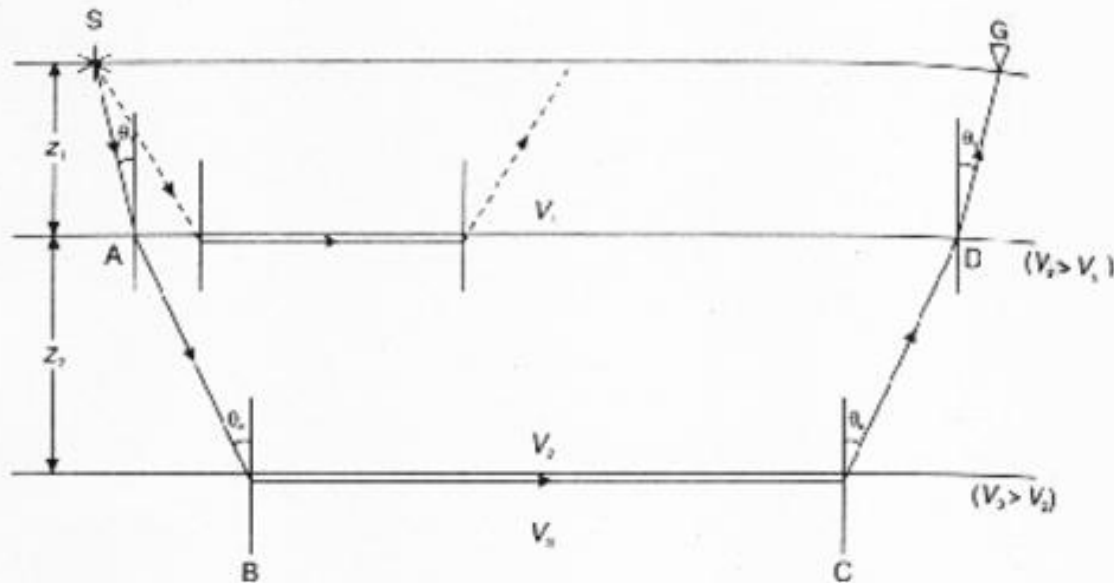
$$v_1 < v_2 < v_3 < \dots < v_N$$

Travel time

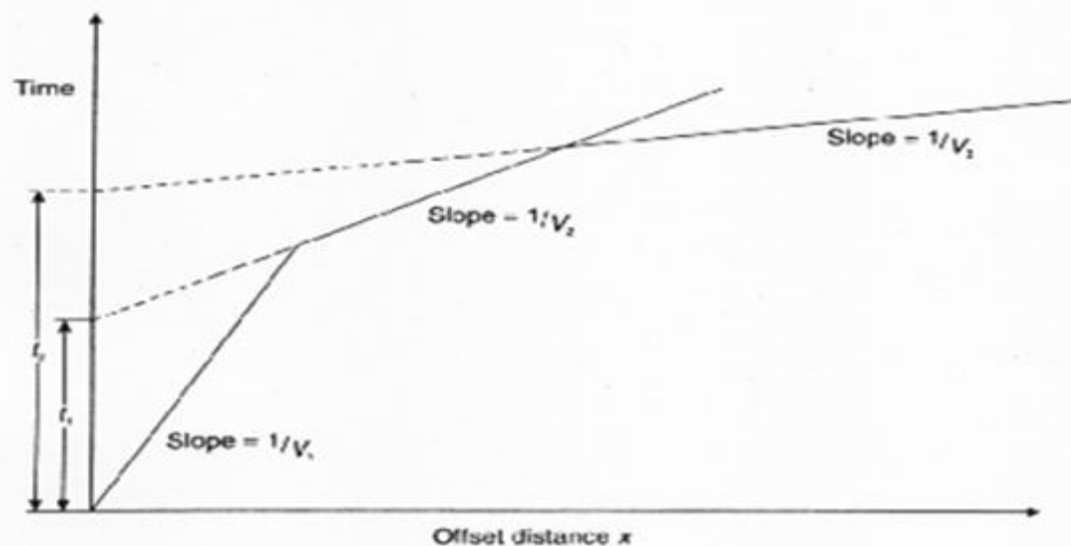
$$T_N = \frac{x}{v_N} + \frac{2}{v_N} \sum_{k=1}^{N-1} (z_k - z_{k-1}) \sqrt{(v_N/v_k)^2 - 1}$$



(A)



(B)



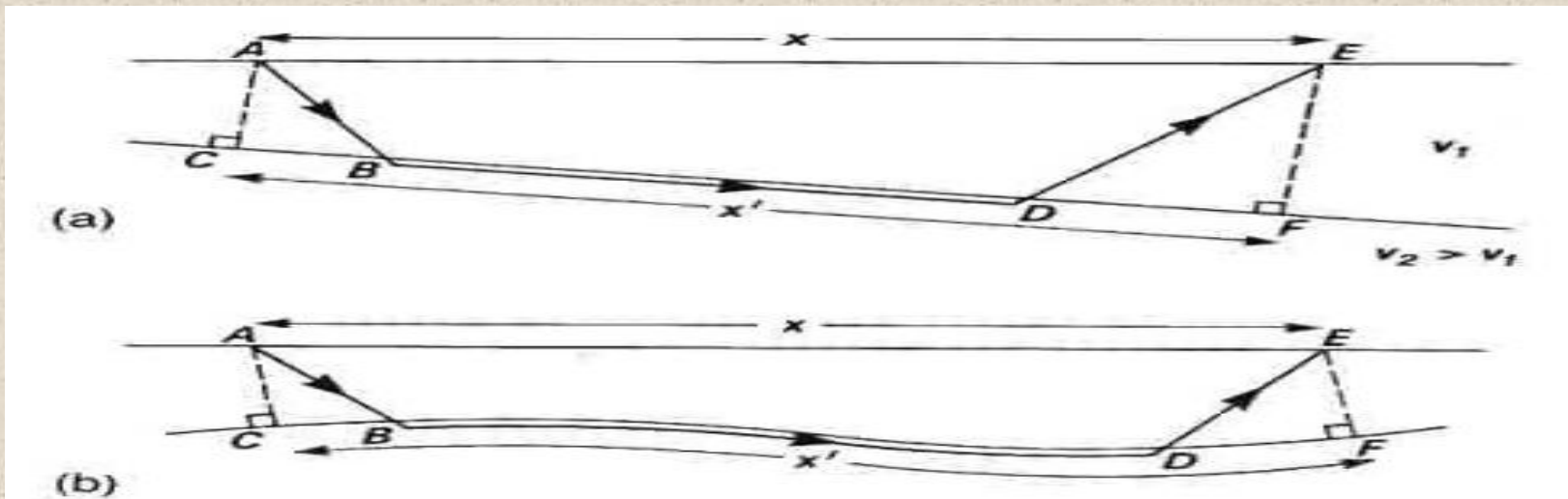
(A) Simple raypaths diagram for refracted rays, and (B) their respective travel time-distance graphs for a three-layer case with horizontal planar interfaces.

Delay Time Concept

For irregular travel time curves, e.g. due to bedrock topography or glacial fill, much analysis is based on delay times.

Total Delay Time is defined as *the difference in travel time along actual ray path and projection of ray path along refracting interface*:

$$\delta = T_{AB} - T_{CF}$$



$$T_{CF} = \frac{CF}{V_2}$$

Total delay time is the delay time at shot plus delay time at geophone:

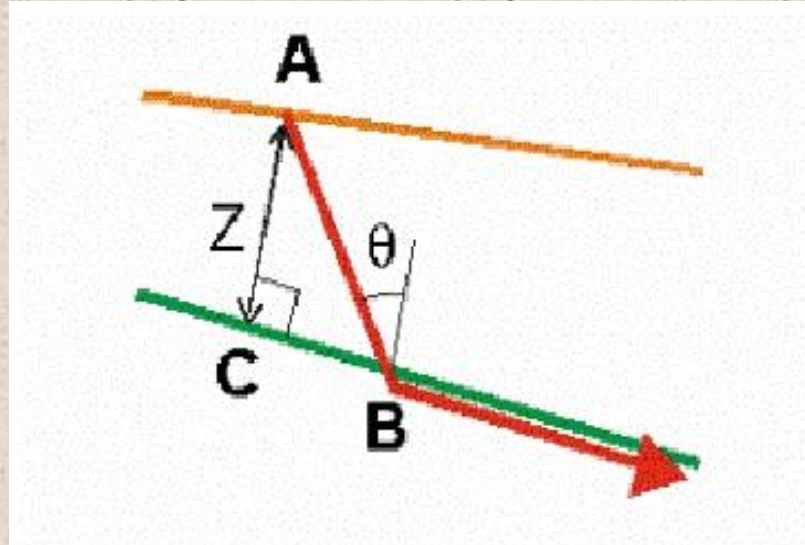
$$\Delta t = \left(\frac{AB}{V_1} - \frac{CB}{V_2} \right) + \left(\frac{DE}{V_1} - \frac{DF}{V_2} \right) = \Delta t_s + \Delta t_g \approx T_{AB} - \frac{x}{V_2}$$

For small dips, can assume $x=x'$, and:

$$\Delta t = T_{AB} - \frac{x'}{V_2}$$

Calculation of Refractor Depth from Delay Time

If velocities of both layers are known, then refractor depth at point A can be calculated from delay time at point A:



$$\delta_A = \frac{AB}{V_1} - \frac{BC}{V_2}$$

Using the triangle ABC to get lengths in terms of Z:

$$\mathcal{A} = \frac{Z}{V_1 \cos \theta} - \frac{Z \tan \theta}{V_2}$$

$$= \frac{Z}{V_1 \cos \theta} \left(1 - \frac{V_1 \sin \theta}{V_2} \right)$$

Using Snell's law to express angles in terms of velocities:

$$\mathcal{A} = \frac{Z}{V_1 \left(1 - \frac{V_1^2}{V_2^2} \right)^{\frac{1}{2}}} \left(1 - \frac{V_1^2}{V_2^2} \right)$$

Simplifying:

$$\mathcal{A} = \frac{Z (V_2^2 - V_1^2)^{\frac{1}{2}}}{V_1 V_2}$$

So, refractor depth at A is:

$$Z = \frac{\mathcal{A} V_1 V_2}{(V_2^2 - V_1^2)^{\frac{1}{2}}}$$